

B-L extended Standard Model with inverse Seesaw
Lagrangian, Rotations and Interactions for eigenstates 'EWSB'
including Renormalization Group Equations
including one-loop Self-Energies

SARAH 4.6.0

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References: [arXiv: 1309.7223](#) , [Comput.Phys.Commun.184:1792-1809,2011 \(1207.0906\)](#) , [Comput.Phys.Commun.182:833,2011 \(1002.0840\)](#) , [Comput.Phys.Commun.181:1077-1086,2010 \(0909.2863\)](#) , [arXiv: 0806.0538](#)

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1 Fields

1.1 Gauge Fields

Name	$SU(N)$	Coupling	Name
B	$U(1)$	g_1	hypercharge
W	$SU(2)$	g_2	left
g	$SU(3)$	g_3	color
VBp	$U(1)$	g_B	BminusL

1.2 Matter Superfields

Name	Spin	Generations	$(U(1) \otimes SU(2) \otimes SU(3) \otimes U(1))$
H	0	1	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1}, 0)$
χ	0	1	$(0, \mathbf{1}, \mathbf{1}, \frac{1}{2})$
q	$\frac{1}{2}$	3	$(\frac{1}{6}, \mathbf{2}, \mathbf{3}, \frac{1}{6})$
l	$\frac{1}{2}$	3	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1}, -\frac{1}{2})$
d	$\frac{1}{2}$	3	$(\frac{1}{3}, \mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{6})$
u	$\frac{1}{2}$	3	$(-\frac{2}{3}, \mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{6})$
e	$\frac{1}{2}$	3	$(1, \mathbf{1}, \mathbf{1}, \frac{1}{2})$
vR	$\frac{1}{2}$	3	$(0, \mathbf{1}, \mathbf{1}, \frac{1}{2})$
s1	$\frac{1}{2}$	3	$(0, \mathbf{1}, \mathbf{1}, 1)$
s2	$\frac{1}{2}$	3	$(0, \mathbf{1}, \mathbf{1}, -1)$

2 Lagrangian

2.1 Input Lagrangian for Eigenstates GaugeES

$$L = 0 \tag{1}$$

2.2 Gauge fixing terms

2.2.1 Gauge fixing terms for eigenstates 'GaugeES'

$$L_{GF} = -\frac{1}{2}|\partial_\mu B|^2 \xi_B^{-1} - \frac{1}{2}|\partial_\mu g|^2 \xi_g^{-1} - \frac{1}{2}|\partial_\mu \text{VBp}|^2 \xi_{\text{VBp}}^{-1} - \frac{1}{2}|\partial_\mu W|^2 \xi_W^{-1} \tag{2}$$

2.2.2 Gauge fixing terms for eigenstates 'EWSB'

$$L_{GF} = -\frac{1}{2}|\partial_\mu g|^2 \xi_g^{-1} - \frac{1}{2}|\partial_\mu \gamma|^2 \xi_\gamma^{-1} - \frac{i}{2}g_2 H^- v \xi_{W^-} + \partial_\mu W^-|^2 \xi_{W^-}^{-1}$$

$$\begin{aligned}
& -\frac{1}{2}\frac{1}{2}\left(2\partial_\mu Z + \xi_Z\left(\left(g_1\text{sigmaH}v - g_{YB}\text{sigmaX}x\right)\cos\Theta'_W\sin\Theta_W + g_2\text{sigmaH}v\cos\Theta_W\cos\Theta'_W + \left(g_B\text{sigmaX}x - g_{BY}\text{sigma}\right.\right.\right. \\
& \left.\left.\left.-\frac{1}{2}\frac{1}{2}\left(2\partial_\mu Z' - \xi_{Z'}\left(\left(g_1\text{sigmaH}v\sin\Theta_W + g_2\text{sigmaH}v\cos\Theta_W - g_{YB}\text{sigmaX}x\sin\Theta_W\right)\sin\Theta'_W + \left(-g_B\text{sigmaX}x + g_{BY}\text{sigma}\right.\right.\right.\right. \\
& \left.\left.\left.\left.\right.\right.\right) \quad (3)
\end{aligned}$$

2.3 Fields integrated out

None

3 Renormalization Group Equations

3.1 Gauge Couplings

$$\begin{aligned}
\beta_{g_1}^{(1)} &= \frac{1}{40}\left(164g_1^3 + 64\sqrt{10}g_1^2g_{YB} + g_1\left(164g_{BY}^2 + 32\sqrt{10}g_{BY}g_B + 405g_{YB}^2\right) + g_{BY}\left(32\sqrt{10}g_{BY} + 405g_B\right)g_{YB}\right) \quad (4) \\
\beta_{g_1}^{(2)} &= \frac{1}{400}\left(1592g_1^5 + 3184g_1^3g_{BY}^2 + 1592g_1g_{BY}^4 + 984\sqrt{10}g_1^3g_{BY}g_B + 984\sqrt{10}g_1g_{BY}^3g_B + 920g_1^3g_B^2\right. \\
&+ 2760g_1g_{BY}^2g_B^2 + 280\sqrt{10}g_1g_{BY}g_B^3 + 1312\sqrt{10}g_1^4g_{YB} + 1640\sqrt{10}g_1^2g_{BY}^2g_{YB} \\
&+ 328\sqrt{10}g_{BY}^4g_{YB} + 6440g_1^2g_{BY}g_Bg_{YB} + 2760g_{BY}^3g_Bg_{YB} + 560\sqrt{10}g_1^2g_B^2g_{YB} \\
&+ 840\sqrt{10}g_{BY}^2g_B^2g_{YB} + 12425g_{BY}g_B^3g_{YB} + 5520g_1^3g_{YB}^2 + 3680g_1g_{BY}^2g_{YB}^2 \\
&+ 1400\sqrt{10}g_1g_{BY}g_Bg_{YB}^2 + 12425g_1g_B^2g_{YB}^2 + 1120\sqrt{10}g_1^2g_{YB}^3 + 280\sqrt{10}g_{BY}^2g_{YB}^3 \\
&+ 12425g_{BY}g_Bg_{YB}^3 + 12425g_1g_{YB}^4 + 1080g_1^3g_2^2 + 1080g_1g_{BY}^2g_2^2 + 360\sqrt{10}g_1g_{BY}g_Bg_2^2 \\
&+ 720\sqrt{10}g_1^2g_{YB}g_2^2 + 360\sqrt{10}g_{BY}^2g_{YB}g_2^2 + 1800g_{BY}g_Bg_{YB}g_2^2 + 1800g_1g_{YB}^2g_2^2 \\
&+ 3520g_1^3g_3^2 + 3520g_1g_{BY}^2g_3^2 + 320\sqrt{10}g_1g_{BY}g_Bg_3^2 + 640\sqrt{10}g_1^2g_{YB}g_3^2 \\
&+ 320\sqrt{10}g_{BY}^2g_{YB}g_3^2 + 1600g_{BY}g_Bg_{YB}g_3^2 + 1600g_1g_{YB}^2g_3^2 \\
&- 20\left(10g_1^3 - 2\sqrt{10}g_1^2g_{YB} + g_1\left(10g_{BY}^2 + 10g_{YB}^2 - \sqrt{10}g_{BY}g_B\right) + g_{BY}\left(10g_B - \sqrt{10}g_{BY}\right)g_{YB}\right)\text{Tr}\left(Y_dY_d^\dagger\right) \\
&- 60\left(10g_1^3 + 6\sqrt{10}g_1^2g_{YB} + g_1\left(10g_{BY}^2 + 10g_{YB}^2 + 3\sqrt{10}g_{BY}g_B\right) + g_{BY}\left(10g_B + 3\sqrt{10}g_{BY}\right)g_{YB}\right)\text{Tr}\left(Y_eY_e^\dagger\right) \\
&- 750g_{BY}g_Bg_{YB}\text{Tr}\left(Y_SY_S^\dagger\right) - 750g_1g_{YB}^2\text{Tr}\left(Y_SY_S^\dagger\right) - 680g_1^3\text{Tr}\left(Y_uY_u^\dagger\right) - 680g_1g_{BY}^2\text{Tr}\left(Y_uY_u^\dagger\right) \\
&- 100\sqrt{10}g_1g_{BY}g_B\text{Tr}\left(Y_uY_u^\dagger\right) - 200\sqrt{10}g_1^2g_{YB}\text{Tr}\left(Y_uY_u^\dagger\right) \\
&- 100\sqrt{10}g_{BY}^2g_{YB}\text{Tr}\left(Y_uY_u^\dagger\right) - 200g_{BY}g_Bg_{YB}\text{Tr}\left(Y_uY_u^\dagger\right) - 200g_1g_{YB}^2\text{Tr}\left(Y_uY_u^\dagger\right) \\
&- 120g_1^3\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 120g_1g_{BY}^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 60\sqrt{10}g_1g_{BY}g_B\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \\
&- 120\sqrt{10}g_1^2g_{YB}\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 60\sqrt{10}g_{BY}^2g_{YB}\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 600g_{BY}g_Bg_{YB}\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \\
&- 600g_1g_{YB}^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \quad (5) \\
\beta_{g_{BY}}^{(1)} &= \frac{1}{40}\left(4g_1^2\left(41g_{BY} + 8\sqrt{10}g_B\right) + g_1\left(32\sqrt{10}g_{BY} + 405g_B\right)g_{YB} + g_{BY}\left(164g_{BY}^2 + 405g_B^2 + 64\sqrt{10}g_{BY}g_B\right)\right) \quad (6)
\end{aligned}$$

$$\begin{aligned}
\beta_{g_{BY}}^{(2)} = & \frac{1}{400} \left(1592g_1^4g_{BY} + 3184g_1^2g_{BY}^3 + 1592g_{BY}^5 + 328\sqrt{10}g_1^4g_B + 1640\sqrt{10}g_1^2g_{BY}^2g_B \right. \\
& + 1312\sqrt{10}g_{BY}^4g_B + 3680g_1^2g_{BY}g_B^2 + 5520g_{BY}^3g_B^2 + 280\sqrt{10}g_1^2g_B^3 \\
& + 1120\sqrt{10}g_{BY}^2g_B^3 + 12425g_{BY}g_B^4 + 984\sqrt{10}g_1^3g_{BY}g_{YB} + 984\sqrt{10}g_1g_{BY}^3g_{YB} \\
& + 2760g_1^3g_Bg_{YB} + 6440g_1g_{BY}^2g_Bg_{YB} + 1400\sqrt{10}g_1g_{BY}g_B^2g_{YB} + 12425g_1g_B^3g_{YB} + 2760g_1^2g_{BY}g_{YB}^2 \\
& + 920g_{BY}^3g_{YB}^2 + 840\sqrt{10}g_1^2g_Bg_{YB}^2 + 560\sqrt{10}g_{BY}^2g_Bg_{YB}^2 + 12425g_{BY}g_B^2g_{YB}^2 \\
& + 280\sqrt{10}g_1g_{BY}g_{YB}^3 + 12425g_1g_Bg_{YB}^3 + 1080g_1^2g_{BY}g_2^2 + 1080g_{BY}^3g_2^2 + 360\sqrt{10}g_1^2g_Bg_2^2 \\
& + 720\sqrt{10}g_{BY}^2g_Bg_2^2 + 1800g_{BY}g_B^2g_2^2 + 360\sqrt{10}g_1g_{BY}g_{YB}g_2^2 + 1800g_1g_Bg_{YB}g_2^2 \\
& + 3520g_1^2g_{BY}g_3^2 + 3520g_{BY}^3g_3^2 + 320\sqrt{10}g_1^2g_Bg_3^2 + 640\sqrt{10}g_{BY}^2g_Bg_3^2 \\
& + 1600g_{BY}g_B^2g_3^2 + 320\sqrt{10}g_1g_{BY}g_{YB}g_3^2 + 1600g_1g_Bg_{YB}g_3^2 \\
& - 20 \left(2g_{BY} \left(5g_B^2 + 5g_{BY}^2 - \sqrt{10}g_{BY}g_B \right) + g_1 \left(10g_Bg_{YB} - \sqrt{10}g_{BY}g_{YB} \right) + g_1^2 \left(10g_{BY} - \sqrt{10}g_B \right) \right) \text{Tr} \left(Y_d Y_d^\dagger \right) \\
& - 60 \left(2g_{BY} \left(3\sqrt{10}g_{BY}g_B + 5g_B^2 + 5g_{BY}^2 \right) + g_1 \left(10g_B + 3\sqrt{10}g_{BY} \right) g_{YB} + g_1^2 \left(10g_{BY} + 3\sqrt{10}g_B \right) \right) \text{Tr} \left(Y_e Y_e^\dagger \right) \\
& - 750g_{BY}g_B^2 \text{Tr} \left(Y_S Y_S^\dagger \right) - 750g_1g_Bg_{YB} \text{Tr} \left(Y_S Y_S^\dagger \right) - 680g_1^2g_{BY} \text{Tr} \left(Y_u Y_u^\dagger \right) - 680g_{BY}^3 \text{Tr} \left(Y_u Y_u^\dagger \right) \\
& - 100\sqrt{10}g_1^2g_B \text{Tr} \left(Y_u Y_u^\dagger \right) - 200\sqrt{10}g_{BY}^2g_B \text{Tr} \left(Y_u Y_u^\dagger \right) - 200g_{BY}g_B^2 \text{Tr} \left(Y_u Y_u^\dagger \right) \\
& - 100\sqrt{10}g_1g_{BY}g_{YB} \text{Tr} \left(Y_u Y_u^\dagger \right) - 200g_1g_Bg_{YB} \text{Tr} \left(Y_u Y_u^\dagger \right) - 120g_1^2g_{BY} \text{Tr} \left(Y_\nu Y_\nu^\dagger \right) \\
& - 120g_{BY}^3 \text{Tr} \left(Y_\nu Y_\nu^\dagger \right) - 60\sqrt{10}g_1^2g_B \text{Tr} \left(Y_\nu Y_\nu^\dagger \right) - 120\sqrt{10}g_{BY}^2g_B \text{Tr} \left(Y_\nu Y_\nu^\dagger \right) \\
& - 600g_{BY}g_B^2 \text{Tr} \left(Y_\nu Y_\nu^\dagger \right) - 60\sqrt{10}g_1g_{BY}g_{YB} \text{Tr} \left(Y_\nu Y_\nu^\dagger \right) - 600g_1g_Bg_{YB} \text{Tr} \left(Y_\nu Y_\nu^\dagger \right) \Big) \tag{7}
\end{aligned}$$

$$\beta_{g_2}^{(1)} = -\frac{19}{6}g_2^3 \tag{8}$$

$$\begin{aligned}
\beta_{g_2}^{(2)} = & \frac{1}{30}g_2^3 \left(27g_1^2 + 27g_{BY}^2 + 18\sqrt{10}g_{BY}g_B + 45g_B^2 + 18\sqrt{10}g_1g_{YB} + 45g_{YB}^2 + 175g_2^2 + 360g_3^2 - 45\text{Tr} \left(Y_d Y_d^\dagger \right) \right. \\
& \left. - 15\text{Tr} \left(Y_e Y_e^\dagger \right) - 45\text{Tr} \left(Y_u Y_u^\dagger \right) - 15\text{Tr} \left(Y_\nu Y_\nu^\dagger \right) \right) \tag{9}
\end{aligned}$$

$$\beta_{g_3}^{(1)} = -7g_3^3 \tag{10}$$

$$\begin{aligned}
\beta_{g_3}^{(2)} = & \frac{1}{10}g_3^3 \left(11g_1^2 + 11g_{BY}^2 + 2\sqrt{10}g_{BY}g_B + 5g_B^2 + 2\sqrt{10}g_1g_{YB} + 5g_{YB}^2 + 45g_2^2 - 260g_3^2 - 20\text{Tr} \left(Y_d Y_d^\dagger \right) \right. \\
& \left. - 20\text{Tr} \left(Y_u Y_u^\dagger \right) \right) \tag{11}
\end{aligned}$$

$$\begin{aligned}
\beta_{g_B}^{(1)} = & \frac{1}{40} \left(164g_{BY}^2g_B + 4g_{BY} \left(16\sqrt{10}g_B^2 + g_{YB} \left(41g_1 + 8\sqrt{10}g_{YB} \right) \right) + g_B \left(405g_B^2 + g_{YB} \left(32\sqrt{10}g_1 + 405g_{YB} \right) \right) \right) \tag{12}
\end{aligned}$$

$$\begin{aligned}
\beta_{g_B}^{(2)} = & \frac{1}{400} \left(1592g_1^2g_{BY}^2g_B + 1592g_{BY}^4g_B + 656\sqrt{10}g_1^2g_{BY}g_B^2 + 1312\sqrt{10}g_{BY}^3g_B^2 + 920g_1^2g_B^3 \right. \\
& + 5520g_{BY}^2g_B^3 + 1120\sqrt{10}g_{BY}g_B^4 + 12425g_B^5 + 1592g_1^3g_{BY}g_{YB} + 1592g_1g_{BY}^3g_{YB} \\
& + 328\sqrt{10}g_1^3g_Bg_{YB} + 1640\sqrt{10}g_1g_{BY}^2g_Bg_{YB} + 6440g_1g_{BY}g_B^2g_{YB} + 840\sqrt{10}g_1g_B^3g_{YB} \Big)
\end{aligned}$$

$$\begin{aligned}
& + 984\sqrt{10}g_1^2g_{BY}g_{YB}^2 + 328\sqrt{10}g_{BY}^3g_{YB}^2 + 2760g_1^2g_{BY}g_{YB}^2 + 3680g_{BY}^2g_{BY}g_{YB}^2 \\
& + 1400\sqrt{10}g_{BY}g_B^2g_{YB}^2 + 24850g_B^3g_{YB}^2 + 2760g_1g_{BY}g_{YB}^3 + 840\sqrt{10}g_1g_Bg_{YB}^3 \\
& + 280\sqrt{10}g_{BY}g_{YB}^4 + 12425g_Bg_{YB}^4 + 1080g_{BY}^2g_{BY}g_{YB}^2 + 720\sqrt{10}g_{BY}g_B^2g_{YB}^2 + 1800g_B^3g_{YB}^2 \\
& + 1080g_1g_{BY}g_{YB}g_{YB}^2 + 360\sqrt{10}g_1g_Bg_{YB}g_{YB}^2 + 360\sqrt{10}g_{BY}g_{YB}^2g_{YB}^2 + 1800g_Bg_{YB}^2g_{YB}^2 \\
& + 3520g_{BY}^2g_Bg_{YB}^3 + 640\sqrt{10}g_{BY}g_B^2g_{YB}^3 + 1600g_B^3g_{YB}^3 + 3520g_1g_{BY}g_{YB}g_{YB}^3 \\
& + 320\sqrt{10}g_1g_Bg_{YB}g_{YB}^3 + 320\sqrt{10}g_{BY}g_{YB}^2g_{YB}^3 + 1600g_Bg_{YB}^2g_{YB}^3 \\
& - 20\left(10g_{BY}^2g_B - 2\sqrt{10}g_{BY}g_B^2 + 10g_B^3 + 10g_1g_{BY}g_{YB} - \sqrt{10}g_1g_Bg_{YB} - \sqrt{10}g_{BY}g_{YB}^2\right. \\
& \left. + 10g_Bg_{YB}^2\right)\text{Tr}\left(Y_dY_d^\dagger\right) \\
& - 60\left(10g_{BY}^2g_B + 6\sqrt{10}g_{BY}g_B^2 + 10g_B^3 + 10g_1g_{BY}g_{YB} + 3\sqrt{10}g_1g_Bg_{YB} + 3\sqrt{10}g_{BY}g_{YB}^2\right. \\
& \left. + 10g_Bg_{YB}^2\right)\text{Tr}\left(Y_eY_e^\dagger\right) \\
& - 750g_B^3\text{Tr}\left(Y_SY_S^\dagger\right) - 750g_Bg_{YB}^2\text{Tr}\left(Y_SY_S^\dagger\right) - 680g_{BY}^2g_B\text{Tr}\left(Y_uY_u^\dagger\right) \\
& - 200\sqrt{10}g_{BY}g_B^2\text{Tr}\left(Y_uY_u^\dagger\right) - 200g_B^3\text{Tr}\left(Y_uY_u^\dagger\right) - 680g_1g_{BY}g_{YB}\text{Tr}\left(Y_uY_u^\dagger\right) \\
& - 100\sqrt{10}g_1g_Bg_{YB}\text{Tr}\left(Y_uY_u^\dagger\right) - 100\sqrt{10}g_{BY}g_{YB}^2\text{Tr}\left(Y_uY_u^\dagger\right) - 200g_Bg_{YB}^2\text{Tr}\left(Y_uY_u^\dagger\right) \\
& - 120g_{BY}^2g_B\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 120\sqrt{10}g_{BY}g_B^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 600g_B^3\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \\
& - 120g_1g_{BY}g_{YB}\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 60\sqrt{10}g_1g_Bg_{YB}\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 60\sqrt{10}g_{BY}g_{YB}^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \\
& - 600g_Bg_{YB}^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right)
\end{aligned} \tag{13}$$

$$\beta_{g_{YB}}^{(1)} = \frac{1}{40}\left(164g_1^2g_{YB} + 4g_1\left(41g_{BY}g_B + 8\sqrt{10}\left(2g_{YB}^2 + g_B^2\right)\right) + g_{YB}\left(32\sqrt{10}g_{BY}g_B + 405\left(g_B^2 + g_{YB}^2\right)\right)\right) \tag{14}$$

$$\begin{aligned}
\beta_{g_{YB}}^{(2)} & = \frac{1}{400}\left(1592g_1^3g_{BY}g_B + 1592g_1g_B^3g_{BY}g_B + 328\sqrt{10}g_1^3g_B^2 + 984\sqrt{10}g_1g_B^2g_{BY}g_B^2 + 2760g_1g_{BY}g_B^3\right. \\
& + 280\sqrt{10}g_1g_B^4 + 1592g_1^4g_{YB} + 1592g_1^2g_{BY}^2g_{YB} + 1640\sqrt{10}g_1^2g_{BY}g_Bg_{YB} \\
& + 328\sqrt{10}g_{BY}^3g_Bg_{YB} + 3680g_1^2g_B^2g_{YB} + 2760g_{BY}^2g_B^2g_{YB} + 840\sqrt{10}g_{BY}g_B^3g_{YB} + 12425g_B^4g_{YB} \\
& + 1312\sqrt{10}g_1^3g_{YB}^2 + 656\sqrt{10}g_1g_{BY}^2g_{YB}^2 + 6440g_1g_{BY}g_Bg_{YB}^2 + 1400\sqrt{10}g_1g_B^2g_{YB}^2 \\
& + 5520g_1^2g_{YB}^3 + 920g_{BY}^2g_{YB}^3 + 840\sqrt{10}g_{BY}g_Bg_{YB}^3 + 24850g_B^2g_{YB}^3 + 1120\sqrt{10}g_1g_{YB}^4 + 12425g_{YB}^5 \\
& + 1080g_1g_{BY}g_Bg_{YB}^2 + 360\sqrt{10}g_1g_B^2g_{YB}^2 + 1080g_1^2g_{YB}g_{YB}^2 + 360\sqrt{10}g_{BY}g_Bg_{YB}g_{YB}^2 \\
& + 1800g_B^2g_{YB}g_{YB}^2 + 720\sqrt{10}g_1g_{YB}^2g_{YB}^2 + 1800g_{YB}^3g_{YB}^2 + 3520g_1g_{BY}g_Bg_{YB}^3 \\
& + 320\sqrt{10}g_1g_B^2g_{YB}^3 + 3520g_1^2g_{YB}g_{YB}^3 + 320\sqrt{10}g_{BY}g_Bg_{YB}g_{YB}^3 + 1600g_B^2g_{YB}g_{YB}^3 \\
& + 640\sqrt{10}g_1g_{YB}^2g_{YB}^3 + 1600g_{YB}^3g_{YB}^3 \\
& - 20\left(10g_1^2g_{YB} + 10g_{YB}\left(g_B^2 + g_{YB}^2\right) + g_1\left(10g_{BY}g_B - \sqrt{10}\left(2g_{YB}^2 + g_B^2\right)\right) - \sqrt{10}g_{BY}g_Bg_{YB}\right)\text{Tr}\left(Y_dY_d^\dagger\right) \\
& - 60\left(10g_1^2g_{YB} + g_1\left(10g_{BY}g_B + 3\sqrt{10}\left(2g_{YB}^2 + g_B^2\right)\right) + g_{YB}\left(10\left(g_B^2 + g_{YB}^2\right) + 3\sqrt{10}g_{BY}g_B\right)\right)\text{Tr}\left(Y_eY_e^\dagger\right)
\end{aligned}$$

$$\begin{aligned}
& -750g_B^2g_{YB}\text{Tr}\left(Y_S Y_S^\dagger\right) - 750g_{YB}^3\text{Tr}\left(Y_S Y_S^\dagger\right) - 680g_1g_{BY}g_B\text{Tr}\left(Y_u Y_u^\dagger\right) \\
& - 100\sqrt{10}g_1g_B^2\text{Tr}\left(Y_u Y_u^\dagger\right) - 680g_1^2g_{YB}\text{Tr}\left(Y_u Y_u^\dagger\right) - 100\sqrt{10}g_{BY}g_Bg_{YB}\text{Tr}\left(Y_u Y_u^\dagger\right) \\
& - 200g_B^2g_{YB}\text{Tr}\left(Y_u Y_u^\dagger\right) - 200\sqrt{10}g_1g_{YB}^2\text{Tr}\left(Y_u Y_u^\dagger\right) - 200g_{YB}^3\text{Tr}\left(Y_u Y_u^\dagger\right) \\
& - 120g_1g_{BY}g_B\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 60\sqrt{10}g_1g_B^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 120g_1^2g_{YB}\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \\
& - 60\sqrt{10}g_{BY}g_Bg_{YB}\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 600g_B^2g_{YB}\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 120\sqrt{10}g_1g_{YB}^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \\
& - 600g_{YB}^3\text{Tr}\left(Y_\nu Y_\nu^\dagger\right)
\end{aligned} \tag{15}$$

3.2 Quartic scalar couplings

$$\begin{aligned}
\beta_{\lambda_3}^{(1)} &= \frac{1}{32}\left(27g_B^4 + 54g_B^2g_{YB}^2 + 27g_{YB}^4 - 144g_B^2\lambda_3 - 144g_{YB}^2\lambda_3 + 640\lambda_3^2 + 16\lambda_5^2 + 128\lambda_3\text{Tr}\left(Y_S Y_S^\dagger\right)\right. \\
& \left. - 64\text{Tr}\left(Y_S Y_S^\dagger Y_S Y_S^\dagger\right)\right)
\end{aligned} \tag{16}$$

$$\begin{aligned}
\beta_{\lambda_3}^{(2)} &= -\frac{1503}{160}g_{BY}^2g_B^4 - 18\sqrt{\frac{2}{5}}g_{BY}g_B^5 - \frac{837}{32}g_B^6 - \frac{1503}{160}g_1^2g_B^2g_{YB}^2 - \frac{1503}{160}g_{BY}^2g_B^2g_{YB}^2 - 18\sqrt{\frac{2}{5}}g_{BY}g_B^3g_{YB}^2 - \frac{2079}{64}g_B^4g_{YB}^2 \\
& - 18\sqrt{\frac{2}{5}}g_1g_B^2g_{YB}^3 - \frac{1503}{160}g_1^2g_{YB}^4 - \frac{2079}{64}g_B^2g_{YB}^4 - 18\sqrt{\frac{2}{5}}g_1g_{YB}^5 - \frac{837}{32}g_{YB}^6 + \frac{633}{40}g_{BY}^2g_B^2\lambda_3 \\
& + 6\sqrt{10}g_{BY}g_B^3\lambda_3 + \frac{837}{16}g_B^4\lambda_3 + \frac{633}{40}g_1^2g_{YB}^2\lambda_3 + \frac{441}{16}g_B^2g_{YB}^2\lambda_3 + 6\sqrt{10}g_1g_{YB}^3\lambda_3 + \frac{837}{16}g_{YB}^4\lambda_3 \\
& + 42g_B^2\lambda_3^2 + 42g_{YB}^2\lambda_3^2 - 240\lambda_3^3 + \frac{9}{8}g_{BY}^2g_B^2\lambda_5 + \frac{9}{4}g_1g_{BY}g_Bg_{YB}\lambda_5 + \frac{9}{8}g_1^2g_{YB}^2\lambda_5 + \frac{3}{5}g_1^2\lambda_5^2 \\
& + \frac{3}{5}g_{BY}^2\lambda_5^2 + 3g_2^2\lambda_5^2 - 5\lambda_3\lambda_5^2 - \lambda_5^3 - 3\lambda_5^2\text{Tr}\left(Y_d Y_d^\dagger\right) - \lambda_5^2\text{Tr}\left(Y_e Y_e^\dagger\right) \\
& - \frac{225}{16}g_B^4\text{Tr}\left(Y_S Y_S^\dagger\right) - \frac{225}{8}g_B^2g_{YB}^2\text{Tr}\left(Y_S Y_S^\dagger\right) - \frac{225}{16}g_{YB}^4\text{Tr}\left(Y_S Y_S^\dagger\right) + \frac{75}{4}g_B^2\lambda_3\text{Tr}\left(Y_S Y_S^\dagger\right) \\
& + \frac{75}{4}g_{YB}^2\lambda_3\text{Tr}\left(Y_S Y_S^\dagger\right) - 40\lambda_3^2\text{Tr}\left(Y_S Y_S^\dagger\right) - 3\lambda_5^2\text{Tr}\left(Y_u Y_u^\dagger\right) - \lambda_5^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \\
& - 6g_B^2\text{Tr}\left(Y_S Y_S^\dagger Y_S Y_S^\dagger\right) - 6g_{YB}^2\text{Tr}\left(Y_S Y_S^\dagger Y_S Y_S^\dagger\right) + 2\lambda_3\text{Tr}\left(Y_S Y_S^\dagger Y_S Y_S^\dagger\right) - 6\lambda_3\text{Tr}\left(Y_S Y_S^\dagger Y_\nu^T Y_\nu^*\right) \\
& + 8\text{Tr}\left(Y_S Y_S^\dagger Y_S Y_S^\dagger Y_S Y_S^\dagger\right) + 4\text{Tr}\left(Y_S Y_S^\dagger Y_\nu^T Y_\nu^* Y_S Y_S^\dagger\right)
\end{aligned} \tag{17}$$

$$\begin{aligned}
\beta_{\lambda_5}^{(1)} &= +\frac{27}{20}g_{BY}^2g_B^2 + \frac{27}{10}g_1g_{BY}g_Bg_{YB} + \frac{27}{20}g_1^2g_{YB}^2 - \frac{9}{10}g_1^2\lambda_5 - \frac{9}{10}g_{BY}^2\lambda_5 - \frac{9}{4}g_B^2\lambda_5 - \frac{9}{4}g_{YB}^2\lambda_5 - \frac{9}{2}g_2^2\lambda_5 \\
& + 6\lambda_1\lambda_5 + 8\lambda_3\lambda_5 + 2\lambda_5^2 + 6\lambda_5\text{Tr}\left(Y_d Y_d^\dagger\right) + 2\lambda_5\text{Tr}\left(Y_e Y_e^\dagger\right) + 2\lambda_5\text{Tr}\left(Y_S Y_S^\dagger\right) + 6\lambda_5\text{Tr}\left(Y_u Y_u^\dagger\right) \\
& + 2\lambda_5\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 8\text{Tr}\left(Y_S Y_S^\dagger Y_\nu^T Y_\nu^*\right)
\end{aligned} \tag{18}$$

$$\beta_{\lambda_5}^{(2)} = -\frac{81}{80}g_1^2g_{BY}^2g_B^2 - \frac{6417}{400}g_{BY}^4g_B^2 - \frac{144}{5}\sqrt{\frac{2}{5}}g_{BY}^3g_B^3 - \frac{6291}{160}g_{BY}^2g_B^4 - \frac{3411}{200}g_1^3g_{BY}g_Bg_{YB} - \frac{3411}{200}g_1g_{BY}^3g_Bg_{YB}$$

$$\begin{aligned}
& -\frac{144}{5}\sqrt{\frac{2}{5}}g_1g_{BY}^2g_B^2g_{YB} - \frac{837}{20}g_1g_{BY}g_B^3g_{YB} - \frac{6417}{400}g_1^4g_{YB}^2 - \frac{81}{80}g_1^2g_{BY}^2g_{YB}^2 - \frac{144}{5}\sqrt{\frac{2}{5}}g_1^2g_{BY}g_Bg_{YB}^2 \\
& - \frac{81}{32}g_1^2g_B^2g_{YB}^2 - \frac{81}{32}g_{BY}^2g_B^2g_{YB}^2 - \frac{144}{5}\sqrt{\frac{2}{5}}g_1^3g_{YB}^3 - \frac{837}{20}g_1g_{BY}g_Bg_{YB}^3 - \frac{6291}{160}g_1^2g_{YB}^4 - \frac{81}{16}g_{BY}^2g_B^2g_2^2 \\
& - \frac{81}{8}g_1g_{BY}g_Bg_{YB}g_2^2 - \frac{81}{16}g_1^2g_{YB}^2g_2^2 + \frac{27}{4}g_{BY}^2g_B^2\lambda_1 + \frac{27}{2}g_1g_{BY}g_Bg_{YB}\lambda_1 + \frac{27}{4}g_1^2g_{YB}^2\lambda_1 \\
& + 9g_{BY}^2g_B^2\lambda_3 + 18g_1g_{BY}g_Bg_{YB}\lambda_3 + 9g_1^2g_{YB}^2\lambda_3 + \frac{1671}{400}g_1^4\lambda_5 + \frac{81}{40}g_1^2g_{BY}^2\lambda_5 + \frac{1671}{400}g_{BY}^4\lambda_5 \\
& + 6\sqrt{\frac{2}{5}}g_{BY}^3g_B\lambda_5 + \frac{2571}{160}g_{BY}^2g_B^2\lambda_5 + 3\sqrt{10}g_{BY}g_B^3\lambda_5 + \frac{729}{32}g_B^4\lambda_5 + 6\sqrt{\frac{2}{5}}g_1^3g_{YB}\lambda_5 + \frac{9}{10}g_1g_{BY}g_Bg_{YB}\lambda_5 \\
& + \frac{2571}{160}g_1^2g_{YB}^2\lambda_5 + \frac{225}{32}g_B^2g_{YB}^2\lambda_5 + 3\sqrt{10}g_1g_{YB}^3\lambda_5 + \frac{729}{32}g_{YB}^4\lambda_5 + \frac{9}{8}g_1^2g_2^2\lambda_5 + \frac{9}{8}g_{BY}^2g_2^2\lambda_5 \\
& - \frac{145}{16}g_2^4\lambda_5 + \frac{36}{5}g_1^2\lambda_1\lambda_5 + \frac{36}{5}g_{BY}^2\lambda_1\lambda_5 + 36g_2^2\lambda_1\lambda_5 - 15\lambda_1^2\lambda_5 + 24g_B^2\lambda_3\lambda_5 + 24g_{YB}^2\lambda_3\lambda_5 \\
& - 40\lambda_3^2\lambda_5 + \frac{3}{10}g_1^2\lambda_5^2 + \frac{3}{10}g_{BY}^2\lambda_5^2 + \frac{3}{4}g_B^2\lambda_5^2 + \frac{3}{4}g_{YB}^2\lambda_5^2 + \frac{3}{2}g_2^2\lambda_5^2 - 18\lambda_1\lambda_5^2 - 24\lambda_3\lambda_5^2 \\
& - \frac{11}{4}\lambda_5^3 \\
& + \frac{1}{20}\left(-90g_B^4 + 45g_1^2g_{YB}^2 + 18\sqrt{10}g_1g_{YB}^3 - 90g_{YB}^4\right. \\
& \left.+ g_{BY}g_B\left(18\sqrt{10}g_B^2 + 90g_1g_{YB} + \sqrt{10}\left(18g_{YB}^2 - 5\lambda_5\right)\right) + 25g_1^2\lambda_5 - 5\sqrt{10}g_1g_{YB}\lambda_5\right. \\
& \left.+ 25g_{YB}^2\lambda_5 + 225g_2^2\lambda_5 + 800g_3^2\lambda_5 - 720\lambda_1\lambda_5 - 120\lambda_5^2 + 5g_{BY}^2\left(5\lambda_5 + 9g_B^2\right)\right. \\
& \left.+ g_B^2\left(-180g_{YB}^2 + 18\sqrt{10}g_1g_{YB} + 25\lambda_5\right)\right)\text{Tr}\left(Y_dY_d^\dagger\right) \\
& - \frac{1}{20}\left(270g_B^4 + 225g_1^2g_{YB}^2 + 162\sqrt{10}g_1g_{YB}^3 + 270g_{YB}^4\right. \\
& \left.+ 9g_{BY}g_B\left(18\sqrt{10}g_B^2 + 50g_1g_{YB} + \sqrt{10}\left(18g_{YB}^2 - 5\lambda_5\right)\right) + 3g_B^2\left(180g_{YB}^2 - 25\lambda_5 + 54\sqrt{10}g_1g_{YB}\right)\right. \\
& \left.+ 75g_{BY}^2\left(3g_B^2 - \lambda_5\right) - 75g_1^2\lambda_5 - 45\sqrt{10}g_1g_{YB}\lambda_5 - 75g_{YB}^2\lambda_5 - 75g_2^2\lambda_5 + 240\lambda_1\lambda_5 + 40\lambda_5^2\right)\text{Tr}\left(Y_eY_e^\dagger\right) \\
& - \frac{45}{4}g_{BY}^2g_B^2\text{Tr}\left(Y_SY_S^\dagger\right) - \frac{45}{2}g_1g_{BY}g_Bg_{YB}\text{Tr}\left(Y_SY_S^\dagger\right) - \frac{45}{4}g_1^2g_{YB}^2\text{Tr}\left(Y_SY_S^\dagger\right) \\
& + \frac{75}{8}g_B^2\lambda_5\text{Tr}\left(Y_SY_S^\dagger\right) + \frac{75}{8}g_{YB}^2\lambda_5\text{Tr}\left(Y_SY_S^\dagger\right) - 16\lambda_3\lambda_5\text{Tr}\left(Y_SY_S^\dagger\right) - 2\lambda_5^2\text{Tr}\left(Y_SY_S^\dagger\right) \\
& - \frac{171}{20}g_{BY}^2g_B^2\text{Tr}\left(Y_uY_u^\dagger\right) - 9\sqrt{\frac{5}{2}}g_{BY}g_B^3\text{Tr}\left(Y_uY_u^\dagger\right) - \frac{9}{2}g_B^4\text{Tr}\left(Y_uY_u^\dagger\right) - \frac{171}{10}g_1g_{BY}g_Bg_{YB}\text{Tr}\left(Y_uY_u^\dagger\right) \\
& - 9\sqrt{\frac{5}{2}}g_1g_B^2g_{YB}\text{Tr}\left(Y_uY_u^\dagger\right) - \frac{171}{20}g_1^2g_{YB}^2\text{Tr}\left(Y_uY_u^\dagger\right) - 9\sqrt{\frac{5}{2}}g_{BY}g_Bg_{YB}^2\text{Tr}\left(Y_uY_u^\dagger\right) \\
& - 9g_B^2g_{YB}^2\text{Tr}\left(Y_uY_u^\dagger\right) - 9\sqrt{\frac{5}{2}}g_1g_{YB}^3\text{Tr}\left(Y_uY_u^\dagger\right) - \frac{9}{2}g_{YB}^4\text{Tr}\left(Y_uY_u^\dagger\right) + \frac{17}{4}g_1^2\lambda_5\text{Tr}\left(Y_uY_u^\dagger\right) \\
& + \frac{17}{4}g_{BY}^2\lambda_5\text{Tr}\left(Y_uY_u^\dagger\right) + \frac{5}{2}\sqrt{\frac{5}{2}}g_{BY}g_B\lambda_5\text{Tr}\left(Y_uY_u^\dagger\right) + \frac{5}{4}g_B^2\lambda_5\text{Tr}\left(Y_uY_u^\dagger\right)
\end{aligned}$$

$$\begin{aligned}
& + \frac{5}{2}\sqrt{\frac{5}{2}}g_1g_{YB}\lambda_5\text{Tr}(Y_uY_u^\dagger) + \frac{5}{4}g_{YB}^2\lambda_5\text{Tr}(Y_uY_u^\dagger) + \frac{45}{4}g_2^2\lambda_5\text{Tr}(Y_uY_u^\dagger) + 40g_3^2\lambda_5\text{Tr}(Y_uY_u^\dagger) \\
& - 36\lambda_1\lambda_5\text{Tr}(Y_uY_u^\dagger) - 6\lambda_5^2\text{Tr}(Y_uY_u^\dagger) - \frac{9}{20}g_{BY}^2g_B^2\text{Tr}(Y_\nu Y_\nu^\dagger) \\
& - 27\frac{1}{\sqrt{10}}g_{BY}g_B^3\text{Tr}(Y_\nu Y_\nu^\dagger) - \frac{27}{2}g_B^4\text{Tr}(Y_\nu Y_\nu^\dagger) - \frac{9}{10}g_1g_{BY}g_Bg_{YB}\text{Tr}(Y_\nu Y_\nu^\dagger) \\
& - 27\frac{1}{\sqrt{10}}g_1g_B^2g_{YB}\text{Tr}(Y_\nu Y_\nu^\dagger) - \frac{9}{20}g_1^2g_{YB}^2\text{Tr}(Y_\nu Y_\nu^\dagger) - 27\frac{1}{\sqrt{10}}g_{BY}g_Bg_{YB}^2\text{Tr}(Y_\nu Y_\nu^\dagger) \\
& - 27g_B^2g_{YB}^2\text{Tr}(Y_\nu Y_\nu^\dagger) - 27\frac{1}{\sqrt{10}}g_1g_{YB}^3\text{Tr}(Y_\nu Y_\nu^\dagger) - \frac{27}{2}g_{YB}^4\text{Tr}(Y_\nu Y_\nu^\dagger) \\
& + \frac{3}{4}g_1^2\lambda_5\text{Tr}(Y_\nu Y_\nu^\dagger) + \frac{3}{4}g_{BY}^2\lambda_5\text{Tr}(Y_\nu Y_\nu^\dagger) + \frac{3}{2}\sqrt{\frac{5}{2}}g_{BY}g_B\lambda_5\text{Tr}(Y_\nu Y_\nu^\dagger) + \frac{15}{4}g_B^2\lambda_5\text{Tr}(Y_\nu Y_\nu^\dagger) \\
& + \frac{3}{2}\sqrt{\frac{5}{2}}g_1g_{YB}\lambda_5\text{Tr}(Y_\nu Y_\nu^\dagger) + \frac{15}{4}g_{YB}^2\lambda_5\text{Tr}(Y_\nu Y_\nu^\dagger) + \frac{15}{4}g_2^2\lambda_5\text{Tr}(Y_\nu Y_\nu^\dagger) - 12\lambda_1\lambda_5\text{Tr}(Y_\nu Y_\nu^\dagger) \\
& - 2\lambda_5^2\text{Tr}(Y_\nu Y_\nu^\dagger) - \frac{27}{2}\lambda_5\text{Tr}(Y_dY_d^\dagger Y_dY_d^\dagger) - 21\lambda_5\text{Tr}(Y_dY_u^\dagger Y_uY_d^\dagger) - \frac{9}{2}\lambda_5\text{Tr}(Y_eY_e^\dagger Y_eY_e^\dagger) \\
& - 3\lambda_5\text{Tr}(Y_SY_S^\dagger Y_SY_S^\dagger) - 6\sqrt{\frac{2}{5}}g_{BY}g_B\text{Tr}(Y_SY_S^\dagger Y_\nu^T Y_\nu^*) - 18g_B^2\text{Tr}(Y_SY_S^\dagger Y_\nu^T Y_\nu^*) \\
& - 6\sqrt{\frac{2}{5}}g_1g_{YB}\text{Tr}(Y_SY_S^\dagger Y_\nu^T Y_\nu^*) - 18g_{YB}^2\text{Tr}(Y_SY_S^\dagger Y_\nu^T Y_\nu^*) + \frac{7}{2}\lambda_5\text{Tr}(Y_SY_S^\dagger Y_\nu^T Y_\nu^*) \\
& - \frac{27}{2}\lambda_5\text{Tr}(Y_uY_u^\dagger Y_uY_u^\dagger) - \frac{9}{2}\lambda_5\text{Tr}(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger) - 7\lambda_5\text{Tr}(Y_\nu Y_\nu^\dagger Y_e^T Y_e^*) + 12\text{Tr}(Y_SY_S^\dagger Y_SY_S^\dagger Y_\nu^T Y_\nu^*) \\
& + 8\text{Tr}(Y_SY_S^\dagger Y_\nu^T Y_\nu^* Y_SY_S^\dagger) + 28\text{Tr}(Y_SY_S^\dagger Y_\nu^T Y_\nu^* Y_\nu^T Y_\nu^*) - 4\text{Tr}(Y_\nu Y_S^* Y_S^T Y_\nu^\dagger Y_e^T Y_e^*)
\end{aligned} \tag{19}$$

$$\begin{aligned}
\beta_{\lambda_1}^{(1)} & = + \frac{27}{100}g_1^4 + \frac{27}{50}g_1^2g_{BY}^2 + \frac{27}{100}g_{BY}^4 + \frac{9}{10}g_1^2g_2^2 + \frac{9}{10}g_{BY}^2g_2^2 + \frac{9}{4}g_2^4 - \frac{9}{5}g_1^2\lambda_1 - \frac{9}{5}g_{BY}^2\lambda_1 - 9g_2^2\lambda_1 + 12\lambda_1^2 \\
& + \frac{1}{2}\lambda_5^2 + 12\lambda_1\text{Tr}(Y_dY_d^\dagger) + 4\lambda_1\text{Tr}(Y_eY_e^\dagger) + 12\lambda_1\text{Tr}(Y_uY_u^\dagger) + 4\lambda_1\text{Tr}(Y_\nu Y_\nu^\dagger) - 12\text{Tr}(Y_dY_d^\dagger Y_dY_d^\dagger) \\
& - 4\text{Tr}(Y_eY_e^\dagger Y_eY_e^\dagger) - 12\text{Tr}(Y_uY_u^\dagger Y_uY_u^\dagger) - 4\text{Tr}(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger)
\end{aligned} \tag{20}$$

$$\begin{aligned}
\beta_{\lambda_1}^{(2)} & = - \frac{3411}{1000}g_1^6 - \frac{4221}{1000}g_1^4g_{BY}^2 - \frac{4221}{1000}g_1^2g_{BY}^4 - \frac{3411}{1000}g_{BY}^6 - \frac{144}{25}\sqrt{\frac{2}{5}}g_1^2g_{BY}^3g_B - \frac{144}{25}\sqrt{\frac{2}{5}}g_{BY}^5g_B - \frac{2943}{400}g_1^2g_{BY}^2g_B^2 - \frac{2943}{400}g_{BY}^4g_B^2 \\
& - \frac{144}{25}\sqrt{\frac{2}{5}}g_1^5g_{YB} - \frac{144}{25}\sqrt{\frac{2}{5}}g_1^3g_{BY}^2g_{YB} - \frac{2943}{400}g_1^4g_{YB}^2 - \frac{2943}{400}g_1^2g_{BY}^2g_{YB}^2 - \frac{1677}{200}g_1^4g_2^2 - \frac{27}{4}g_1^2g_{BY}^2g_2^2 \\
& - \frac{1677}{200}g_{BY}^4g_2^2 - \frac{48}{5}\sqrt{\frac{2}{5}}g_{BY}^3g_Bg_2^2 - \frac{981}{80}g_{BY}^2g_B^2g_2^2 - \frac{48}{5}\sqrt{\frac{2}{5}}g_1^3g_{YB}g_2^2 - \frac{981}{80}g_1^2g_{YB}^2g_2^2 - \frac{289}{40}g_1^2g_2^4 \\
& - \frac{289}{40}g_{BY}^2g_2^4 + \frac{305}{8}g_2^6 + \frac{1887}{200}g_1^4\lambda_1 + \frac{621}{100}g_1^2g_{BY}^2\lambda_1 + \frac{1887}{200}g_{BY}^4\lambda_1 + 12\sqrt{\frac{2}{5}}g_{BY}^3g_B\lambda_1 + \frac{1233}{80}g_{BY}^2g_B^2\lambda_1 \\
& + 12\sqrt{\frac{2}{5}}g_1^3g_{YB}\lambda_1 + \frac{1233}{80}g_1^2g_{YB}^2\lambda_1 + \frac{117}{20}g_1^2g_2^2\lambda_1 + \frac{117}{20}g_{BY}^2g_2^2\lambda_1 - \frac{73}{8}g_2^4\lambda_1 + \frac{54}{5}g_1^2\lambda_1^2 + \frac{54}{5}g_{BY}^2\lambda_1^2 \\
& + 54g_2^2\lambda_1^2 - 78\lambda_1^3 + \frac{9}{8}g_{BY}^2g_B^2\lambda_5 + \frac{9}{4}g_1g_{BY}g_Bg_{YB}\lambda_5 + \frac{9}{8}g_1^2g_{YB}^2\lambda_5 + \frac{3}{2}g_B^2\lambda_5^2 + \frac{3}{2}g_{YB}^2\lambda_5^2
\end{aligned}$$

$$\begin{aligned}
& -\frac{5}{2}\lambda_1\lambda_5^2 - \lambda_3^3 \\
& + \frac{1}{50}\left(45g_1^4 + 45g_{BY}^4 + 18\sqrt{10}g_{BY}^3g_B + 18\sqrt{10}g_1^3g_{YB} - 5\sqrt{10}g_{BY}g_B(18g_2^2 + 5\lambda_1)\right) \\
& + g_{BY}^2\left(125\lambda_1 + 270g_2^2 - 90g_B^2\right) + g_1^2\left(125\lambda_1 + 18\sqrt{10}g_{BY}g_B + 270g_2^2 + 90g_{BY}^2 - 90g_{YB}^2\right) \\
& + 25\left(\left(-144\lambda_1 + 160g_3^2 + 5g_B^2 + 5g_{YB}^2\right)\lambda_1 + 45g_2^2\lambda_1 - 9g_2^4\right) \\
& + g_1g_{YB}\left(-180g_{BY}g_B + 18\sqrt{10}g_{BY}^2 - 5\sqrt{10}(18g_2^2 + 5\lambda_1)\right)\text{Tr}\left(Y_dY_d^\dagger\right) \\
& - \frac{3}{50}\left(75g_1^4 + 75g_{BY}^4 + 54\sqrt{10}g_{BY}^3g_B + 54\sqrt{10}g_1^3g_{YB}\right) \\
& + g_1^2\left(-110g_2^2 - 125\lambda_1 + 150g_{BY}^2 + 54\sqrt{10}g_{BY}g_B + 90g_{YB}^2\right) + 5g_{BY}^2\left(18g_B^2 - 22g_2^2 - 25\lambda_1\right) \\
& - 15\sqrt{10}g_{BY}g_B\left(2g_2^2 + 5\lambda_1\right) + 3g_1g_{YB}\left(18\sqrt{10}g_{BY}^2 - 5\sqrt{10}(2g_2^2 + 5\lambda_1) + 60g_{BY}g_B\right) \\
& + 25\left(-5g_2^2\lambda_1 + \lambda_1\left(16\lambda_1 - 5g_B^2 - 5g_{YB}^2\right) + g_2^4\right)\text{Tr}\left(Y_eY_e^\dagger\right) \\
& - \lambda_5^2\text{Tr}\left(Y_SY_S^\dagger\right) - \frac{171}{50}g_1^4\text{Tr}\left(Y_uY_u^\dagger\right) - \frac{171}{25}g_1^2g_{BY}^2\text{Tr}\left(Y_uY_u^\dagger\right) - \frac{171}{50}g_{BY}^4\text{Tr}\left(Y_uY_u^\dagger\right) \\
& - 9\sqrt{\frac{2}{5}}g_1^2g_{BY}g_B\text{Tr}\left(Y_uY_u^\dagger\right) - 9\sqrt{\frac{2}{5}}g_{BY}^3g_B\text{Tr}\left(Y_uY_u^\dagger\right) - \frac{9}{5}g_{BY}^2g_B^2\text{Tr}\left(Y_uY_u^\dagger\right) \\
& - 9\sqrt{\frac{2}{5}}g_1^3g_{YB}\text{Tr}\left(Y_uY_u^\dagger\right) - 9\sqrt{\frac{2}{5}}g_1g_{BY}^2g_{YB}\text{Tr}\left(Y_uY_u^\dagger\right) - \frac{18}{5}g_1g_{BY}g_Bg_{YB}\text{Tr}\left(Y_uY_u^\dagger\right) \\
& - \frac{9}{5}g_1^2g_{YB}^2\text{Tr}\left(Y_uY_u^\dagger\right) + \frac{63}{5}g_1^2g_2^2\text{Tr}\left(Y_uY_u^\dagger\right) + \frac{63}{5}g_{BY}^2g_2^2\text{Tr}\left(Y_uY_u^\dagger\right) \\
& + 9\sqrt{\frac{2}{5}}g_{BY}g_Bg_2^2\text{Tr}\left(Y_uY_u^\dagger\right) + 9\sqrt{\frac{2}{5}}g_1g_{YB}g_2^2\text{Tr}\left(Y_uY_u^\dagger\right) - \frac{9}{2}g_2^4\text{Tr}\left(Y_uY_u^\dagger\right) + \frac{17}{2}g_1^2\lambda_1\text{Tr}\left(Y_uY_u^\dagger\right) \\
& + \frac{17}{2}g_{BY}^2\lambda_1\text{Tr}\left(Y_uY_u^\dagger\right) + 5\sqrt{\frac{5}{2}}g_{BY}g_B\lambda_1\text{Tr}\left(Y_uY_u^\dagger\right) + \frac{5}{2}g_B^2\lambda_1\text{Tr}\left(Y_uY_u^\dagger\right) \\
& + 5\sqrt{\frac{5}{2}}g_1g_{YB}\lambda_1\text{Tr}\left(Y_uY_u^\dagger\right) + \frac{5}{2}g_{YB}^2\lambda_1\text{Tr}\left(Y_uY_u^\dagger\right) + \frac{45}{2}g_2^2\lambda_1\text{Tr}\left(Y_uY_u^\dagger\right) + 80g_3^2\lambda_1\text{Tr}\left(Y_uY_u^\dagger\right) \\
& - 72\lambda_1^2\text{Tr}\left(Y_uY_u^\dagger\right) - \frac{9}{50}g_1^4\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - \frac{9}{25}g_1^2g_{BY}^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - \frac{9}{50}g_{BY}^4\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \\
& - \frac{27}{5}\sqrt{\frac{2}{5}}g_1^2g_{BY}g_B\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - \frac{27}{5}\sqrt{\frac{2}{5}}g_{BY}^3g_B\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - \frac{27}{5}g_{BY}^2g_B^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \\
& - \frac{27}{5}\sqrt{\frac{2}{5}}g_1^3g_{YB}\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - \frac{27}{5}\sqrt{\frac{2}{5}}g_1g_{BY}^2g_{YB}\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - \frac{54}{5}g_1g_{BY}g_Bg_{YB}\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \\
& - \frac{27}{5}g_1^2g_{YB}^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - \frac{3}{5}g_1^2g_2^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - \frac{3}{5}g_{BY}^2g_2^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \\
& - 9\sqrt{\frac{2}{5}}g_{BY}g_Bg_2^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 9\sqrt{\frac{2}{5}}g_1g_{YB}g_2^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - \frac{3}{2}g_2^4\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) + \frac{3}{2}g_1^2\lambda_1\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \\
& + \frac{3}{2}g_{BY}^2\lambda_1\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) + 3\sqrt{\frac{5}{2}}g_{BY}g_B\lambda_1\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) + \frac{15}{2}g_B^2\lambda_1\text{Tr}\left(Y_\nu Y_\nu^\dagger\right)
\end{aligned}$$

$$\begin{aligned}
& + 3\sqrt{\frac{5}{2}}g_1g_{YB}\lambda_1\text{Tr}(Y_\nu Y_\nu^\dagger) + \frac{15}{2}g_{YB}^2\lambda_1\text{Tr}(Y_\nu Y_\nu^\dagger) + \frac{15}{2}g_2^2\lambda_1\text{Tr}(Y_\nu Y_\nu^\dagger) - 24\lambda_1^2\text{Tr}(Y_\nu Y_\nu^\dagger) \\
& + \frac{8}{5}g_1^2\text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) + \frac{8}{5}g_{BY}^2\text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) + 2\sqrt{\frac{2}{5}}g_{BY}g_B\text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) - 2g_B^2\text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) \\
& + 2\sqrt{\frac{2}{5}}g_1g_{YB}\text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) - 2g_{YB}^2\text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) - 64g_3^2\text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) - 3\lambda_1\text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) \\
& - 42\lambda_1\text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) - \frac{24}{5}g_1^2\text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) - \frac{24}{5}g_{BY}^2\text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) \\
& - 18\sqrt{\frac{2}{5}}g_{BY}g_B\text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) - 6g_B^2\text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) - 18\sqrt{\frac{2}{5}}g_1g_{YB}\text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) \\
& - 6g_{YB}^2\text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) - \lambda_1\text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) - 3\lambda_1\text{Tr}(Y_S Y_S^\dagger Y_\nu^T Y_\nu^*) - \frac{16}{5}g_1^2\text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger) \\
& - \frac{16}{5}g_{BY}^2\text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger) - 2\sqrt{10}g_{BY}g_B\text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger) - 2g_B^2\text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger) \\
& - 2\sqrt{10}g_1g_{YB}\text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger) - 2g_{YB}^2\text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger) - 64g_3^2\text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger) \\
& - 3\lambda_1\text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger) - 6\sqrt{\frac{2}{5}}g_{BY}g_B\text{Tr}(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger) - 6g_B^2\text{Tr}(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger) - 6\sqrt{\frac{2}{5}}g_1g_{YB}\text{Tr}(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger) \\
& - 6g_{YB}^2\text{Tr}(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger) - \lambda_1\text{Tr}(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger) - 14\lambda_1\text{Tr}(Y_\nu Y_\nu^\dagger Y_e^T Y_e^*) + 60\text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger Y_d Y_d^\dagger) \\
& + 12\text{Tr}(Y_d Y_d^\dagger Y_d Y_u^\dagger Y_u Y_d^\dagger) - 24\text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger Y_d Y_d^\dagger) - 12\text{Tr}(Y_d Y_u^\dagger Y_u Y_u^\dagger Y_u Y_d^\dagger) \\
& + 20\text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger Y_e Y_e^\dagger) + 4\text{Tr}(Y_S Y_S^\dagger Y_\nu^T Y_\nu^* Y_\nu^T Y_\nu^*) + 60\text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger Y_u Y_u^\dagger) + 20\text{Tr}(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger) \\
& - 4\text{Tr}(Y_\nu Y_\nu^\dagger Y_e^T Y_e^* Y_\nu Y_\nu^\dagger) - 4\text{Tr}(Y_\nu Y_\nu^\dagger Y_e^T Y_e^* Y_e^T Y_e^*)
\end{aligned} \tag{21}$$

3.3 Yukawa Couplings

$$\begin{aligned}
\beta_{Y_S}^{(1)} &= -\frac{1}{8}Y_S\left(45(g_B^2 + g_{YB}^2) - 8\text{Tr}(Y_S Y_S^\dagger)\right) + Y_S Y_S^\dagger Y_S + Y_\nu^T Y_\nu^* Y_S \\
\beta_{Y_S}^{(2)} &= \frac{1}{80}\left(2\left(10\left(7Y_S Y_S^\dagger Y_S Y_S^\dagger Y_S - Y_\nu^T Y_e^\dagger Y_e Y_\nu^* Y_S - Y_\nu^T Y_\nu^* Y_\nu^T Y_\nu^* Y_S - Y_S Y_S^\dagger Y_\nu^T Y_\nu^* Y_S\right)\right.\right. \\
& + 5Y_S Y_S^\dagger Y_S\left(-12\text{Tr}(Y_S Y_S^\dagger) - 64\lambda_3 + 87g_B^2 + 87g_{YB}^2\right) \\
& + Y_\nu^T Y_\nu^* Y_S\left(51g_1^2 + 51g_{BY}^2 + 69\sqrt{10}g_{BY}g_B + 300g_B^2 + 69\sqrt{10}g_1g_{YB} + 300g_{YB}^2 + 255g_2^2 - 80\lambda_5\right. \\
& \left. - 180\text{Tr}(Y_d Y_d^\dagger) - 60\text{Tr}(Y_e Y_e^\dagger) - 180\text{Tr}(Y_u Y_u^\dagger) - 60\text{Tr}(Y_\nu Y_\nu^\dagger)\right) \\
& + Y_S\left(375(g_B^2 + g_{YB}^2)\text{Tr}(Y_S Y_S^\dagger)\right) \\
& + 2\left(699g_{BY}^2g_B^2 + 264\sqrt{10}g_{BY}g_B^3 + 1665g_B^4 + 1398g_1g_{BY}g_Bg_{YB} + 264\sqrt{10}g_1g_B^2g_{YB} + 699g_1^2g_{YB}^2\right. \\
& \left. + 264\sqrt{10}g_{BY}g_Bg_{YB}^2 + 3330g_B^2g_{YB}^2 + 264\sqrt{10}g_1g_{YB}^3 + 1665g_{YB}^4 + 160\lambda_3^2 + 10\lambda_5^2\right)
\end{aligned} \tag{22}$$

$$- 60\text{Tr}\left(Y_S Y_S^\dagger Y_S Y_S^\dagger\right) - 60\text{Tr}\left(Y_S Y_S^\dagger Y_\nu^T Y_\nu^*\right)\right) \quad (23)$$

$$\begin{aligned} \beta_{Y_d}^{(1)} = & \frac{1}{20} \left(30 \left(- Y_d Y_u^\dagger Y_u + Y_d Y_d^\dagger Y_d \right) \right. \\ & + Y_d \left(- 5g_1^2 - 5g_{BY}^2 + \sqrt{10}g_{BY}g_B - 5g_B^2 + \sqrt{10}g_1g_{YB} - 5g_{YB}^2 - 45g_2^2 - 160g_3^2 + 60\text{Tr}\left(Y_d Y_d^\dagger\right) \right. \\ & \left. \left. + 20\text{Tr}\left(Y_e Y_e^\dagger\right) + 60\text{Tr}\left(Y_u Y_u^\dagger\right) + 20\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \right) \right) \quad (24) \end{aligned}$$

$$\begin{aligned} \beta_{Y_d}^{(2)} = & + \frac{1}{80} \left(20 \left(11Y_d Y_u^\dagger Y_u Y_u^\dagger Y_u - 4Y_d Y_d^\dagger Y_d Y_u^\dagger Y_u + 6Y_d Y_d^\dagger Y_d Y_d^\dagger Y_d - Y_d Y_u^\dagger Y_u Y_d^\dagger Y_d \right) \right. \\ & + Y_d Y_d^\dagger Y_d \left(187g_1^2 + 187g_{BY}^2 + 7\sqrt{10}g_{BY}g_B + 40g_B^2 + 7\sqrt{10}g_1g_{YB} + 40g_{YB}^2 + 675g_2^2 + 1280g_3^2 - 480\lambda_1 \right. \\ & - 540\text{Tr}\left(Y_d Y_d^\dagger\right) - 180\text{Tr}\left(Y_e Y_e^\dagger\right) - 540\text{Tr}\left(Y_u Y_u^\dagger\right) - 180\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \\ & - Y_d Y_u^\dagger Y_u \left(79g_1^2 + 79g_{BY}^2 + 25\sqrt{10}g_{BY}g_B + 40g_B^2 + 25\sqrt{10}g_1g_{YB} + 40g_{YB}^2 - 45g_2^2 + 1280g_3^2 \right. \\ & \left. - 300\text{Tr}\left(Y_d Y_d^\dagger\right) - 100\text{Tr}\left(Y_e Y_e^\dagger\right) - 300\text{Tr}\left(Y_u Y_u^\dagger\right) - 100\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \right) \\ & - \frac{1}{4800} Y_d \left(1016g_1^4 + 2032g_1^2g_{BY}^2 + 1016g_{BY}^4 + 1016\sqrt{10}g_1^2g_{BY}g_B + 1016\sqrt{10}g_{BY}^3g_B + 1290g_1^2g_B^2 \right. \\ & - 570g_{BY}^2g_B^2 + 445\sqrt{10}g_{BY}g_B^3 - 10250g_B^4 + 1016\sqrt{10}g_1^3g_{YB} + 1016\sqrt{10}g_1g_{BY}^2g_{YB} \\ & - 3720g_1g_{BY}g_Bg_{YB} + 445\sqrt{10}g_1g_B^2g_{YB} - 570g_1^2g_{YB}^2 + 1290g_{BY}^2g_{YB}^2 + 445\sqrt{10}g_{BY}g_Bg_{YB}^2 \\ & - 20500g_B^2g_{YB}^2 + 445\sqrt{10}g_1g_{YB}^3 - 10250g_{YB}^4 + 6480g_1^2g_2^2 + 6480g_{BY}^2g_2^2 \\ & + 540\sqrt{10}g_{BY}g_Bg_2^2 - 1350g_B^2g_2^2 + 540\sqrt{10}g_1g_{YB}g_2^2 - 1350g_{YB}^2g_2^2 + 27600g_2^4 \\ & - 9920g_1^2g_3^2 - 9920g_{BY}^2g_3^2 - 320\sqrt{10}g_{BY}g_Bg_3^2 + 1600g_B^2g_3^2 - 320\sqrt{10}g_1g_{YB}g_3^2 \\ & + 1600g_{YB}^2g_3^2 - 43200g_2^2g_3^2 + 518400g_3^4 - 7200\lambda_1^2 - 600\lambda_5^2 \\ & - 600 \left(5 \left(32g_3^2 + 9g_2^2 + g_B^2 + g_{YB}^2 \right) + 5g_1^2 + 5g_{BY}^2 - \sqrt{10}g_1g_{YB} - \sqrt{10}g_{BY}g_B \right) \text{Tr}\left(Y_d Y_d^\dagger\right) \\ & - 1800 \left(3\sqrt{10}g_1g_{YB} + 3\sqrt{10}g_{BY}g_B + 5g_1^2 + 5 \left(g_B^2 + g_{YB}^2 + g_2^2 \right) + 5g_{BY}^2 \right) \text{Tr}\left(Y_e Y_e^\dagger\right) \\ & - 10200g_1^2 \text{Tr}\left(Y_u Y_u^\dagger\right) - 10200g_{BY}^2 \text{Tr}\left(Y_u Y_u^\dagger\right) - 3000\sqrt{10}g_{BY}g_B \text{Tr}\left(Y_u Y_u^\dagger\right) \\ & - 3000g_B^2 \text{Tr}\left(Y_u Y_u^\dagger\right) - 3000\sqrt{10}g_1g_{YB} \text{Tr}\left(Y_u Y_u^\dagger\right) - 3000g_{YB}^2 \text{Tr}\left(Y_u Y_u^\dagger\right) \\ & - 27000g_2^2 \text{Tr}\left(Y_u Y_u^\dagger\right) - 96000g_3^2 \text{Tr}\left(Y_u Y_u^\dagger\right) - 1800g_1^2 \text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 1800g_{BY}^2 \text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \\ & - 1800\sqrt{10}g_{BY}g_B \text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 9000g_B^2 \text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 1800\sqrt{10}g_1g_{YB} \text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \\ & - 9000g_{YB}^2 \text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 9000g_2^2 \text{Tr}\left(Y_\nu Y_\nu^\dagger\right) + 32400\text{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger\right) - 7200\text{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger\right) \\ & + 10800\text{Tr}\left(Y_e Y_e^\dagger Y_e Y_e^\dagger\right) + 3600\text{Tr}\left(Y_S Y_S^\dagger Y_\nu^T Y_\nu^*\right) + 32400\text{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger\right) + 10800\text{Tr}\left(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger\right) \\ & \left. - 2400\text{Tr}\left(Y_\nu Y_\nu^\dagger Y_e^T Y_e^*\right) \right) \quad (25) \end{aligned}$$

$$\beta_{Y_e}^{(1)} = \frac{1}{20} \left(30 \left(- Y_e Y_\nu^* Y_\nu^T + Y_e Y_e^\dagger Y_e \right) \right)$$

$$\begin{aligned}
& - Y_e \left(45g_1^2 + 45g_{BY}^2 + 27\sqrt{10}g_{BY}g_B + 45g_B^2 + 27\sqrt{10}g_1g_{YB} + 45g_{YB}^2 + 45g_2^2 - 60\text{Tr}\left(Y_d Y_d^\dagger\right) \right. \\
& \left. - 20\text{Tr}\left(Y_e Y_e^\dagger\right) - 60\text{Tr}\left(Y_u Y_u^\dagger\right) - 20\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \right) \tag{26}
\end{aligned}$$

$$\begin{aligned}
\beta_{Y_e}^{(2)} = & \frac{1}{1600} \left(20 \left(10 \left(12Y_e Y_e^\dagger Y_e Y_e^\dagger Y_e - 8Y_e Y_e^\dagger Y_e Y_\nu^* Y_\nu^T + 7Y_e Y_\nu^* Y_S Y_S^\dagger Y_\nu^T - 2Y_e Y_\nu^* Y_\nu^T Y_e^\dagger Y_e \right. \right. \right. \\
& \left. \left. + 22Y_e Y_\nu^* Y_\nu^T Y_\nu^* Y_\nu^T \right) \right. \\
& + 3Y_e Y_e^\dagger Y_e \left(129g_1^2 + 129g_{BY}^2 + 57\sqrt{10}g_{BY}g_B + 120g_B^2 + 57\sqrt{10}g_1g_{YB} + 120g_{YB}^2 + 225g_2^2 - 160\lambda_1 \right. \\
& \left. - 180\text{Tr}\left(Y_d Y_d^\dagger\right) - 60\text{Tr}\left(Y_e Y_e^\dagger\right) - 180\text{Tr}\left(Y_u Y_u^\dagger\right) - 60\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \right) \\
& + Y_e Y_\nu^* Y_\nu^T \left(-135g_1^2 - 135g_{BY}^2 - 117\sqrt{10}g_{BY}g_B - 360g_B^2 - 117\sqrt{10}g_1g_{YB} - 360g_{YB}^2 + 45g_2^2 \right. \\
& \left. + 300\text{Tr}\left(Y_d Y_d^\dagger\right) + 100\text{Tr}\left(Y_e Y_e^\dagger\right) + 300\text{Tr}\left(Y_u Y_u^\dagger\right) + 100\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \right) \\
& + Y_e \left(10968g_1^4 + 21936g_1^2 g_{BY}^2 + 10968g_{BY}^4 + 11496\sqrt{10}g_1^2 g_{BY}g_B + 11496\sqrt{10}g_{BY}^3 g_B + 14130g_1^2 g_B^2 \right. \\
& + 65670g_{BY}^2 g_B^2 + 22485\sqrt{10}g_{BY}g_B^3 + 30150g_B^4 + 11496\sqrt{10}g_1^3 g_{YB} + 11496\sqrt{10}g_1 g_{BY}^2 g_{YB} \\
& + 103080g_1 g_{BY} g_B g_{YB} + 22485\sqrt{10}g_1 g_B^2 g_{YB} + 65670g_1^2 g_{YB}^2 + 14130g_{BY}^2 g_{YB}^2 \\
& + 22485\sqrt{10}g_{BY}g_B g_{YB}^2 + 60300g_B^2 g_{YB}^2 + 22485\sqrt{10}g_1 g_{YB}^3 + 30150g_{YB}^4 + 2160g_1^2 g_2^2 \\
& + 2160g_{BY}^2 g_2^2 + 2700\sqrt{10}g_{BY}g_B g_2^2 + 4050g_B^2 g_2^2 + 2700\sqrt{10}g_1 g_{YB} g_2^2 + 4050g_{YB}^2 g_2^2 \\
& \left. - 9200g_2^4 + 2400\lambda_1^2 + 200\lambda_5^2 \right) \\
& + 200 \left(5 \left(32g_3^2 + 9g_2^2 + g_B^2 + g_{YB}^2 \right) + 5g_1^2 + 5g_{BY}^2 - \sqrt{10}g_1g_{YB} - \sqrt{10}g_{BY}g_B \right) \text{Tr}\left(Y_d Y_d^\dagger\right) \\
& + 600 \left(3\sqrt{10}g_1g_{YB} + 3\sqrt{10}g_{BY}g_B + 5g_1^2 + 5 \left(g_B^2 + g_{YB}^2 + g_2^2 \right) + 5g_{BY}^2 \right) \text{Tr}\left(Y_e Y_e^\dagger\right) \\
& + 3400g_1^2 \text{Tr}\left(Y_u Y_u^\dagger\right) + 3400g_{BY}^2 \text{Tr}\left(Y_u Y_u^\dagger\right) + 1000\sqrt{10}g_{BY}g_B \text{Tr}\left(Y_u Y_u^\dagger\right) + 1000g_B^2 \text{Tr}\left(Y_u Y_u^\dagger\right) \\
& + 1000\sqrt{10}g_1g_{YB} \text{Tr}\left(Y_u Y_u^\dagger\right) + 1000g_{YB}^2 \text{Tr}\left(Y_u Y_u^\dagger\right) + 9000g_2^2 \text{Tr}\left(Y_u Y_u^\dagger\right) + 32000g_3^2 \text{Tr}\left(Y_u Y_u^\dagger\right) \\
& + 600g_1^2 \text{Tr}\left(Y_\nu Y_\nu^\dagger\right) + 600g_{BY}^2 \text{Tr}\left(Y_\nu Y_\nu^\dagger\right) + 600\sqrt{10}g_{BY}g_B \text{Tr}\left(Y_\nu Y_\nu^\dagger\right) + 3000g_B^2 \text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \\
& + 600\sqrt{10}g_1g_{YB} \text{Tr}\left(Y_\nu Y_\nu^\dagger\right) + 3000g_{YB}^2 \text{Tr}\left(Y_\nu Y_\nu^\dagger\right) + 3000g_2^2 \text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 10800\text{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger\right) \\
& + 2400\text{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger\right) - 3600\text{Tr}\left(Y_e Y_e^\dagger Y_e Y_e^\dagger\right) - 1200\text{Tr}\left(Y_S Y_S^\dagger Y_\nu^T Y_\nu^*\right) - 10800\text{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger\right) \\
& \left. - 3600\text{Tr}\left(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger\right) + 800\text{Tr}\left(Y_\nu Y_\nu^\dagger Y_e^T Y_e^*\right) \right) \tag{27}
\end{aligned}$$

$$\begin{aligned}
\beta_{Y_\nu}^{(1)} = & \frac{1}{20} \left(10 \left(-3Y_e^T Y_e^* Y_\nu + 3Y_\nu Y_\nu^\dagger Y_\nu + Y_\nu Y_S^* Y_S^T \right) \right. \\
& - Y_\nu \left(9g_1^2 + 9g_{BY}^2 + 9\sqrt{10}g_{BY}g_B + 45g_B^2 + 9\sqrt{10}g_1g_{YB} + 45g_{YB}^2 + 45g_2^2 - 60\text{Tr}\left(Y_d Y_d^\dagger\right) \right. \\
& \left. - 20\text{Tr}\left(Y_e Y_e^\dagger\right) - 60\text{Tr}\left(Y_u Y_u^\dagger\right) - 20\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \right) \tag{28}
\end{aligned}$$

$$\beta_{Y_\nu}^{(2)} = \frac{1}{1600} \left(10 \left(Y_\nu Y_S^* Y_S^T \left(-120\text{Tr}\left(Y_S Y_S^\dagger\right) + 144\sqrt{10}g_1g_{YB} + 144\sqrt{10}g_{BY}g_B - 160\lambda_5 + 825g_B^2 + 825g_{YB}^2 \right) \right) \right)$$

$$\begin{aligned}
& -2\left(10\left(-12Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu + 2Y_\nu Y_\nu^\dagger Y_e^T Y_e^* Y_\nu + Y_\nu Y_S^* Y_S^T Y_\nu^\dagger Y_\nu + Y_\nu Y_S^* Y_S^T Y_S^* Y_S^T\right.\right. \\
& + 8Y_e^T Y_e^* Y_\nu Y_\nu^\dagger Y_\nu - 22Y_e^T Y_e^* Y_e^T Y_e^* Y_\nu \left.\left.)\right) \\
& + Y_e^T Y_e^* Y_\nu \left(243g_1^2 + 243g_{BY}^2 + 171\sqrt{10}g_{BY}g_B + 360g_B^2 + 171\sqrt{10}g_1g_{YB} + 360g_{YB}^2 - 45g_2^2 - 300\text{Tr}\left(Y_d Y_d^\dagger\right)\right. \\
& - 100\text{Tr}\left(Y_e Y_e^\dagger\right) - 300\text{Tr}\left(Y_u Y_u^\dagger\right) - 100\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \left.\left.)\right) \\
& + 6Y_\nu Y_\nu^\dagger Y_\nu \left(93g_1^2 + 93g_{BY}^2 + 39\sqrt{10}g_{BY}g_B + 120g_B^2 + 39\sqrt{10}g_1g_{YB} + 120g_{YB}^2 + 225g_2^2 - 160\lambda_1\right. \\
& - 180\text{Tr}\left(Y_d Y_d^\dagger\right) - 60\text{Tr}\left(Y_e Y_e^\dagger\right) - 180\text{Tr}\left(Y_u Y_u^\dagger\right) - 60\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \left.\left.)\right) \\
& + Y_\nu \left(840g_1^4 + 1680g_1^2g_{BY}^2 + 840g_{BY}^4 + 3024\sqrt{10}g_1^2g_{BY}g_B + 3024\sqrt{10}g_{BY}^3g_B + 5610g_1^2g_B^2\right. \\
& + 25050g_{BY}^2g_B^2 + 10695\sqrt{10}g_{BY}g_B^3 + 30150g_B^4 + 3024\sqrt{10}g_1^3g_{YB} + 3024\sqrt{10}g_1g_{BY}^2g_{YB} \\
& + 38880g_1g_{BY}g_Bg_{YB} + 10695\sqrt{10}g_1g_B^2g_{YB} + 25050g_1^2g_{YB}^2 + 5610g_{BY}^2g_{YB}^2 + 10695\sqrt{10}g_{BY}g_Bg_{YB}^2 \\
& + 60300g_B^2g_{YB}^2 + 10695\sqrt{10}g_1g_{YB}^3 + 30150g_{YB}^4 - 2160g_1^2g_2^2 - 2160g_{BY}^2g_2^2 \\
& + 540\sqrt{10}g_{BY}g_Bg_2^2 + 4050g_B^2g_2^2 + 540\sqrt{10}g_1g_{YB}g_2^2 + 4050g_{YB}^2g_2^2 - 9200g_2^4 + 2400\lambda_1^2 \\
& + 200\lambda_5^2 \\
& + 200\left(5\left(32g_3^2 + 9g_2^2 + g_B^2 + g_{YB}^2\right) + 5g_1^2 + 5g_{BY}^2 - \sqrt{10}g_1g_{YB} - \sqrt{10}g_{BY}g_B\right)\text{Tr}\left(Y_d Y_d^\dagger\right) \\
& + 600\left(3\sqrt{10}g_1g_{YB} + 3\sqrt{10}g_{BY}g_B + 5g_1^2 + 5\left(g_B^2 + g_{YB}^2 + g_2^2\right) + 5g_{BY}^2\right)\text{Tr}\left(Y_e Y_e^\dagger\right) \\
& + 3400g_1^2\text{Tr}\left(Y_u Y_u^\dagger\right) + 3400g_{BY}^2\text{Tr}\left(Y_u Y_u^\dagger\right) + 1000\sqrt{10}g_{BY}g_B\text{Tr}\left(Y_u Y_u^\dagger\right) + 1000g_B^2\text{Tr}\left(Y_u Y_u^\dagger\right) \\
& + 1000\sqrt{10}g_1g_{YB}\text{Tr}\left(Y_u Y_u^\dagger\right) + 1000g_{YB}^2\text{Tr}\left(Y_u Y_u^\dagger\right) + 9000g_2^2\text{Tr}\left(Y_u Y_u^\dagger\right) + 32000g_3^2\text{Tr}\left(Y_u Y_u^\dagger\right) \\
& + 600g_1^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) + 600g_{BY}^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) + 600\sqrt{10}g_{BY}g_B\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) + 3000g_B^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \\
& + 600\sqrt{10}g_1g_{YB}\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) + 3000g_{YB}^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) + 3000g_2^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 10800\text{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger\right) \\
& + 2400\text{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger\right) - 3600\text{Tr}\left(Y_e Y_e^\dagger Y_e Y_e^\dagger\right) - 1200\text{Tr}\left(Y_S Y_S^\dagger Y_\nu^T Y_\nu^*\right) - 10800\text{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger\right) \\
& - 3600\text{Tr}\left(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger\right) + 800\text{Tr}\left(Y_\nu Y_\nu^\dagger Y_e^T Y_e^*\right) \left.\left.)\right) \tag{29}
\end{aligned}$$

$$\begin{aligned}
\beta_{Y_u}^{(1)} &= \frac{1}{20}\left(30\left(-Y_u Y_d^\dagger Y_d + Y_u Y_u^\dagger Y_u\right)\right. \\
& - Y_u \left(17g_1^2 + 17g_{BY}^2 + 5\sqrt{10}g_{BY}g_B + 5g_B^2 + 5\sqrt{10}g_1g_{YB} + 5g_{YB}^2 + 45g_2^2 + 160g_3^2 - 60\text{Tr}\left(Y_d Y_d^\dagger\right)\right. \\
& \left. - 20\text{Tr}\left(Y_e Y_e^\dagger\right) - 60\text{Tr}\left(Y_u Y_u^\dagger\right) - 20\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \left.\left.)\right) \tag{30}
\end{aligned}$$

$$\begin{aligned}
\beta_{Y_u}^{(2)} &= \frac{1}{4800}\left(-60\left(-20\left(11Y_u Y_d^\dagger Y_d Y_d^\dagger Y_d - 4Y_u Y_u^\dagger Y_u Y_d^\dagger Y_d + 6Y_u Y_u^\dagger Y_u Y_u^\dagger Y_u - Y_u Y_d^\dagger Y_d Y_u^\dagger Y_u\right)\right.\right. \\
& - Y_u Y_u^\dagger Y_u \left(223g_1^2 + 223g_{BY}^2 + 25\sqrt{10}g_{BY}g_B + 40g_B^2 + 25\sqrt{10}g_1g_{YB} + 40g_{YB}^2 + 675g_2^2 + 1280g_3^2 - 480\lambda_1\right. \\
& \left. - 540\text{Tr}\left(Y_d Y_d^\dagger\right) - 180\text{Tr}\left(Y_e Y_e^\dagger\right) - 540\text{Tr}\left(Y_u Y_u^\dagger\right) - 180\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \left.\left.)\right)
\end{aligned}$$

$$\begin{aligned}
& + Y_u Y_d^\dagger Y_d \left(43g_1^2 + 43g_{BY}^2 + 7\sqrt{10}g_{BY}g_B + 40g_B^2 + 7\sqrt{10}g_1g_{YB} + 40g_{YB}^2 - 45g_2^2 + 1280g_3^2 - 300\text{Tr}\left(Y_d Y_d^\dagger\right) \right. \\
& \left. - 100\text{Tr}\left(Y_e Y_e^\dagger\right) - 300\text{Tr}\left(Y_u Y_u^\dagger\right) - 100\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \right) \\
& + Y_u \left(9496g_1^4 + 18992g_1^2g_{BY}^2 + 9496g_{BY}^4 + 8032\sqrt{10}g_1^2g_{BY}g_B + 8032\sqrt{10}g_{BY}^3g_B + 8190g_1^2g_B^2 \right. \\
& + 44070g_{BY}^2g_B^2 + 11825\sqrt{10}g_{BY}g_B^3 + 10250g_B^4 + 8032\sqrt{10}g_1^3g_{YB} + 8032\sqrt{10}g_1g_{BY}^2g_{YB} \\
& + 71760g_1g_{BY}g_Bg_{YB} + 11825\sqrt{10}g_1g_B^2g_{YB} + 44070g_1^2g_{YB}^2 + 8190g_{BY}^2g_{YB}^2 + 11825\sqrt{10}g_{BY}g_Bg_{YB}^2 \\
& + 20500g_B^2g_{YB}^2 + 11825\sqrt{10}g_1g_{YB}^3 + 10250g_{YB}^4 - 2160g_1^2g_2^2 - 2160g_{BY}^2g_2^2 \\
& + 1620\sqrt{10}g_{BY}g_Bg_2^2 + 1350g_B^2g_2^2 + 1620\sqrt{10}g_1g_{YB}g_2^2 + 1350g_{YB}^2g_2^2 - 27600g_2^4 \\
& + 6080g_1^2g_3^2 + 6080g_{BY}^2g_3^2 - 1600\sqrt{10}g_{BY}g_Bg_3^2 - 1600g_B^2g_3^2 - 1600\sqrt{10}g_1g_{YB}g_3^2 \\
& \left. - 1600g_{YB}^2g_3^2 + 43200g_2^2g_3^2 - 518400g_3^4 + 7200\lambda_1^2 + 600\lambda_5^2 \right. \\
& + 600 \left(5 \left(32g_3^2 + 9g_2^2 + g_B^2 + g_{YB}^2 \right) + 5g_1^2 + 5g_{BY}^2 - \sqrt{10}g_1g_{YB} - \sqrt{10}g_{BY}g_B \right) \text{Tr}\left(Y_d Y_d^\dagger\right) \\
& + 1800 \left(3\sqrt{10}g_1g_{YB} + 3\sqrt{10}g_{BY}g_B + 5g_1^2 + 5 \left(g_B^2 + g_{YB}^2 + g_2^2 \right) + 5g_{BY}^2 \right) \text{Tr}\left(Y_e Y_e^\dagger\right) \\
& + 10200g_1^2\text{Tr}\left(Y_u Y_u^\dagger\right) + 10200g_{BY}^2\text{Tr}\left(Y_u Y_u^\dagger\right) + 3000\sqrt{10}g_{BY}g_B\text{Tr}\left(Y_u Y_u^\dagger\right) + 3000g_B^2\text{Tr}\left(Y_u Y_u^\dagger\right) \\
& + 3000\sqrt{10}g_1g_{YB}\text{Tr}\left(Y_u Y_u^\dagger\right) + 3000g_{YB}^2\text{Tr}\left(Y_u Y_u^\dagger\right) + 27000g_2^2\text{Tr}\left(Y_u Y_u^\dagger\right) + 96000g_3^2\text{Tr}\left(Y_u Y_u^\dagger\right) \\
& + 1800g_1^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) + 1800g_{BY}^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) + 1800\sqrt{10}g_{BY}g_B\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) + 9000g_B^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \\
& + 1800\sqrt{10}g_1g_{YB}\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) + 9000g_{YB}^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) + 9000g_2^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 32400\text{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger\right) \\
& + 7200\text{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger\right) - 10800\text{Tr}\left(Y_e Y_e^\dagger Y_e Y_e^\dagger\right) - 3600\text{Tr}\left(Y_S Y_S^\dagger Y_\nu^T Y_\nu^*\right) - 32400\text{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger\right) \\
& \left. - 10800\text{Tr}\left(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger\right) + 2400\text{Tr}\left(Y_\nu Y_\nu^\dagger Y_e^T Y_e^*\right) \right) \quad (31)
\end{aligned}$$

3.4 Fermion Mass Terms

$$\beta_{\mu_s 1}^{(1)} = -9 \left(g_B^2 + g_{YB}^2 \right) \mu_s 1 \quad (32)$$

$$\beta_{\mu_s 1}^{(2)} = \frac{3}{80} \left(1875g_B^4 + 320\sqrt{10}g_{BY}g_B^3 - 360g_B^2g_{YB}^2 + 844g_{BY}^2g_B^2 + g_{YB}^2 \left(1875g_{YB}^2 + 320\sqrt{10}g_1g_{YB} + 844g_1^2 \right) \right) \mu_s 1 \quad (33)$$

$$\beta_{\mu_s}^{(1)} = \frac{1}{2} \left(-18 \left(g_B^2 + g_{YB}^2 \right) \mu_s + \mu_s Y_S^\dagger Y_S + Y_S^T Y_S^* \mu_s \right) \quad (34)$$

$$\begin{aligned}
\beta_{\mu_s}^{(2)} &= \frac{1}{160} \left(6 \left(1875g_B^4 + 320\sqrt{10}g_{BY}g_B^3 - 360g_B^2g_{YB}^2 + 844g_{BY}^2g_B^2 + g_{YB}^2 \left(1875g_{YB}^2 + 320\sqrt{10}g_1g_{YB} + 844g_1^2 \right) \right) \mu_s \right. \\
&\quad \left. - 5 \left(4 \left(\mu_s Y_S^\dagger Y_S Y_S^\dagger Y_S + 2\mu_s Y_S^\dagger Y_\nu^T Y_\nu^* Y_S + 2Y_S^T Y_\nu^\dagger Y_\nu Y_S^* \mu_s - 16Y_S^T Y_S^* \mu_s Y_S^\dagger Y_S \right. \right. \right. \\
&\quad \left. \left. \left. + Y_S^T Y_S^* Y_S^T Y_S^* \mu_s \right) \right. \right. \\
&\quad \left. \left. + 3\mu_s Y_S^\dagger Y_S \left(59 \left(g_B^2 + g_{YB}^2 \right) + 8\text{Tr}\left(Y_S Y_S^\dagger\right) \right) + 3Y_S^T Y_S^* \mu_s \left(59 \left(g_B^2 + g_{YB}^2 \right) + 8\text{Tr}\left(Y_S Y_S^\dagger\right) \right) \right) \right) \quad (35)
\end{aligned}$$

3.5 Scalar Mass Terms

$$\beta_{\mu_2}^{(1)} = -16\text{Tr}\left(\mu_s Y_S^\dagger Y_S \mu_s^*\right) + 2\lambda_5 \mu_1 + 2\mu_2 \text{Tr}\left(Y_S Y_S^\dagger\right) + 8\lambda_3 \mu_2 - \frac{9}{4}g_B^2 \mu_2 - \frac{9}{4}g_{Y_B}^2 \mu_2 \quad (36)$$

$$\begin{aligned} \beta_{\mu_2}^{(2)} = & + \frac{9}{4}g_{BY}^2 g_B^2 \mu_1 + \frac{9}{2}g_1 g_{BY} g_B g_{Y_B} \mu_1 + \frac{9}{4}g_1^2 g_{Y_B}^2 \mu_1 + \frac{12}{5}g_1^2 \lambda_5 \mu_1 + \frac{12}{5}g_{BY}^2 \lambda_5 \mu_1 + 12g_2^2 \lambda_5 \mu_1 \\ & - 2\lambda_5^2 \mu_1 + \frac{633}{80}g_{BY}^2 g_B^2 \mu_2 + 3\sqrt{10}g_{BY} g_B^3 \mu_2 + \frac{729}{32}g_B^4 \mu_2 + \frac{633}{80}g_1^2 g_{Y_B}^2 \mu_2 + \frac{225}{32}g_B^2 g_{Y_B}^2 \mu_2 \\ & + 3\sqrt{10}g_1 g_{Y_B}^3 \mu_2 + \frac{729}{32}g_{Y_B}^4 \mu_2 + 24g_B^2 \lambda_3 \mu_2 + 24g_{Y_B}^2 \lambda_3 \mu_2 - 40\lambda_3^2 \mu_2 - \frac{1}{2}\lambda_5^2 \mu_2 \\ & + 36\left(g_B^2 + g_{Y_B}^2\right)^2 \text{Tr}\left(\mu_s \mu_s^*\right) + 36\left(g_B^2 + g_{Y_B}^2\right)^2 \text{Tr}\left(\mu_s 1 \mu_s^*\right) - 12\lambda_5 \mu_1 \text{Tr}\left(Y_d Y_d^\dagger\right) - 4\lambda_5 \mu_1 \text{Tr}\left(Y_e Y_e^\dagger\right) \\ & + \frac{75}{8}g_B^2 \mu_2 \text{Tr}\left(Y_S Y_S^\dagger\right) + \frac{75}{8}g_{Y_B}^2 \mu_2 \text{Tr}\left(Y_S Y_S^\dagger\right) - 16\lambda_3 \mu_2 \text{Tr}\left(Y_S Y_S^\dagger\right) - 12\lambda_5 \mu_1 \text{Tr}\left(Y_u Y_u^\dagger\right) \\ & - 4\lambda_5 \mu_1 \text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 72g_B^2 \text{Tr}\left(\mu_s Y_S^\dagger Y_S \mu_s^*\right) - 72g_{Y_B}^2 \text{Tr}\left(\mu_s Y_S^\dagger Y_S \mu_s^*\right) - 3\mu_2 \text{Tr}\left(Y_S Y_S^\dagger Y_S Y_S^\dagger\right) \\ & - 3\mu_2 \text{Tr}\left(Y_S Y_S^\dagger Y_\nu^T Y_\nu^* Y_\nu^* Y_\nu^T Y_\nu^*\right) + 40\text{Tr}\left(\mu_s Y_S^\dagger Y_S Y_S^\dagger Y_S \mu_s^*\right) + 24\text{Tr}\left(\mu_s Y_S^\dagger Y_S \mu_s^* Y_S^T Y_S^*\right) \\ & + 16\text{Tr}\left(\mu_s Y_S^\dagger Y_\nu^T Y_\nu^* Y_S \mu_s^*\right) \end{aligned} \quad (37)$$

$$\begin{aligned} \beta_{\mu_1}^{(1)} = & -\frac{9}{10}g_1^2 \mu_1 - \frac{9}{10}g_{BY}^2 \mu_1 - \frac{9}{2}g_2^2 \mu_1 + 6\lambda_1 \mu_1 + \lambda_5 \mu_2 + 6\mu_1 \text{Tr}\left(Y_d Y_d^\dagger\right) + 2\mu_1 \text{Tr}\left(Y_e Y_e^\dagger\right) \\ & + 6\mu_1 \text{Tr}\left(Y_u Y_u^\dagger\right) + 2\mu_1 \text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \end{aligned} \quad (38)$$

$$\begin{aligned} \beta_{\mu_1}^{(2)} = & + \frac{1671}{400}g_1^4 \mu_1 + \frac{81}{40}g_1^2 g_{BY}^2 \mu_1 + \frac{1671}{400}g_{BY}^4 \mu_1 + 6\sqrt{\frac{2}{5}}g_{BY}^3 g_B \mu_1 + \frac{1233}{160}g_{BY}^2 g_B^2 \mu_1 + 6\sqrt{\frac{2}{5}}g_1^3 g_{Y_B} \mu_1 + \frac{1233}{160}g_1^2 g_{Y_B}^2 \mu_1 \\ & + \frac{9}{8}g_1^2 g_2^2 \mu_1 + \frac{9}{8}g_{BY}^2 g_2^2 \mu_1 - \frac{145}{16}g_2^4 \mu_1 + \frac{36}{5}g_1^2 \lambda_1 \mu_1 + \frac{36}{5}g_{BY}^2 \lambda_1 \mu_1 + 36g_2^2 \lambda_1 \mu_1 - 15\lambda_1^2 \mu_1 \\ & - \frac{1}{4}\lambda_5^2 \mu_1 + \frac{9}{8}g_{BY}^2 g_B^2 \mu_2 + \frac{9}{4}g_1 g_{BY} g_B g_{Y_B} \mu_2 + \frac{9}{8}g_1^2 g_{Y_B}^2 \mu_2 + 3g_B^2 \lambda_5 \mu_2 + 3g_{Y_B}^2 \lambda_5 \mu_2 - \lambda_5^2 \mu_2 \\ & + \frac{72}{5}\left(g_1 g_{Y_B} + g_{BY} g_B\right)^2 \text{Tr}\left(\mu_s \mu_s^*\right) + \frac{72}{5}\left(g_1 g_{Y_B} + g_{BY} g_B\right)^2 \text{Tr}\left(\mu_s 1 \mu_s^*\right) + \frac{5}{4}g_1^2 \mu_1 \text{Tr}\left(Y_d Y_d^\dagger\right) \\ & + \frac{5}{4}g_{BY}^2 \mu_1 \text{Tr}\left(Y_d Y_d^\dagger\right) - \frac{1}{2}\sqrt{\frac{5}{2}}g_{BY} g_B \mu_1 \text{Tr}\left(Y_d Y_d^\dagger\right) + \frac{5}{4}g_B^2 \mu_1 \text{Tr}\left(Y_d Y_d^\dagger\right) \\ & - \frac{1}{2}\sqrt{\frac{5}{2}}g_1 g_{Y_B} \mu_1 \text{Tr}\left(Y_d Y_d^\dagger\right) + \frac{5}{4}g_{Y_B}^2 \mu_1 \text{Tr}\left(Y_d Y_d^\dagger\right) + \frac{45}{4}g_2^2 \mu_1 \text{Tr}\left(Y_d Y_d^\dagger\right) + 40g_3^2 \mu_1 \text{Tr}\left(Y_d Y_d^\dagger\right) \\ & - 36\lambda_1 \mu_1 \text{Tr}\left(Y_d Y_d^\dagger\right) + \frac{15}{4}g_1^2 \mu_1 \text{Tr}\left(Y_e Y_e^\dagger\right) + \frac{15}{4}g_{BY}^2 \mu_1 \text{Tr}\left(Y_e Y_e^\dagger\right) + \frac{9}{2}\sqrt{\frac{5}{2}}g_{BY} g_B \mu_1 \text{Tr}\left(Y_e Y_e^\dagger\right) \\ & + \frac{15}{4}g_B^2 \mu_1 \text{Tr}\left(Y_e Y_e^\dagger\right) + \frac{9}{2}\sqrt{\frac{5}{2}}g_1 g_{Y_B} \mu_1 \text{Tr}\left(Y_e Y_e^\dagger\right) + \frac{15}{4}g_{Y_B}^2 \mu_1 \text{Tr}\left(Y_e Y_e^\dagger\right) + \frac{15}{4}g_2^2 \mu_1 \text{Tr}\left(Y_e Y_e^\dagger\right) \\ & - 12\lambda_1 \mu_1 \text{Tr}\left(Y_e Y_e^\dagger\right) - 2\lambda_5 \mu_2 \text{Tr}\left(Y_S Y_S^\dagger\right) + \frac{17}{4}g_1^2 \mu_1 \text{Tr}\left(Y_u Y_u^\dagger\right) + \frac{17}{4}g_{BY}^2 \mu_1 \text{Tr}\left(Y_u Y_u^\dagger\right) \end{aligned}$$

$$\begin{aligned}
& + \frac{5}{2}\sqrt{\frac{5}{2}}g_{BY}g_B\mu_1\text{Tr}\left(Y_uY_u^\dagger\right) + \frac{5}{4}g_B^2\mu_1\text{Tr}\left(Y_uY_u^\dagger\right) + \frac{5}{2}\sqrt{\frac{5}{2}}g_1g_{YB}\mu_1\text{Tr}\left(Y_uY_u^\dagger\right) \\
& + \frac{5}{4}g_{YB}^2\mu_1\text{Tr}\left(Y_uY_u^\dagger\right) + \frac{45}{4}g_2^2\mu_1\text{Tr}\left(Y_uY_u^\dagger\right) + 40g_3^2\mu_1\text{Tr}\left(Y_uY_u^\dagger\right) - 36\lambda_1\mu_1\text{Tr}\left(Y_uY_u^\dagger\right) \\
& + \frac{3}{4}g_1^2\mu_1\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) + \frac{3}{4}g_{BY}^2\mu_1\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) + \frac{3}{2}\sqrt{\frac{5}{2}}g_{BY}g_B\mu_1\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) + \frac{15}{4}g_B^2\mu_1\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \\
& + \frac{3}{2}\sqrt{\frac{5}{2}}g_1g_{YB}\mu_1\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) + \frac{15}{4}g_{YB}^2\mu_1\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) + \frac{15}{4}g_2^2\mu_1\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 12\lambda_1\mu_1\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \\
& - \frac{27}{2}\mu_1\text{Tr}\left(Y_dY_d^\dagger Y_dY_d^\dagger\right) - 21\mu_1\text{Tr}\left(Y_dY_u^\dagger Y_uY_d^\dagger\right) - \frac{9}{2}\mu_1\text{Tr}\left(Y_eY_e^\dagger Y_eY_e^\dagger\right) - \frac{3}{2}\mu_1\text{Tr}\left(Y_SY_S^\dagger Y_\nu^T Y_\nu^*\right) \\
& - \frac{27}{2}\mu_1\text{Tr}\left(Y_uY_u^\dagger Y_uY_u^\dagger\right) - \frac{9}{2}\mu_1\text{Tr}\left(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger\right) - 7\mu_1\text{Tr}\left(Y_\nu Y_\nu^\dagger Y_e^T Y_e^*\right) \\
& + 16\text{Tr}\left(\mu_s Y_S^\dagger Y_\nu^T Y_\nu^* Y_S \mu_s^*\right)
\end{aligned} \tag{39}$$

3.6 Vacuum expectation values

$$\begin{aligned}
\beta_v^{(1)} &= \frac{1}{20}v\left(9g_1^2 + 9g_{BY}^2 + 45g_2^2 + 3g_1^2\text{Xi} + 3g_{BY}^2\text{Xi} + 15g_2^2\text{Xi} - 60\text{Tr}\left(Y_dY_d^\dagger\right) - 20\text{Tr}\left(Y_eY_e^\dagger\right) - 60\text{Tr}\left(Y_uY_u^\dagger\right) \right. \\
& \left. - 20\text{Tr}\left(Y_\nu Y_\nu^\dagger\right)\right)
\end{aligned} \tag{40}$$

$$\begin{aligned}
\beta_v^{(2)} &= \frac{1}{1600}v\left(-2586g_1^4 - 108g_1^2g_{BY}^2 - 2586g_{BY}^4 - 960\sqrt{10}g_{BY}^3g_B - 6165g_{BY}^2g_B^2 - 960\sqrt{10}g_1^3g_{YB} \right. \\
& - 6165g_1^2g_{YB}^2 - 540g_1^2g_2^2 - 540g_{BY}^2g_2^2 + 13550g_4^2 - 2400\lambda_1^2 - 200\lambda_5^2 + 36g_1^4\text{Xi} + 72g_1^2g_{BY}^2\text{Xi} \\
& + 36g_{BY}^4\text{Xi} + 360g_1^2g_2^2\text{Xi} + 360g_{BY}^2g_2^2\text{Xi} + 4500g_2^4\text{Xi} + 36g_1^4\text{Xi}^2 + 72g_1^2g_{BY}^2\text{Xi}^2 + 36g_{BY}^4\text{Xi}^2 + 360g_1^2g_2^2\text{Xi}^2 \\
& + 360g_{BY}^2g_2^2\text{Xi}^2 - 900g_2^4\text{Xi}^2 \\
& - 40\left(5\left(160g_3^2 + 18g_2^2\text{Xi} + 45g_2^2 + 5g_B^2 + 5g_{YB}^2\right) - 5\sqrt{10}g_1g_{YB} - 5\sqrt{10}g_{BY}g_B + g_1^2\left(18\text{Xi} + 25\right) + g_{BY}^2\left(18\text{Xi} + 25\right)\right)\text{Tr}\left(Y_dY_d^\dagger\right) \\
& - 120\left(15\sqrt{10}g_1g_{YB} + 15\sqrt{10}g_{BY}g_B + 5\left(5g_B^2 + 5g_{YB}^2 + g_2^2\left(2\text{Xi} + 5\right)\right) + g_1^2\left(2\text{Xi} + 25\right) + g_{BY}^2\left(2\text{Xi} + 25\right)\right)\text{Tr}\left(Y_eY_e^\dagger\right) \\
& - 3400g_1^2\text{Tr}\left(Y_uY_u^\dagger\right) - 3400g_{BY}^2\text{Tr}\left(Y_uY_u^\dagger\right) - 1000\sqrt{10}g_{BY}g_B\text{Tr}\left(Y_uY_u^\dagger\right) - 1000g_B^2\text{Tr}\left(Y_uY_u^\dagger\right) \\
& - 1000\sqrt{10}g_1g_{YB}\text{Tr}\left(Y_uY_u^\dagger\right) - 1000g_{YB}^2\text{Tr}\left(Y_uY_u^\dagger\right) - 9000g_2^2\text{Tr}\left(Y_uY_u^\dagger\right) \\
& - 32000g_3^2\text{Tr}\left(Y_uY_u^\dagger\right) - 720g_1^2\text{Xi}\text{Tr}\left(Y_uY_u^\dagger\right) - 720g_{BY}^2\text{Xi}\text{Tr}\left(Y_uY_u^\dagger\right) - 3600g_2^2\text{Xi}\text{Tr}\left(Y_uY_u^\dagger\right) \\
& - 600g_1^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 600g_{BY}^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 600\sqrt{10}g_{BY}g_B\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 3000g_B^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \\
& - 600\sqrt{10}g_1g_{YB}\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 3000g_{YB}^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 3000g_2^2\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) \\
& - 240g_1^2\text{Xi}\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 240g_{BY}^2\text{Xi}\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 1200g_2^2\text{Xi}\text{Tr}\left(Y_\nu Y_\nu^\dagger\right) + 10800\text{Tr}\left(Y_dY_d^\dagger Y_dY_d^\dagger\right) \\
& - 2400\text{Tr}\left(Y_dY_u^\dagger Y_uY_d^\dagger\right) + 3600\text{Tr}\left(Y_eY_e^\dagger Y_eY_e^\dagger\right) + 1200\text{Tr}\left(Y_SY_S^\dagger Y_\nu^T Y_\nu^*\right) + 10800\text{Tr}\left(Y_uY_u^\dagger Y_uY_u^\dagger\right)
\end{aligned}$$

$$+ 3600\text{Tr}\left(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger\right) - 800\text{Tr}\left(Y_\nu Y_\nu^\dagger Y_e^T Y_e^*\right) \quad (41)$$

$$\beta_x^{(1)} = -x\text{Tr}\left(Y_S Y_S^\dagger\right) \quad (42)$$

$$\beta_x^{(2)} = -\frac{1}{16}x\left(4\left(16\lambda_3^2 - 6\text{Tr}\left(Y_S Y_S^\dagger Y_\nu^T Y_\nu^*\right) - 6\text{Tr}\left(Y_S Y_S^\dagger Y_S Y_S^\dagger\right) + \lambda_5^2\right) + 75\left(g_B^2 + g_{YB}^2\right)\text{Tr}\left(Y_S Y_S^\dagger\right)\right) \quad (43)$$

4 Field Rotations

4.1 Rotations in gauge sector for eigenstates 'EWSB'

$$\begin{pmatrix} B_\rho \\ W_{3\rho} \\ \text{VBp}(\{\text{lt}1\}) \end{pmatrix} = Z^{\gamma ZZ'} \begin{pmatrix} \gamma_\rho \\ Z_\rho \\ Z'_\rho \end{pmatrix} \quad (44)$$

$$\begin{pmatrix} W_{1\rho} \\ W_{2\rho} \end{pmatrix} = Z^W \begin{pmatrix} W_\rho^- \\ W_\rho^- \end{pmatrix} \quad (45)$$

$$(46)$$

The mixing matrices are parametrized by

$$Z^{\gamma ZZ'} = \begin{pmatrix} \cos \Theta_W & -\cos \Theta'_W \sin \Theta_W & \sin \Theta_W \sin \Theta'_W \\ \sin \Theta_W & \cos \Theta_W \cos \Theta'_W & -\cos \Theta_W \sin \Theta'_W \\ 0 & \sin \Theta'_W & \cos \Theta'_W \end{pmatrix} \quad (47)$$

$$Z^W = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -i\frac{1}{\sqrt{2}} & i\frac{1}{\sqrt{2}} \end{pmatrix} \quad (48)$$

$$(49)$$

4.2 Rotations in Mass sector for eigenstates 'EWSB'

4.2.1 Mass Matrices for Scalars

- **Mass matrix for Higgs**, Basis: $(\text{phiH}, \text{phiX}), (\text{phiH}, \text{phiX})$

$$m_h^2 = \begin{pmatrix} \frac{1}{4}(6\lambda_1 v^2 + \lambda_5 x^2) + \mu_1 & \frac{1}{2}\lambda_5 v x \\ \frac{1}{2}\lambda_5 v x & 3\lambda_3 x^2 + \frac{1}{4}\lambda_5 v^2 + \mu_2 \end{pmatrix} \quad (50)$$

This matrix is diagonalized by Z^H :

$$Z^H m_h^2 Z^{H,\dagger} = m_{2,h}^{dia} \quad (51)$$

with

$$\text{phiH} = \sum_j Z_{j1}^H h_j, \quad \text{phiX} = \sum_j Z_{j2}^H h_j \quad (52)$$

- **Mass matrix for Pseudo-Scalar Higgs**, Basis: $(\text{sigmaH}, \text{sigmaX}), (\text{sigmaH}, \text{sigmaX})$

$$m_{A_h}^2 = \begin{pmatrix} \frac{1}{4}(2\lambda_1 v^2 + \lambda_5 x^2) + \mu_1 & 0 \\ 0 & \frac{1}{4}\lambda_5 v^2 + \lambda_3 x^2 + \mu_2 \end{pmatrix} + \xi_Z m^2(Z) + \xi_{Z'} m^2(Z') \quad (53)$$

Gauge fixing contributions:

$$m^2(\xi_Z) = \begin{pmatrix} m_{\text{sigmaHsigmaH}} & m_{\text{sigmaXsigmaH}} \\ m_{\text{sigmaHsigmaX}} & m_{\text{sigmaXsigmaX}} \end{pmatrix} \quad (54)$$

$$m_{\text{sigmaHsigmaH}} = \frac{1}{4}v^2 \left(\cos \Theta'_W (g_1 \sin \Theta_W + g_2 \cos \Theta_W) - g_{BY} \sin \Theta'_W \right)^2 \quad (55)$$

$$m_{\text{sigmaHsigmaX}} = -\frac{1}{4}vx \left(\cos \Theta'_W (g_1 \sin \Theta_W + g_2 \cos \Theta_W) - g_{BY} \sin \Theta'_W \right) \left(-g_B \sin \Theta'_W + g_{YB} \cos \Theta'_W \sin \Theta_W \right) \quad (56)$$

$$m_{\text{sigmaXsigmaX}} = \frac{1}{4}x^2 \left(-g_B \sin \Theta'_W + g_{YB} \cos \Theta'_W \sin \Theta_W \right)^2 \quad (57)$$

$$m^2(\xi_{Z'}) = \begin{pmatrix} m_{\text{sigmaHsigmaH}} & m_{\text{sigmaXsigmaH}} \\ m_{\text{sigmaHsigmaX}} & m_{\text{sigmaXsigmaX}} \end{pmatrix} \quad (58)$$

$$m_{\text{sigmaHsigmaH}} = \frac{1}{4}v^2 \left((g_1 \sin \Theta_W + g_2 \cos \Theta_W) \sin \Theta'_W + g_{BY} \cos \Theta'_W \right)^2 \quad (59)$$

$$m_{\text{sigmaHsigmaX}} = -\frac{1}{4}vx \left(g_B \cos \Theta'_W + g_{YB} \sin \Theta_W \sin \Theta'_W \right) \left((g_1 \sin \Theta_W + g_2 \cos \Theta_W) \sin \Theta'_W + g_{BY} \cos \Theta'_W \right) \quad (60)$$

$$m_{\text{sigmaXsigmaX}} = \frac{1}{4}x^2 \left(g_B \cos \Theta'_W + g_{YB} \sin \Theta_W \sin \Theta'_W \right)^2 \quad (61)$$

This matrix is diagonalized by Z^A :

$$Z^A m_{A_h}^2 Z^{A,\dagger} = m_{2,A_h}^{dia} \quad (62)$$

with

$$\text{sigmaH} = \sum_j Z_{j1}^A A_{h,j}, \quad \text{sigmaX} = \sum_j Z_{j2}^A A_{h,j} \quad (63)$$

4.2.2 Mass Matrices for Fermions

- **Mass matrix for Down-Quarks**, Basis: $(\text{dL}(\{\text{cm1}\}), (\text{conj}(\text{dR}(\{\text{cn1}\}))))$

$$m_d = \left(\frac{1}{\sqrt{2}} v \delta_{\alpha_1 \beta_1} Y_d^T \right) \quad (64)$$

This matrix is diagonalized by U_L^d and U_R^d

$$U_L^{d,*} m_d U_R^{d,\dagger} = m_d^{dia} \quad (65)$$

with

$$\text{dL}(\{\text{gt1}, \text{ct1}\}) = \sum_{t_2} U_{L,ji}^{d,*} \text{FDL}(\{\text{gt2}, \text{ct1}\}) \quad (66)$$

$$\text{dR}(\{\text{gt1}, \text{ct1}\}) = \sum_{t_2} U_{R,ij}^d \text{conj}(\text{FDR}(\{\text{gt2}, \text{ct1}\})) \quad (67)$$

- **Mass matrix for Up-Quarks**, Basis: $(\text{uL}(\{\text{cm1}\}), (\text{conj}(\text{uR}(\{\text{cn1}\}))))$

$$m_u = \left(\frac{1}{\sqrt{2}} v \delta_{\alpha_1 \beta_1} Y_u^T \right) \quad (68)$$

This matrix is diagonalized by U_L^u and U_R^u

$$U_L^{u,*} m_u U_R^{u,\dagger} = m_u^{dia} \quad (69)$$

with

$$\text{uL}(\{\text{gt1}, \text{ct1}\}) = \sum_{t_2} U_{L,ji}^{u,*} \text{FUL}(\{\text{gt2}, \text{ct1}\}) \quad (70)$$

$$\text{uR}(\{\text{gt1}, \text{ct1}\}) = \sum_{t_2} U_{R,ij}^u \text{conj}(\text{FUR}(\{\text{gt2}, \text{ct1}\})) \quad (71)$$

- **Mass matrix for Leptons**, Basis: $(\text{eL}), (\text{conj}(\text{eR}))$

$$m_e = \left(\frac{1}{\sqrt{2}} v Y_e^T \right) \quad (72)$$

This matrix is diagonalized by U_L^e and U_R^e

$$U_L^{e,*} m_e U_R^{e,\dagger} = m_e^{dia} \quad (73)$$

with

$$\text{eL}(\{\text{gt1}\}) = \sum_{t_2} U_{L,ji}^{e,*} \text{FEL}(\{\text{gt2}\}) \quad (74)$$

$$\text{eR}(\{\text{gt1}\}) = \sum_{t_2} U_{R,ij}^e \text{conj}(\text{FER}(\{\text{gt2}\})) \quad (75)$$

- **Mass matrix for Neutrinos**, Basis: $(\nu_L, \text{conj}(\nu_R), S_2), (\nu_L, \text{conj}(\nu_R), S_2)$

$$m_\nu = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}}vY_\nu & 0 \\ \frac{1}{\sqrt{2}}vY_\nu^T & 0 & \frac{1}{\sqrt{2}}xY_S \\ 0 & \frac{1}{\sqrt{2}}xY_S^T & 2\mu_s \end{pmatrix} \quad (76)$$

This matrix is diagonalized by U^V :

$$U^{V,*}m_\nu U^{V,\dagger} = m_\nu^{dia} \quad (77)$$

with

$$\nu_L(\{\text{gt1}\}) = \sum_j U_{ji}^{V,*} F_{vm}(\{\text{gt2}\}), \quad \nu_R(\{\text{gt1}\}) = \sum_j U_{ji}^V \text{conj}(F_{vm}(\{\text{gt2}\})), \quad S_2(\{\text{gt1}\}) = \sum_j U_{ji}^{V,*} F_{vm}(\{\text{gt2}\}) \quad (78)$$

- **Mass matrix for s neutrino**, Basis: $(S_1), (S_1)$

$$m_{S_1} = \begin{pmatrix} 2\mu_s 1 \end{pmatrix} \quad (79)$$

This matrix is diagonalized by SS :

$$SS^*m_{S_1}SS^\dagger = m_{S_1}^{dia} \quad (80)$$

with

$$S_1(\{\text{gt1}\}) = \sum_j SS_{ji}^* F_{s1}(\{\text{gt2}\}) \quad (81)$$

5 Vacuum Expectation Values

$$H^0 = \frac{1}{\sqrt{2}}\text{phiH} + \frac{1}{\sqrt{2}}v + i\frac{1}{\sqrt{2}}\text{sigmaH} \quad (82)$$

$$X_1 = \frac{1}{\sqrt{2}}\text{phiX} + \frac{1}{\sqrt{2}}x + i\frac{1}{\sqrt{2}}\text{sigmaX} \quad (83)$$

6 Tadpole Equations

$$\frac{\partial V}{\partial \text{phiH}} = \frac{1}{4}v(2\lambda_1 v^2 + 4\mu_1 + \lambda_5 x^2) \quad (84)$$

$$\frac{\partial V}{\partial \text{phiX}} = \left(\frac{1}{4}\lambda_5 v^2 + \mu_2\right)x + \lambda_3 x^3 \quad (85)$$

7 Particle content for eigenstates 'EWSB'

Name	Type	complex/real	Generations	Indices
H^-	Scalar	complex	1	
h	Scalar	real	2	generation, 2
A_h	Scalar	real	2	generation, 2
d	Fermion	Dirac	3	generation, 3, color, 3
u	Fermion	Dirac	3	generation, 3, color, 3
e	Fermion	Dirac	3	generation, 3
ν	Fermion	Majorana	9	generation, 9
S_1	Fermion	Majorana	3	generation, 3
g	Vector	real	1	color, 8, lorentz, 4
γ	Vector	real	1	lorentz, 4
Z	Vector	real	1	lorentz, 4
Z'	Vector	real	1	lorentz, 4
W^-	Vector	complex	1	lorentz, 4
η^G	Ghost	real	1	color, 8
η^γ	Ghost	real	1	
η^Z	Ghost	real	1	
$\eta^{Z'}$	Ghost	real	1	
η^-	Ghost	complex	1	
η^+	Ghost	complex	1	

8 One Loop Self-Energy and One Loop Tadpoles for eigenstates 'EWSB'

8.1 One Loop Self-Energy

- Self-Energy for Higgs (h)

$$\begin{aligned}
\Pi_{i,j}(p^2) = & +2\left(-\frac{1}{2}\text{rMS} + B_0(p^2, 0, 0)\right)\Gamma_{\tilde{h}_j, \gamma, \gamma}^* \Gamma_{\tilde{h}_i, \gamma, \gamma} + 4\left(-\frac{1}{2}\text{rMS} + B_0(p^2, 0, m_Z^2)\right)\Gamma_{\tilde{h}_j, Z, \gamma}^* \Gamma_{\tilde{h}_i, Z, \gamma} + 2\left(-\frac{1}{2}\text{rMS} + B_0(p^2, m_Z^2, m_{Z'}^2)\right)\Gamma_{\tilde{h}_j, Z', \gamma}^* \Gamma_{\tilde{h}_i, Z', \gamma} \\
& + 4\left(-\frac{1}{2}\text{rMS} + B_0(p^2, 0, m_{Z'}^2)\right)\Gamma_{\tilde{h}_j, Z', \gamma}^* \Gamma_{\tilde{h}_i, Z', \gamma} + 4\left(-\frac{1}{2}\text{rMS} + B_0(p^2, m_Z^2, m_{Z'}^2)\right)\Gamma_{\tilde{h}_j, Z', Z}^* \Gamma_{\tilde{h}_i, Z', Z} + 2\left(-\frac{1}{2}\text{rMS} + B_0(p^2, m_{H^-}^2, m_{H^-}^2)\right)\Gamma_{\tilde{h}_j, H^-, * , H^-}^* \Gamma_{\tilde{h}_i, H^-, * , H^-} \\
& + B_0(p^2, m_{H^-}^2, m_{H^-}^2)\Gamma_{\tilde{h}_j, H^-, * , H^-}^* \Gamma_{\tilde{h}_i, H^-, * , H^-} + 4\left(-\frac{1}{2}\text{rMS} + B_0(p^2, m_{W^-}^2, m_{W^-}^2)\right)\Gamma_{\tilde{h}_j, W^+, W^-}^* \Gamma_{\tilde{h}_i, W^+, W^-} \\
& - B_0(p^2, m_{\eta^-}^2, m_{\eta^-}^2)\Gamma_{\tilde{h}_i, \eta^-, \eta^-} \Gamma_{\tilde{h}_j, \eta^-, \eta^-} - B_0(p^2, m_{\eta^+}^2, m_{\eta^+}^2)\Gamma_{\tilde{h}_i, \eta^+, \eta^+} \Gamma_{\tilde{h}_j, \eta^+, \eta^+}
\end{aligned}$$

$$\begin{aligned}
& - B_0(p^2, m_{\eta^Z}^2, m_{\eta^Z}^2) \Gamma_{\tilde{h}_i, \tilde{\eta}^Z, \eta^Z} \Gamma_{\tilde{h}_j, \tilde{\eta}^Z, \eta^Z} - 2B_0(p^2, m_{\eta^Z}^2, m_{\eta^{Z'}}^2) \Gamma_{\tilde{h}_i, \tilde{\eta}^{Z'}, \eta^Z} \Gamma_{\tilde{h}_j, \tilde{\eta}^{Z'}, \eta^Z} \\
& - B_0(p^2, m_{\eta^{Z'}}^2, m_{\eta^{Z'}}^2) \Gamma_{\tilde{h}_i, \tilde{\eta}^{Z'}, \eta^{Z'}} \Gamma_{\tilde{h}_j, \tilde{\eta}^{Z'}, \eta^{Z'}} - A_0(m_{H^-}^2) \Gamma_{\tilde{h}_i, \tilde{h}_j, H^-, *, H^-} \\
& + 2\Gamma_{\tilde{h}_j, W^+, H^-}^* \Gamma_{\tilde{h}_i, W^+, H^-} F_0(p^2, m_{H^-}^2, m_{W^-}^2) + 4\Gamma_{\tilde{h}_i, \tilde{h}_j, W^+, W^-} \left(-\frac{1}{2} \text{rMS} m_{W^-}^2 + A_0(m_{W^-}^2) \right) \\
& + 2\Gamma_{\tilde{h}_i, \tilde{h}_j, Z} \left(-\frac{1}{2} \text{rMS} m_Z^2 + A_0(m_Z^2) \right) + 2\Gamma_{\tilde{h}_i, \tilde{h}_j, Z', Z'} \left(-\frac{1}{2} \text{rMS} m_{Z'}^2 + A_0(m_{Z'}^2) \right) - \frac{1}{2} \sum_{a=1}^2 A_0(m_{A_{h,a}}^2) \Gamma_{\tilde{h}_i, \tilde{h}_j, A_{h,a}, A_{h,a}} \\
& - \frac{1}{2} \sum_{a=1}^2 A_0(m_{h_a}^2) \Gamma_{\tilde{h}_i, \tilde{h}_j, h_a, h_a} + \frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^2 B_0(p^2, m_{A_{h,a}}^2, m_{A_{h,b}}^2) \Gamma_{\tilde{h}_j, A_{h,a}, A_{h,b}}^* \Gamma_{\tilde{h}_i, A_{h,a}, A_{h,b}} \\
& + \frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^2 B_0(p^2, m_{h_a}^2, m_{h_b}^2) \Gamma_{\tilde{h}_j, h_a, h_b}^* \Gamma_{\tilde{h}_i, h_a, h_b} \\
& - 6 \sum_{a=1}^3 m_{d_a} \sum_{b=1}^3 B_0(p^2, m_{d_a}^2, m_{d_b}^2) m_{d_b} \left(\Gamma_{\tilde{h}_j, \bar{d}_a, d_b}^{L*} \Gamma_{\tilde{h}_i, \bar{d}_a, d_b}^R + \Gamma_{\tilde{h}_j, \bar{d}_a, d_b}^{R*} \Gamma_{\tilde{h}_i, \bar{d}_a, d_b}^L \right) \\
& + 3 \sum_{a=1}^3 \sum_{b=1}^3 G_0(p^2, m_{d_a}^2, m_{d_b}^2) \left(\Gamma_{\tilde{h}_j, \bar{d}_a, d_b}^{L*} \Gamma_{\tilde{h}_i, \bar{d}_a, d_b}^L + \Gamma_{\tilde{h}_j, \bar{d}_a, d_b}^{R*} \Gamma_{\tilde{h}_i, \bar{d}_a, d_b}^R \right) \\
& - 2 \sum_{a=1}^3 m_{e_a} \sum_{b=1}^3 B_0(p^2, m_{e_a}^2, m_{e_b}^2) m_{e_b} \left(\Gamma_{\tilde{h}_j, \bar{e}_a, e_b}^{L*} \Gamma_{\tilde{h}_i, \bar{e}_a, e_b}^R + \Gamma_{\tilde{h}_j, \bar{e}_a, e_b}^{R*} \Gamma_{\tilde{h}_i, \bar{e}_a, e_b}^L \right) \\
& + \sum_{a=1}^3 \sum_{b=1}^3 G_0(p^2, m_{e_a}^2, m_{e_b}^2) \left(\Gamma_{\tilde{h}_j, \bar{e}_a, e_b}^{L*} \Gamma_{\tilde{h}_i, \bar{e}_a, e_b}^L + \Gamma_{\tilde{h}_j, \bar{e}_a, e_b}^{R*} \Gamma_{\tilde{h}_i, \bar{e}_a, e_b}^R \right) \\
& - 6 \sum_{a=1}^3 m_{u_a} \sum_{b=1}^3 B_0(p^2, m_{u_a}^2, m_{u_b}^2) m_{u_b} \left(\Gamma_{\tilde{h}_j, \bar{u}_a, u_b}^{L*} \Gamma_{\tilde{h}_i, \bar{u}_a, u_b}^R + \Gamma_{\tilde{h}_j, \bar{u}_a, u_b}^{R*} \Gamma_{\tilde{h}_i, \bar{u}_a, u_b}^L \right) \\
& + 3 \sum_{a=1}^3 \sum_{b=1}^3 G_0(p^2, m_{u_a}^2, m_{u_b}^2) \left(\Gamma_{\tilde{h}_j, \bar{u}_a, u_b}^{L*} \Gamma_{\tilde{h}_i, \bar{u}_a, u_b}^L + \Gamma_{\tilde{h}_j, \bar{u}_a, u_b}^{R*} \Gamma_{\tilde{h}_i, \bar{u}_a, u_b}^R \right) \\
& - \sum_{a=1}^9 m_{\nu_a} \sum_{b=1}^9 B_0(p^2, m_{\nu_a}^2, m_{\nu_b}^2) m_{\nu_b} \left(\Gamma_{\tilde{h}_j, \nu_a, \nu_b}^{L*} \Gamma_{\tilde{h}_i, \nu_a, \nu_b}^R + \Gamma_{\tilde{h}_j, \nu_a, \nu_b}^{R*} \Gamma_{\tilde{h}_i, \nu_a, \nu_b}^L \right) \\
& + \frac{1}{2} \sum_{a=1}^9 \sum_{b=1}^9 G_0(p^2, m_{\nu_a}^2, m_{\nu_b}^2) \left(\Gamma_{\tilde{h}_j, \nu_a, \nu_b}^{L*} \Gamma_{\tilde{h}_i, \nu_a, \nu_b}^L + \Gamma_{\tilde{h}_j, \nu_a, \nu_b}^{R*} \Gamma_{\tilde{h}_i, \nu_a, \nu_b}^R \right) \\
& + \sum_{b=1}^2 \Gamma_{\tilde{h}_j, \gamma, A_{h,b}}^* \Gamma_{\tilde{h}_i, \gamma, A_{h,b}} F_0(p^2, m_{A_{h,b}}^2, 0) + \sum_{b=1}^2 \Gamma_{\tilde{h}_j, Z, A_{h,b}}^* \Gamma_{\tilde{h}_i, Z, A_{h,b}} F_0(p^2, m_{A_{h,b}}^2, m_Z^2) \\
& + \sum_{b=1}^2 \Gamma_{\tilde{h}_j, Z', A_{h,b}}^* \Gamma_{\tilde{h}_i, Z', A_{h,b}} F_0(p^2, m_{A_{h,b}}^2, m_{Z'}^2)
\end{aligned} \tag{86}$$

• Self-Energy for Pseudo-Scalar Higgs (A_h)

$$\begin{aligned}
\Pi_{i,j}(p^2) = & -B_0(p^2, m_{\eta^-}^2, m_{\eta^-}^2) \Gamma_{\check{A}_{h,i}, \check{\eta}^-, \eta^-} \Gamma_{\check{A}_{h,j}, \check{\eta}^-, \eta^-} - B_0(p^2, m_{\eta^+}^2, m_{\eta^+}^2) \Gamma_{\check{A}_{h,i}, \check{\eta}^+, \eta^+} \Gamma_{\check{A}_{h,j}, \check{\eta}^+, \eta^+} \\
& - A_0(m_{H^-}^2) \Gamma_{\check{A}_{h,i}, \check{A}_{h,j}, H^-, * , H^-} + 2\Gamma_{\check{A}_{h,j}, W^+, H^-}^* \Gamma_{\check{A}_{h,i}, W^+, H^-} F_0(p^2, m_{H^-}^2, m_{W^-}^2) \\
& + 4\Gamma_{\check{A}_{h,i}, \check{A}_{h,j}, W^+, W^-} \left(-\frac{1}{2} \text{rMS} m_{W^-}^2 + A_0(m_{W^-}^2) \right) + 2\Gamma_{\check{A}_{h,i}, \check{A}_{h,j}, Z, Z} \left(-\frac{1}{2} \text{rMS} m_Z^2 + A_0(m_Z^2) \right) \\
& + 2\Gamma_{\check{A}_{h,i}, \check{A}_{h,j}, Z', Z'} \left(-\frac{1}{2} \text{rMS} m_{Z'}^2 + A_0(m_{Z'}^2) \right) - \frac{1}{2} \sum_{a=1}^2 A_0(m_{\check{A}_{h,a}}^2) \Gamma_{\check{A}_{h,i}, \check{A}_{h,j}, A_{h,a}, A_{h,a}} \\
& - \frac{1}{2} \sum_{a=1}^2 A_0(m_{h_a}^2) \Gamma_{\check{A}_{h,i}, \check{A}_{h,j}, h_a, h_a} + \sum_{a=1}^2 \sum_{b=1}^2 B_0(p^2, m_{h_a}^2, m_{A_{h,b}}^2) \Gamma_{\check{A}_{h,j}, h_a, A_{h,b}}^* \Gamma_{\check{A}_{h,i}, h_a, A_{h,b}} \\
& - 6 \sum_{a=1}^3 m_{d_a} \sum_{b=1}^3 B_0(p^2, m_{d_a}^2, m_{d_b}^2) m_{d_b} \left(\Gamma_{\check{A}_{h,j}, \check{d}_a, d_b}^{L*} \Gamma_{\check{A}_{h,i}, \check{d}_a, d_b}^R + \Gamma_{\check{A}_{h,j}, \check{d}_a, d_b}^{R*} \Gamma_{\check{A}_{h,i}, \check{d}_a, d_b}^L \right) \\
& + 3 \sum_{a=1}^3 \sum_{b=1}^3 G_0(p^2, m_{d_a}^2, m_{d_b}^2) \left(\Gamma_{\check{A}_{h,j}, \check{d}_a, d_b}^{L*} \Gamma_{\check{A}_{h,i}, \check{d}_a, d_b}^L + \Gamma_{\check{A}_{h,j}, \check{d}_a, d_b}^{R*} \Gamma_{\check{A}_{h,i}, \check{d}_a, d_b}^R \right) \\
& - 2 \sum_{a=1}^3 m_{e_a} \sum_{b=1}^3 B_0(p^2, m_{e_a}^2, m_{e_b}^2) m_{e_b} \left(\Gamma_{\check{A}_{h,j}, \check{e}_a, e_b}^{L*} \Gamma_{\check{A}_{h,i}, \check{e}_a, e_b}^R + \Gamma_{\check{A}_{h,j}, \check{e}_a, e_b}^{R*} \Gamma_{\check{A}_{h,i}, \check{e}_a, e_b}^L \right) \\
& + \sum_{a=1}^3 \sum_{b=1}^3 G_0(p^2, m_{e_a}^2, m_{e_b}^2) \left(\Gamma_{\check{A}_{h,j}, \check{e}_a, e_b}^{L*} \Gamma_{\check{A}_{h,i}, \check{e}_a, e_b}^L + \Gamma_{\check{A}_{h,j}, \check{e}_a, e_b}^{R*} \Gamma_{\check{A}_{h,i}, \check{e}_a, e_b}^R \right) \\
& - 6 \sum_{a=1}^3 m_{u_a} \sum_{b=1}^3 B_0(p^2, m_{u_a}^2, m_{u_b}^2) m_{u_b} \left(\Gamma_{\check{A}_{h,j}, \check{u}_a, u_b}^{L*} \Gamma_{\check{A}_{h,i}, \check{u}_a, u_b}^R + \Gamma_{\check{A}_{h,j}, \check{u}_a, u_b}^{R*} \Gamma_{\check{A}_{h,i}, \check{u}_a, u_b}^L \right) \\
& + 3 \sum_{a=1}^3 \sum_{b=1}^3 G_0(p^2, m_{u_a}^2, m_{u_b}^2) \left(\Gamma_{\check{A}_{h,j}, \check{u}_a, u_b}^{L*} \Gamma_{\check{A}_{h,i}, \check{u}_a, u_b}^L + \Gamma_{\check{A}_{h,j}, \check{u}_a, u_b}^{R*} \Gamma_{\check{A}_{h,i}, \check{u}_a, u_b}^R \right) \\
& - \sum_{a=1}^9 m_{\nu_a} \sum_{b=1}^9 B_0(p^2, m_{\nu_a}^2, m_{\nu_b}^2) m_{\nu_b} \left(\Gamma_{\check{A}_{h,j}, \check{\nu}_a, \nu_b}^{L*} \Gamma_{\check{A}_{h,i}, \check{\nu}_a, \nu_b}^R + \Gamma_{\check{A}_{h,j}, \check{\nu}_a, \nu_b}^{R*} \Gamma_{\check{A}_{h,i}, \check{\nu}_a, \nu_b}^L \right) \\
& + \frac{1}{2} \sum_{a=1}^9 \sum_{b=1}^9 G_0(p^2, m_{\nu_a}^2, m_{\nu_b}^2) \left(\Gamma_{\check{A}_{h,j}, \check{\nu}_a, \nu_b}^{L*} \Gamma_{\check{A}_{h,i}, \check{\nu}_a, \nu_b}^L + \Gamma_{\check{A}_{h,j}, \check{\nu}_a, \nu_b}^{R*} \Gamma_{\check{A}_{h,i}, \check{\nu}_a, \nu_b}^R \right) \\
& + \sum_{b=1}^2 \Gamma_{\check{A}_{h,j}, \gamma, h_b}^* \Gamma_{\check{A}_{h,i}, \gamma, h_b} F_0(p^2, m_{h_b}^2, 0) + \sum_{b=1}^2 \Gamma_{\check{A}_{h,j}, Z, h_b}^* \Gamma_{\check{A}_{h,i}, Z, h_b} F_0(p^2, m_{h_b}^2, m_Z^2) \\
& + \sum_{b=1}^2 \Gamma_{\check{A}_{h,j}, Z', h_b}^* \Gamma_{\check{A}_{h,i}, Z', h_b} F_0(p^2, m_{h_b}^2, m_{Z'}^2) \tag{87}
\end{aligned}$$

• Self-Energy for Down-Quarks (d)

$$\begin{aligned}
\Sigma_{i,j}^S(p^2) = & + \sum_{a=1}^2 \sum_{b=1}^3 B_0(p^2, m_{d_b}^2, m_{h_a}^2) \Gamma_{\tilde{d}_j, h_a, d_b}^{L*} m_{d_b} \Gamma_{\tilde{d}_i, h_a, d_b}^R \\
& + \sum_{a=1}^3 m_{d_a} \sum_{b=1}^2 B_0(p^2, m_{d_a}^2, m_{A_{h,b}}^2) \Gamma_{\tilde{d}_j, d_a, A_{h,b}}^{L*} \Gamma_{\tilde{d}_i, d_a, A_{h,b}}^R \\
& + \sum_{b=1}^3 B_0(p^2, m_{u_b}^2, m_{H^-}^2) \Gamma_{\tilde{d}_j, H^-, u_b}^{L*} m_{u_b} \Gamma_{\tilde{d}_i, H^-, u_b}^R - \frac{16}{3} \sum_{b=1}^3 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{d_b}^2, 0) \right) \Gamma_{\tilde{d}_j, g, d_b}^{R*} m_{d_b} \Gamma_{\tilde{d}_i, g, d_b}^L \\
& - 4 \sum_{b=1}^3 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{d_b}^2, 0) \right) \Gamma_{\tilde{d}_j, \gamma, d_b}^{R*} m_{d_b} \Gamma_{\tilde{d}_i, \gamma, d_b}^L \\
& - 4 \sum_{b=1}^3 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{u_b}^2, m_{W^-}^2) \right) \Gamma_{\tilde{d}_j, W^-, u_b}^{R*} m_{u_b} \Gamma_{\tilde{d}_i, W^-, u_b}^L \\
& - 4 \sum_{b=1}^3 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{d_b}^2, m_Z^2) \right) \Gamma_{\tilde{d}_j, Z, d_b}^{R*} m_{d_b} \Gamma_{\tilde{d}_i, Z, d_b}^L \\
& - 4 \sum_{b=1}^3 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{d_b}^2, m_{Z'}^2) \right) \Gamma_{\tilde{d}_j, Z', d_b}^{R*} m_{d_b} \Gamma_{\tilde{d}_i, Z', d_b}^L \tag{88}
\end{aligned}$$

$$\begin{aligned}
\Sigma_{i,j}^R(p^2) = & -\frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, m_{h_a}^2) \Gamma_{\tilde{d}_j, h_a, d_b}^{R*} \Gamma_{\tilde{d}_i, h_a, d_b}^R \\
& - \frac{1}{2} \sum_{a=1}^3 \sum_{b=1}^2 B_1(p^2, m_{d_a}^2, m_{A_{h,b}}^2) \Gamma_{\tilde{d}_j, d_a, A_{h,b}}^{R*} \Gamma_{\tilde{d}_i, d_a, A_{h,b}}^R \\
& - \frac{1}{2} \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, m_{H^-}^2) \Gamma_{\tilde{d}_j, H^-, u_b}^{R*} \Gamma_{\tilde{d}_i, H^-, u_b}^R - \frac{4}{3} \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, 0) \Gamma_{\tilde{d}_j, g, d_b}^{L*} \Gamma_{\tilde{d}_i, g, d_b}^L \\
& - \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, 0) \Gamma_{\tilde{d}_j, \gamma, d_b}^{L*} \Gamma_{\tilde{d}_i, \gamma, d_b}^L - \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, m_{W^-}^2) \Gamma_{\tilde{d}_j, W^-, u_b}^{L*} \Gamma_{\tilde{d}_i, W^-, u_b}^L \\
& - \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, m_Z^2) \Gamma_{\tilde{d}_j, Z, d_b}^{L*} \Gamma_{\tilde{d}_i, Z, d_b}^L - \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, m_{Z'}^2) \Gamma_{\tilde{d}_j, Z', d_b}^{L*} \Gamma_{\tilde{d}_i, Z', d_b}^L \tag{89}
\end{aligned}$$

$$\begin{aligned}
\Sigma_{i,j}^L(p^2) = & -\frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, m_{h_a}^2) \Gamma_{\tilde{d}_j, h_a, d_b}^{L*} \Gamma_{\tilde{d}_i, h_a, d_b}^L \\
& - \frac{1}{2} \sum_{a=1}^3 \sum_{b=1}^2 B_1(p^2, m_{d_a}^2, m_{A_{h,b}}^2) \Gamma_{\tilde{d}_j, d_a, A_{h,b}}^{L*} \Gamma_{\tilde{d}_i, d_a, A_{h,b}}^L \\
& - \frac{1}{2} \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, m_{H^-}^2) \Gamma_{\tilde{d}_j, H^-, u_b}^{L*} \Gamma_{\tilde{d}_i, H^-, u_b}^L - \frac{4}{3} \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, 0) \Gamma_{\tilde{d}_j, g, d_b}^{R*} \Gamma_{\tilde{d}_i, g, d_b}^R
\end{aligned}$$

$$\begin{aligned}
& - \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, 0) \Gamma_{\tilde{d}_j, \gamma, d_b}^{R*} \Gamma_{\tilde{d}_i, \gamma, d_b}^R - \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, m_{W^-}^2) \Gamma_{\tilde{d}_j, W^-, u_b}^{R*} \Gamma_{\tilde{d}_i, W^-, u_b}^R \\
& - \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, m_Z^2) \Gamma_{\tilde{d}_j, Z, d_b}^{R*} \Gamma_{\tilde{d}_i, Z, d_b}^R - \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, m_{Z'}^2) \Gamma_{\tilde{d}_j, Z', d_b}^{R*} \Gamma_{\tilde{d}_i, Z', d_b}^R
\end{aligned} \tag{90}$$

• Self-Energy for Up-Quarks (u)

$$\begin{aligned}
\Sigma_{i,j}^S(p^2) &= + \sum_{a=1}^2 \sum_{b=1}^3 B_0(p^2, m_{u_b}^2, m_{h_a}^2) \Gamma_{\tilde{u}_j, h_a, u_b}^{L*} m_{u_b} \Gamma_{\tilde{u}_i, h_a, u_b}^R \\
& + \sum_{a=1}^3 m_{u_a} \sum_{b=1}^2 B_0(p^2, m_{u_a}^2, m_{A_{h,b}}^2) \Gamma_{\tilde{u}_j, u_a, A_{h,b}}^{L*} \Gamma_{\tilde{u}_i, u_a, A_{h,b}}^R \\
& - \frac{16}{3} \sum_{b=1}^3 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{u_b}^2, 0) \right) \Gamma_{\tilde{u}_j, g, u_b}^{R*} m_{u_b} \Gamma_{\tilde{u}_i, g, u_b}^L - 4 \sum_{b=1}^3 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{u_b}^2, 0) \right) \Gamma_{\tilde{u}_j, \gamma, u_b}^{R*} m_{u_b} \Gamma_{\tilde{u}_i, \gamma, u_b}^L \\
& - 4 \sum_{b=1}^3 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{u_b}^2, m_Z^2) \right) \Gamma_{\tilde{u}_j, Z, u_b}^{R*} m_{u_b} \Gamma_{\tilde{u}_i, Z, u_b}^L \\
& - 4 \sum_{b=1}^3 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{u_b}^2, m_{Z'}^2) \right) \Gamma_{\tilde{u}_j, Z', u_b}^{R*} m_{u_b} \Gamma_{\tilde{u}_i, Z', u_b}^L \\
& + \sum_{b=1}^3 B_0(p^2, m_{d_b}^2, m_{H^-}^2) \Gamma_{\tilde{u}_j, H^-, * , d_b}^{L*} m_{d_b} \Gamma_{\tilde{u}_i, H^-, * , d_b}^R \\
& - 4 \sum_{b=1}^3 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{d_b}^2, m_{W^-}^2) \right) \Gamma_{\tilde{u}_j, W^+, d_b}^{R*} m_{d_b} \Gamma_{\tilde{u}_i, W^+, d_b}^L
\end{aligned} \tag{91}$$

$$\begin{aligned}
\Sigma_{i,j}^R(p^2) &= -\frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, m_{h_a}^2) \Gamma_{\tilde{u}_j, h_a, u_b}^{R*} \Gamma_{\tilde{u}_i, h_a, u_b}^R \\
& - \frac{1}{2} \sum_{a=1}^3 \sum_{b=1}^2 B_1(p^2, m_{u_a}^2, m_{A_{h,b}}^2) \Gamma_{\tilde{u}_j, u_a, A_{h,b}}^{R*} \Gamma_{\tilde{u}_i, u_a, A_{h,b}}^R - \frac{4}{3} \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, 0) \Gamma_{\tilde{u}_j, g, u_b}^{L*} \Gamma_{\tilde{u}_i, g, u_b}^L \\
& - \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, 0) \Gamma_{\tilde{u}_j, \gamma, u_b}^{L*} \Gamma_{\tilde{u}_i, \gamma, u_b}^L - \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, m_Z^2) \Gamma_{\tilde{u}_j, Z, u_b}^{L*} \Gamma_{\tilde{u}_i, Z, u_b}^L \\
& - \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, m_{Z'}^2) \Gamma_{\tilde{u}_j, Z', u_b}^{L*} \Gamma_{\tilde{u}_i, Z', u_b}^L - \frac{1}{2} \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, m_{H^-}^2) \Gamma_{\tilde{u}_j, H^-, * , d_b}^{R*} \Gamma_{\tilde{u}_i, H^-, * , d_b}^R \\
& - \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, m_{W^-}^2) \Gamma_{\tilde{u}_j, W^+, d_b}^{L*} \Gamma_{\tilde{u}_i, W^+, d_b}^L
\end{aligned} \tag{92}$$

$$\begin{aligned}
\Sigma_{i,j}^L(p^2) = & -\frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, m_{h_a}^2) \Gamma_{\tilde{u}_j, h_a, u_b}^{L*} \Gamma_{\tilde{u}_i, h_a, u_b}^L \\
& - \frac{1}{2} \sum_{a=1}^3 \sum_{b=1}^2 B_1(p^2, m_{u_a}^2, m_{A_{h,b}}^2) \Gamma_{\tilde{u}_j, u_a, A_{h,b}}^{L*} \Gamma_{\tilde{u}_i, u_a, A_{h,b}}^L - \frac{4}{3} \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, 0) \Gamma_{\tilde{u}_j, g, u_b}^{R*} \Gamma_{\tilde{u}_i, g, u_b}^R \\
& - \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, 0) \Gamma_{\tilde{u}_j, \gamma, u_b}^{R*} \Gamma_{\tilde{u}_i, \gamma, u_b}^R - \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, m_Z^2) \Gamma_{\tilde{u}_j, Z, u_b}^{R*} \Gamma_{\tilde{u}_i, Z, u_b}^R \\
& - \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, m_{Z'}^2) \Gamma_{\tilde{u}_j, Z', u_b}^{R*} \Gamma_{\tilde{u}_i, Z', u_b}^R - \frac{1}{2} \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, m_{H^-}^2) \Gamma_{\tilde{u}_j, H^-, d_b}^{L*} \Gamma_{\tilde{u}_i, H^-, d_b}^L \\
& - \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, m_{W^-}^2) \Gamma_{\tilde{u}_j, W^-, d_b}^{R*} \Gamma_{\tilde{u}_i, W^-, d_b}^R
\end{aligned} \tag{93}$$

• Self-Energy for Leptons (e)

$$\begin{aligned}
\Sigma_{i,j}^S(p^2) = & + \sum_{a=1}^2 \sum_{b=1}^3 B_0(p^2, m_{e_b}^2, m_{h_a}^2) \Gamma_{\tilde{e}_j, h_a, e_b}^{L*} m_{e_b} \Gamma_{\tilde{e}_i, h_a, e_b}^R \\
& + \sum_{a=1}^3 m_{e_a} \sum_{b=1}^2 B_0(p^2, m_{e_a}^2, m_{A_{h,b}}^2) \Gamma_{\tilde{e}_j, e_a, A_{h,b}}^{L*} \Gamma_{\tilde{e}_i, e_a, A_{h,b}}^R \\
& - 4 \sum_{b=1}^3 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{e_b}^2, 0) \right) \Gamma_{\tilde{e}_j, \gamma, e_b}^{R*} m_{e_b} \Gamma_{\tilde{e}_i, \gamma, e_b}^L - 4 \sum_{b=1}^3 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{e_b}^2, m_Z^2) \right) \Gamma_{\tilde{e}_j, Z, e_b}^{R*} m_{e_b} \Gamma_{\tilde{e}_i, Z, e_b}^L \\
& - 4 \sum_{b=1}^3 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{e_b}^2, m_{Z'}^2) \right) \Gamma_{\tilde{e}_j, Z', e_b}^{R*} m_{e_b} \Gamma_{\tilde{e}_i, Z', e_b}^L \\
& + \sum_{b=1}^9 B_0(p^2, m_{\nu_b}^2, m_{H^-}^2) \Gamma_{\tilde{e}_j, H^-, \nu_b}^{L*} m_{\nu_b} \Gamma_{\tilde{e}_i, H^-, \nu_b}^R \\
& - 4 \sum_{b=1}^9 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{\nu_b}^2, m_{W^-}^2) \right) \Gamma_{\tilde{e}_j, W^-, \nu_b}^{R*} m_{\nu_b} \Gamma_{\tilde{e}_i, W^-, \nu_b}^L
\end{aligned} \tag{94}$$

$$\begin{aligned}
\Sigma_{i,j}^R(p^2) = & -\frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^3 B_1(p^2, m_{e_b}^2, m_{h_a}^2) \Gamma_{\tilde{e}_j, h_a, e_b}^{R*} \Gamma_{\tilde{e}_i, h_a, e_b}^R \\
& - \frac{1}{2} \sum_{a=1}^3 \sum_{b=1}^2 B_1(p^2, m_{e_a}^2, m_{A_{h,b}}^2) \Gamma_{\tilde{e}_j, e_a, A_{h,b}}^{R*} \Gamma_{\tilde{e}_i, e_a, A_{h,b}}^R - \sum_{b=1}^3 B_1(p^2, m_{e_b}^2, 0) \Gamma_{\tilde{e}_j, \gamma, e_b}^{L*} \Gamma_{\tilde{e}_i, \gamma, e_b}^L \\
& - \sum_{b=1}^3 B_1(p^2, m_{e_b}^2, m_Z^2) \Gamma_{\tilde{e}_j, Z, e_b}^{L*} \Gamma_{\tilde{e}_i, Z, e_b}^L - \sum_{b=1}^3 B_1(p^2, m_{e_b}^2, m_{Z'}^2) \Gamma_{\tilde{e}_j, Z', e_b}^{L*} \Gamma_{\tilde{e}_i, Z', e_b}^L
\end{aligned}$$

$$-\frac{1}{2} \sum_{b=1}^9 B_1(p^2, m_{\nu_b}^2, m_{H^-}^2) \Gamma_{\tilde{e}_j, H^-, \nu_b}^{R*} \Gamma_{\tilde{e}_i, H^-, \nu_b}^R - \sum_{b=1}^9 B_1(p^2, m_{\nu_b}^2, m_{W^-}^2) \Gamma_{\tilde{e}_j, W^-, \nu_b}^{L*} \Gamma_{\tilde{e}_i, W^-, \nu_b}^L \quad (95)$$

$$\begin{aligned} \Sigma_{i,j}^L(p^2) = & -\frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^3 B_1(p^2, m_{e_b}^2, m_{h_a}^2) \Gamma_{\tilde{e}_j, h_a, e_b}^{L*} \Gamma_{\tilde{e}_i, h_a, e_b}^L \\ & -\frac{1}{2} \sum_{a=1}^3 \sum_{b=1}^2 B_1(p^2, m_{e_a}^2, m_{A_{h,b}}^2) \Gamma_{\tilde{e}_j, e_a, A_{h,b}}^{L*} \Gamma_{\tilde{e}_i, e_a, A_{h,b}}^L - \sum_{b=1}^3 B_1(p^2, m_{e_b}^2, 0) \Gamma_{\tilde{e}_j, \gamma, e_b}^{R*} \Gamma_{\tilde{e}_i, \gamma, e_b}^R \\ & -\sum_{b=1}^3 B_1(p^2, m_{e_b}^2, m_Z^2) \Gamma_{\tilde{e}_j, Z, e_b}^{R*} \Gamma_{\tilde{e}_i, Z, e_b}^R - \sum_{b=1}^3 B_1(p^2, m_{e_b}^2, m_{Z'}^2) \Gamma_{\tilde{e}_j, Z', e_b}^{R*} \Gamma_{\tilde{e}_i, Z', e_b}^R \\ & -\frac{1}{2} \sum_{b=1}^9 B_1(p^2, m_{\nu_b}^2, m_{H^-}^2) \Gamma_{\tilde{e}_j, H^-, \nu_b}^{L*} \Gamma_{\tilde{e}_i, H^-, \nu_b}^L - \sum_{b=1}^9 B_1(p^2, m_{\nu_b}^2, m_{W^-}^2) \Gamma_{\tilde{e}_j, W^-, \nu_b}^{R*} \Gamma_{\tilde{e}_i, W^-, \nu_b}^R \end{aligned} \quad (96)$$

• **Self-Energy for Neutrinos (ν)**

$$\begin{aligned} \Sigma_{i,j}^S(p^2) = & + \sum_{a=1}^2 \sum_{b=1}^9 B_0(p^2, m_{\nu_b}^2, m_{h_a}^2) \Gamma_{\tilde{\nu}_j, h_a, \nu_b}^{L*} m_{\nu_b} \Gamma_{\tilde{\nu}_i, h_a, \nu_b}^R \\ & + \sum_{a=1}^9 m_{\nu_a} \sum_{b=1}^2 B_0(p^2, m_{\nu_a}^2, m_{A_{h,b}}^2) \Gamma_{\tilde{\nu}_j, \nu_a, A_{h,b}}^{L*} \Gamma_{\tilde{\nu}_i, \nu_a, A_{h,b}}^R \\ & + 2 \sum_{b=1}^3 B_0(p^2, m_{e_b}^2, m_{H^-}^2) \Gamma_{\tilde{\nu}_j, H^-, *, e_b}^{L*} m_{e_b} \Gamma_{\tilde{\nu}_i, H^-, *, e_b}^R \\ & - 8 \sum_{b=1}^3 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{e_b}^2, m_{W^-}^2) \right) \Gamma_{\tilde{\nu}_j, W^+, e_b}^{R*} m_{e_b} \Gamma_{\tilde{\nu}_i, W^+, e_b}^L - 4 \sum_{b=1}^9 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{\nu_b}^2, 0) \right) \Gamma_{\tilde{\nu}_j, \gamma, \nu_b}^{R*} m_{\nu_b} \Gamma_{\tilde{\nu}_i, \gamma}^L \\ & - 4 \sum_{b=1}^9 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{\nu_b}^2, m_Z^2) \right) \Gamma_{\tilde{\nu}_j, Z, \nu_b}^{R*} m_{\nu_b} \Gamma_{\tilde{\nu}_i, Z, \nu_b}^L - 4 \sum_{b=1}^9 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{\nu_b}^2, m_{Z'}^2) \right) \Gamma_{\tilde{\nu}_j, Z', \nu_b}^{R*} m_{\nu_b} \Gamma_{\tilde{\nu}_i, Z', \nu_b}^L \end{aligned} \quad (97)$$

$$\begin{aligned} \Sigma_{i,j}^R(p^2) = & -\frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^9 B_1(p^2, m_{\nu_b}^2, m_{h_a}^2) \Gamma_{\tilde{\nu}_j, h_a, \nu_b}^{R*} \Gamma_{\tilde{\nu}_i, h_a, \nu_b}^R \\ & -\frac{1}{2} \sum_{a=1}^9 \sum_{b=1}^2 B_1(p^2, m_{\nu_a}^2, m_{A_{h,b}}^2) \Gamma_{\tilde{\nu}_j, \nu_a, A_{h,b}}^{R*} \Gamma_{\tilde{\nu}_i, \nu_a, A_{h,b}}^R - \sum_{b=1}^3 B_1(p^2, m_{e_b}^2, m_{H^-}^2) \Gamma_{\tilde{\nu}_j, H^-, *, e_b}^{R*} \Gamma_{\tilde{\nu}_i, H^-, *, e_b}^R \\ & - 2 \sum_{b=1}^3 B_1(p^2, m_{e_b}^2, m_{W^-}^2) \Gamma_{\tilde{\nu}_j, W^+, e_b}^{L*} \Gamma_{\tilde{\nu}_i, W^+, e_b}^L - \sum_{b=1}^9 B_1(p^2, m_{\nu_b}^2, 0) \Gamma_{\tilde{\nu}_j, \gamma, \nu_b}^{L*} \Gamma_{\tilde{\nu}_i, \gamma, \nu_b}^L \\ & - \sum_{b=1}^9 B_1(p^2, m_{\nu_b}^2, m_Z^2) \Gamma_{\tilde{\nu}_j, Z, \nu_b}^{L*} \Gamma_{\tilde{\nu}_i, Z, \nu_b}^L - \sum_{b=1}^9 B_1(p^2, m_{\nu_b}^2, m_{Z'}^2) \Gamma_{\tilde{\nu}_j, Z', \nu_b}^{L*} \Gamma_{\tilde{\nu}_i, Z', \nu_b}^L \end{aligned} \quad (98)$$

$$\begin{aligned}
\Sigma_{i,j}^L(p^2) = & -\frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^9 B_1(p^2, m_{\nu_b}^2, m_{h_a}^2) \Gamma_{\tilde{\nu}_j, h_a, \nu_b}^{L*} \Gamma_{\tilde{\nu}_i, h_a, \nu_b}^L \\
& - \frac{1}{2} \sum_{a=1}^9 \sum_{b=1}^2 B_1(p^2, m_{\nu_a}^2, m_{A_{h,b}}^2) \Gamma_{\tilde{\nu}_j, \nu_a, A_{h,b}}^{L*} \Gamma_{\tilde{\nu}_i, \nu_a, A_{h,b}}^L - \sum_{b=1}^3 B_1(p^2, m_{e_b}^2, m_{H^-}^2) \Gamma_{\tilde{\nu}_j, H^-, e_b}^{L*} \Gamma_{\tilde{\nu}_i, H^-, e_b}^L \\
& - 2 \sum_{b=1}^3 B_1(p^2, m_{e_b}^2, m_{W^-}^2) \Gamma_{\tilde{\nu}_j, W^+, e_b}^{R*} \Gamma_{\tilde{\nu}_i, W^+, e_b}^R - \sum_{b=1}^9 B_1(p^2, m_{\nu_b}^2, 0) \Gamma_{\tilde{\nu}_j, \gamma, \nu_b}^{R*} \Gamma_{\tilde{\nu}_i, \gamma, \nu_b}^R \\
& - \sum_{b=1}^9 B_1(p^2, m_{\nu_b}^2, m_Z^2) \Gamma_{\tilde{\nu}_j, Z, \nu_b}^{R*} \Gamma_{\tilde{\nu}_i, Z, \nu_b}^R - \sum_{b=1}^9 B_1(p^2, m_{\nu_b}^2, m_{Z'}^2) \Gamma_{\tilde{\nu}_j, Z', \nu_b}^{R*} \Gamma_{\tilde{\nu}_i, Z', \nu_b}^R
\end{aligned} \tag{99}$$

• **Self-Energy for s neutrino (S_1)**

$$\begin{aligned}
\Sigma_{i,j}^S(p^2) = & -4 \sum_{b=1}^3 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{S_{1,b}}^2, 0) \right) \Gamma_{\tilde{S}_{1,j}, \gamma, S_{1,b}}^{R*} m_{S_{1,b}} \Gamma_{\tilde{S}_{1,i}, \gamma, S_{1,b}}^L \\
& - 4 \sum_{b=1}^3 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{S_{1,b}}^2, m_Z^2) \right) \Gamma_{\tilde{S}_{1,j}, Z, S_{1,b}}^{R*} m_{S_{1,b}} \Gamma_{\tilde{S}_{1,i}, Z, S_{1,b}}^L \\
& - 4 \sum_{b=1}^3 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{S_{1,b}}^2, m_{Z'}^2) \right) \Gamma_{\tilde{S}_{1,j}, Z', S_{1,b}}^{R*} m_{S_{1,b}} \Gamma_{\tilde{S}_{1,i}, Z', S_{1,b}}^L
\end{aligned} \tag{100}$$

$$\begin{aligned}
\Sigma_{i,j}^R(p^2) = & - \sum_{b=1}^3 B_1(p^2, m_{S_{1,b}}^2, 0) \Gamma_{\tilde{S}_{1,j}, \gamma, S_{1,b}}^{L*} \Gamma_{\tilde{S}_{1,i}, \gamma, S_{1,b}}^L - \sum_{b=1}^3 B_1(p^2, m_{S_{1,b}}^2, m_Z^2) \Gamma_{\tilde{S}_{1,j}, Z, S_{1,b}}^{L*} \Gamma_{\tilde{S}_{1,i}, Z, S_{1,b}}^L \\
& - \sum_{b=1}^3 B_1(p^2, m_{S_{1,b}}^2, m_{Z'}^2) \Gamma_{\tilde{S}_{1,j}, Z', S_{1,b}}^{L*} \Gamma_{\tilde{S}_{1,i}, Z', S_{1,b}}^L
\end{aligned} \tag{101}$$

$$\begin{aligned}
\Sigma_{i,j}^L(p^2) = & - \sum_{b=1}^3 B_1(p^2, m_{S_{1,b}}^2, 0) \Gamma_{\tilde{S}_{1,j}, \gamma, S_{1,b}}^{R*} \Gamma_{\tilde{S}_{1,i}, \gamma, S_{1,b}}^R - \sum_{b=1}^3 B_1(p^2, m_{S_{1,b}}^2, m_Z^2) \Gamma_{\tilde{S}_{1,j}, Z, S_{1,b}}^{R*} \Gamma_{\tilde{S}_{1,i}, Z, S_{1,b}}^R \\
& - \sum_{b=1}^3 B_1(p^2, m_{S_{1,b}}^2, m_{Z'}^2) \Gamma_{\tilde{S}_{1,j}, Z', S_{1,b}}^{R*} \Gamma_{\tilde{S}_{1,i}, Z', S_{1,b}}^R
\end{aligned} \tag{102}$$

• **Self-Energy for Charged Higgs (H^-)**

$$\begin{aligned}
\Pi(p^2) = & +4|\Gamma_{H^-, *, W^-, \gamma}|^2 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, 0, m_{W^-}^2) \right) + 4|\Gamma_{H^-, *, Z, W^-}|^2 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{W^-}^2, m_Z^2) \right) + 4|\Gamma_{H^-, *, Z', W^-}|^2 \\
& - B_0(p^2, m_{\eta^+}^2, m_{\eta^+}^2) \Gamma_{H^-, \eta^+, \eta^+} \Gamma_{H^-, *, \eta^+, \eta^+} - B_0(p^2, m_{\eta^{z'}}^2, m_{\eta^+}^2) \Gamma_{H^-, \eta^+, \eta^{z'}} \Gamma_{H^-, *, \eta^+, \eta^{z'}} \\
& - B_0(p^2, m_{\eta^-}^2, m_{\eta^+}^2) \Gamma_{H^-, \eta^+, \eta^-} \Gamma_{H^-, *, \eta^+, \eta^-} - B_0(p^2, m_{\eta^-}^2, m_{\eta^{z'}}^2) \Gamma_{H^-, \eta^+, \eta^-} \Gamma_{H^-, *, \eta^+, \eta^{z'}} \\
& - A_0(m_{H^-}^2) \Gamma_{H^-, H^-, *, H^-, *, H^-} + |\Gamma_{H^-, *, \gamma, H^-}|^2 F_0(p^2, m_{H^-}^2, 0) + |\Gamma_{H^-, *, Z, H^-}|^2 F_0(p^2, m_{H^-}^2, m_Z^2)
\end{aligned}$$

$$\begin{aligned}
& + |\Gamma_{H^-,*,Z',H^-}|^2 F_0(p^2, m_{H^-}^2, m_{Z'}^2) + 4\Gamma_{H^-,H^-,*,W^+,W^-} \left(-\frac{1}{2} \text{rMS} m_{W^-}^2 + A_0(m_{W^-}^2) \right) \\
& + 2\Gamma_{H^-,H^-,*,Z,Z} \left(-\frac{1}{2} \text{rMS} m_Z^2 + A_0(m_Z^2) \right) + 2\Gamma_{H^-,H^-,*,Z',Z'} \left(-\frac{1}{2} \text{rMS} m_{Z'}^2 + A_0(m_{Z'}^2) \right) \\
& - \frac{1}{2} \sum_{a=1}^2 A_0(m_{A_{h,a}}^2) \Gamma_{H^-,H^-,*,A_{h,a},A_{h,a}} - \frac{1}{2} \sum_{a=1}^2 A_0(m_{h_a}^2) \Gamma_{H^-,H^-,*,h_a,h_a} \\
& + 3 \sum_{a=1}^3 \sum_{b=1}^3 \left(|\Gamma_{H^-,*,\bar{u}_a,d_b}^L|^2 + |\Gamma_{H^-,*,\bar{u}_a,d_b}^R|^2 \right) G_0(p^2, m_{u_a}^2, m_{d_b}^2) \\
& - 6 \sum_{a=1}^3 m_{u_a} \sum_{b=1}^3 B_0(p^2, m_{u_a}^2, m_{d_b}^2) m_{d_b} \left(\Gamma_{H^-,*,\bar{u}_a,d_b}^{L*} \Gamma_{H^-,*,\bar{u}_a,d_b}^R + \Gamma_{H^-,*,\bar{u}_a,d_b}^{R*} \Gamma_{H^-,*,\bar{u}_a,d_b}^L \right) \\
& + \sum_{a=1}^9 \sum_{b=1}^3 \left(|\Gamma_{H^-,*,\nu_a,e_b}^L|^2 + |\Gamma_{H^-,*,\nu_a,e_b}^R|^2 \right) G_0(p^2, m_{\nu_a}^2, m_{e_b}^2) \\
& - 2 \sum_{a=1}^9 m_{\nu_a} \sum_{b=1}^3 B_0(p^2, m_{\nu_a}^2, m_{e_b}^2) m_{e_b} \left(\Gamma_{H^-,*,\nu_a,e_b}^{L*} \Gamma_{H^-,*,\nu_a,e_b}^R + \Gamma_{H^-,*,\nu_a,e_b}^{R*} \Gamma_{H^-,*,\nu_a,e_b}^L \right) \\
& + \sum_{b=1}^2 |\Gamma_{H^-,*,H^-,h_b}|^2 B_0(p^2, m_{H^-}^2, m_{h_b}^2) + \sum_{b=1}^2 |\Gamma_{H^-,*,W^-,A_{h,b}}|^2 F_0(p^2, m_{A_{h,b}}^2, m_{W^-}^2) \\
& + \sum_{b=1}^2 |\Gamma_{H^-,*,W^-,h_b}|^2 F_0(p^2, m_{h_b}^2, m_{W^-}^2) \tag{103}
\end{aligned}$$

• **Self-Energy for Z-Boson** (Z)

$$\begin{aligned}
\Pi(p^2) & = +2|\Gamma_{Z,W^+,H^-}|^2 B_0(p^2, m_{W^-}^2, m_{H^-}^2) + |\Gamma_{Z,\eta^-, \eta^-}|^2 B_{00}(p^2, m_{\eta^-}^2, m_{\eta^-}^2) + |\Gamma_{Z,\eta^+, \eta^+}|^2 B_{00}(p^2, m_{\eta^+}^2, m_{\eta^+}^2) \\
& - 4|\Gamma_{Z,H^-,*,H^-}|^2 B_{00}(p^2, m_{H^-}^2, m_{H^-}^2) + A_0(m_{H^-}^2) \Gamma_{Z,Z,H^-,*,H^-} \\
& - |\Gamma_{Z,W^+,W^-}|^2 \left(10B_{00}(p^2, m_{W^-}^2, m_{W^-}^2) + 2A_0(m_{W^-}^2) - 2\text{rMS} \left(2m_{W^-}^2 - \frac{1}{3}p^2 \right) + B_0(p^2, m_{W^-}^2, m_{W^-}^2) \left(2m_{W^-}^2 + 4p^2 \right) \right) \\
& + \frac{1}{2} \sum_{a=1}^2 A_0(m_{A_{h,a}}^2) \Gamma_{Z,Z,A_{h,a},A_{h,a}} + \frac{1}{2} \sum_{a=1}^2 A_0(m_{h_a}^2) \Gamma_{Z,Z,h_a,h_a} \\
& - 4 \sum_{a=1}^2 \sum_{b=1}^2 |\Gamma_{Z,h_a,A_{h,b}}|^2 B_{00}(p^2, m_{A_{h,b}}^2, m_{h_a}^2) \\
& + 3 \sum_{a=1}^3 \sum_{b=1}^3 \left[\left(|\Gamma_{Z,\bar{d}_a,d_b}^L|^2 + |\Gamma_{Z,\bar{d}_a,d_b}^R|^2 \right) H_0(p^2, m_{d_a}^2, m_{d_b}^2) \right. \\
& \left. + 4B_0(p^2, m_{d_a}^2, m_{d_b}^2) m_{d_a} m_{d_b} \Re \left(\Gamma_{Z,\bar{d}_a,d_b}^{L*} \Gamma_{Z,\bar{d}_a,d_b}^R \right) \right] \\
& + \sum_{a=1}^3 \sum_{b=1}^3 \left[\left(|\Gamma_{Z,\bar{e}_a,e_b}^L|^2 + |\Gamma_{Z,\bar{e}_a,e_b}^R|^2 \right) H_0(p^2, m_{e_a}^2, m_{e_b}^2) \right]
\end{aligned}$$

$$\begin{aligned}
& + 4B_0 \left(p^2, m_{e_a}^2, m_{e_b}^2 \right) m_{e_a} m_{e_b} \Re \left(\Gamma_{Z, \bar{e}_a, e_b}^{L*} \Gamma_{Z, \bar{e}_a, e_b}^R \right) \\
& + 3 \sum_{a=1}^3 \sum_{b=1}^3 \left[\left(|\Gamma_{Z, \bar{u}_a, u_b}^L|^2 + |\Gamma_{Z, \bar{u}_a, u_b}^R|^2 \right) H_0 \left(p^2, m_{u_a}^2, m_{u_b}^2 \right) \right. \\
& + 4B_0 \left(p^2, m_{u_a}^2, m_{u_b}^2 \right) m_{u_a} m_{u_b} \Re \left(\Gamma_{Z, \bar{u}_a, u_b}^{L*} \Gamma_{Z, \bar{u}_a, u_b}^R \right) \\
& + \frac{1}{2} \sum_{a=1}^3 \sum_{b=1}^3 \left[\left(|\Gamma_{Z, S_{1,a}, S_{1,b}}^L|^2 + |\Gamma_{Z, S_{1,a}, S_{1,b}}^R|^2 \right) H_0 \left(p^2, m_{S_{1,a}}^2, m_{S_{1,b}}^2 \right) \right. \\
& + 4B_0 \left(p^2, m_{S_{1,a}}^2, m_{S_{1,b}}^2 \right) m_{S_{1,a}} m_{S_{1,b}} \Re \left(\Gamma_{Z, S_{1,a}, S_{1,b}}^{L*} \Gamma_{Z, S_{1,a}, S_{1,b}}^R \right) \\
& + \frac{1}{2} \sum_{a=1}^9 \sum_{b=1}^9 \left[\left(|\Gamma_{Z, \nu_a, \nu_b}^L|^2 + |\Gamma_{Z, \nu_a, \nu_b}^R|^2 \right) H_0 \left(p^2, m_{\nu_a}^2, m_{\nu_b}^2 \right) \right. \\
& + 4B_0 \left(p^2, m_{\nu_a}^2, m_{\nu_b}^2 \right) m_{\nu_a} m_{\nu_b} \Re \left(\Gamma_{Z, \nu_a, \nu_b}^{L*} \Gamma_{Z, \nu_a, \nu_b}^R \right) \\
& + \sum_{b=1}^2 |\Gamma_{Z, \gamma, h_b}|^2 B_0 \left(p^2, 0, m_{h_b}^2 \right) + \sum_{b=1}^2 |\Gamma_{Z, Z, h_b}|^2 B_0 \left(p^2, m_Z^2, m_{h_b}^2 \right) + \sum_{b=1}^2 |\Gamma_{Z, Z', h_b}|^2 B_0 \left(p^2, m_{Z'}^2, m_{h_b}^2 \right) \\
& + 2\text{rMS} m_{W^-}^2 \Gamma_{Z, Z, W^+, W^-}^1 - A_0 \left(m_{W^-}^2 \right) \left(4\Gamma_{Z, Z, W^+, W^-}^1 + \Gamma_{Z, Z, W^+, W^-}^2 + \Gamma_{Z, Z, W^+, W^-}^3 \right) \tag{104}
\end{aligned}$$

• **Self-Energy for Z'-Boson** (Z')

$$\begin{aligned}
\Pi(p^2) = & + 2|\Gamma_{Z', W^+, H^-}|^2 B_0 \left(p^2, m_{W^-}^2, m_{H^-}^2 \right) + |\Gamma_{Z', \eta^-, \eta^-}|^2 B_{00} \left(p^2, m_{\eta^-}^2, m_{\eta^-}^2 \right) + |\Gamma_{Z', \eta^+, \eta^+}|^2 B_{00} \left(p^2, m_{\eta^+}^2, m_{\eta^+}^2 \right) \\
& - 4|\Gamma_{Z', H^-, *}|^2 B_{00} \left(p^2, m_{H^-}^2, m_{H^-}^2 \right) + A_0 \left(m_{H^-}^2 \right) \Gamma_{Z', Z', H^-, *, H^-} \\
& - |\Gamma_{Z', W^+, W^-}|^2 \left(10B_{00} \left(p^2, m_{W^-}^2, m_{W^-}^2 \right) + 2A_0 \left(m_{W^-}^2 \right) - 2\text{rMS} \left(2m_{W^-}^2 - \frac{1}{3}p^2 \right) + B_0 \left(p^2, m_{W^-}^2, m_{W^-}^2 \right) \left(2m_{W^-}^2 + 4p^2 \right) \right) \\
& + \frac{1}{2} \sum_{a=1}^2 A_0 \left(m_{A_{h,a}}^2 \right) \Gamma_{Z', Z', A_{h,a}, A_{h,a}} + \frac{1}{2} \sum_{a=1}^2 A_0 \left(m_{h_a}^2 \right) \Gamma_{Z', Z', h_a, h_a} \\
& - 4 \sum_{a=1}^2 \sum_{b=1}^2 |\Gamma_{Z', h_a, A_{h,b}}|^2 B_{00} \left(p^2, m_{A_{h,b}}^2, m_{h_a}^2 \right) \\
& + 3 \sum_{a=1}^3 \sum_{b=1}^3 \left[\left(|\Gamma_{Z', \bar{d}_a, d_b}^L|^2 + |\Gamma_{Z', \bar{d}_a, d_b}^R|^2 \right) H_0 \left(p^2, m_{d_a}^2, m_{d_b}^2 \right) \right. \\
& + 4B_0 \left(p^2, m_{d_a}^2, m_{d_b}^2 \right) m_{d_a} m_{d_b} \Re \left(\Gamma_{Z', \bar{d}_a, d_b}^{L*} \Gamma_{Z', \bar{d}_a, d_b}^R \right) \\
& + \sum_{a=1}^3 \sum_{b=1}^3 \left[\left(|\Gamma_{Z', \bar{e}_a, e_b}^L|^2 + |\Gamma_{Z', \bar{e}_a, e_b}^R|^2 \right) H_0 \left(p^2, m_{e_a}^2, m_{e_b}^2 \right) \right. \\
& + 4B_0 \left(p^2, m_{e_a}^2, m_{e_b}^2 \right) m_{e_a} m_{e_b} \Re \left(\Gamma_{Z', \bar{e}_a, e_b}^{L*} \Gamma_{Z', \bar{e}_a, e_b}^R \right) \\
\end{aligned}$$

$$\begin{aligned}
& + 3 \sum_{a=1}^3 \sum_{b=1}^3 \left[\left(|\Gamma_{Z', \bar{u}_a, u_b}^L|^2 + |\Gamma_{Z', \bar{u}_a, u_b}^R|^2 \right) H_0 \left(p^2, m_{u_a}^2, m_{u_b}^2 \right) \right. \\
& + 4B_0 \left(p^2, m_{u_a}^2, m_{u_b}^2 \right) m_{u_a} m_{u_b} \Re \left(\Gamma_{Z', \bar{u}_a, u_b}^{L*} \Gamma_{Z', \bar{u}_a, u_b}^R \right) \left. \right] \\
& + \frac{1}{2} \sum_{a=1}^3 \sum_{b=1}^3 \left[\left(|\Gamma_{Z', S_{1,a}, S_{1,b}}^L|^2 + |\Gamma_{Z', S_{1,a}, S_{1,b}}^R|^2 \right) H_0 \left(p^2, m_{S_{1,a}}^2, m_{S_{1,b}}^2 \right) \right. \\
& + 4B_0 \left(p^2, m_{S_{1,a}}^2, m_{S_{1,b}}^2 \right) m_{S_{1,a}} m_{S_{1,b}} \Re \left(\Gamma_{Z', S_{1,a}, S_{1,b}}^{L*} \Gamma_{Z', S_{1,a}, S_{1,b}}^R \right) \left. \right] \\
& + \frac{1}{2} \sum_{a=1}^9 \sum_{b=1}^9 \left[\left(|\Gamma_{Z', \nu_a, \nu_b}^L|^2 + |\Gamma_{Z', \nu_a, \nu_b}^R|^2 \right) H_0 \left(p^2, m_{\nu_a}^2, m_{\nu_b}^2 \right) \right. \\
& + 4B_0 \left(p^2, m_{\nu_a}^2, m_{\nu_b}^2 \right) m_{\nu_a} m_{\nu_b} \Re \left(\Gamma_{Z', \nu_a, \nu_b}^{L*} \Gamma_{Z', \nu_a, \nu_b}^R \right) \left. \right] \\
& + \sum_{b=1}^2 |\Gamma_{Z', \gamma, h_b}|^2 B_0 \left(p^2, 0, m_{h_b}^2 \right) + \sum_{b=1}^2 |\Gamma_{Z', Z, h_b}|^2 B_0 \left(p^2, m_Z^2, m_{h_b}^2 \right) + \sum_{b=1}^2 |\Gamma_{Z', Z', h_b}|^2 B_0 \left(p^2, m_{Z'}^2, m_{h_b}^2 \right) \\
& + 2\text{rMS} m_{W^-}^2 \Gamma_{Z', Z', W^+, W^-}^1 - A_0 \left(m_{W^-}^2 \right) \left(4\Gamma_{Z', Z', W^+, W^-}^1 + \Gamma_{Z', Z', W^+, W^-}^2 + \Gamma_{Z', Z', W^+, W^-}^3 \right) \quad (105)
\end{aligned}$$

• **Self-Energy for W-Boson** (W^-)

$$\begin{aligned}
\Pi(p^2) & = 2\text{rMS} m_{W^-}^2 \Gamma_{W^-, W^+, W^+, W^-}^1 + 3 \sum_{a=1}^3 \sum_{b=1}^3 \left[\left(|\Gamma_{W^+, \bar{u}_a, d_b}^L|^2 + |\Gamma_{W^+, \bar{u}_a, d_b}^R|^2 \right) H_0 \left(p^2, m_{u_a}^2, m_{d_b}^2 \right) \right. \\
& + 4B_0 \left(p^2, m_{u_a}^2, m_{d_b}^2 \right) m_{d_b} m_{u_a} \Re \left(\Gamma_{W^+, \bar{u}_a, d_b}^{L*} \Gamma_{W^+, \bar{u}_a, d_b}^R \right) \left. \right] - 4 \sum_{b=1}^2 |\Gamma_{W^+, H^-, A_{h,b}}|^2 B_{00} \left(p^2, m_{A_{h,b}}^2, m_{H^-}^2 \right) - 4 \sum_{b=1}^2 |\Gamma_{W^+, H^-, h_b}|^2 \\
& + 4B_0 \left(p^2, m_{\nu_a}^2, m_{e_b}^2 \right) m_{e_b} m_{\nu_a} \Re \left(\Gamma_{W^+, \nu_a, e_b}^{L*} \Gamma_{W^+, \nu_a, e_b}^R \right) \left. \right] + \sum_{b=1}^2 |\Gamma_{W^+, W^-, h_b}|^2 B_0 \left(p^2, m_{W^-}^2, m_{h_b}^2 \right) \quad (106)
\end{aligned}$$

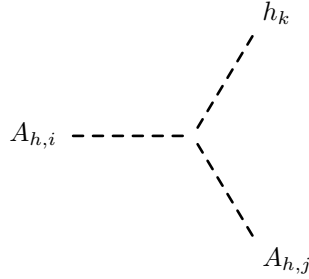
8.2 Tadpoles

$$\begin{aligned}
\delta t_h^{(1)} & = + A_0 \left(m_{\eta^-}^2 \right) \Gamma_{\tilde{h}_i, \eta^-, \eta^-} + A_0 \left(m_{\eta^+}^2 \right) \Gamma_{\tilde{h}_i, \eta^+, \eta^+} + A_0 \left(m_{\eta^Z}^2 \right) \Gamma_{\tilde{h}_i, \eta^Z, \eta^Z} \\
& + A_0 \left(m_{\eta^{Z'}}^2 \right) \Gamma_{\tilde{h}_i, \eta^{Z'}, \eta^{Z'}} - A_0 \left(m_{H^-}^2 \right) \Gamma_{\tilde{h}_i, H^-, H^-} + 4\Gamma_{\tilde{h}_i, W^+, W^-} - \left(-\frac{1}{2} \text{rMS} m_{W^-}^2 + A_0 \left(m_{W^-}^2 \right) \right) \\
& + 2\Gamma_{\tilde{h}_i, Z, Z} \left(-\frac{1}{2} \text{rMS} m_Z^2 + A_0 \left(m_Z^2 \right) \right) + 2\Gamma_{\tilde{h}_i, Z', Z'} \left(-\frac{1}{2} \text{rMS} m_{Z'}^2 + A_0 \left(m_{Z'}^2 \right) \right) - \frac{1}{2} \sum_{a=1}^2 A_0 \left(m_{A_{h,a}}^2 \right) \Gamma_{\tilde{h}_i, A_{h,a}, A_{h,a}} \\
& - \frac{1}{2} \sum_{a=1}^2 A_0 \left(m_{h_a}^2 \right) \Gamma_{\tilde{h}_i, h_a, h_a} + 6 \sum_{a=1}^3 A_0 \left(m_{d_a}^2 \right) m_{d_a} \left(\Gamma_{\tilde{h}_i, \bar{d}_a, d_a}^L + \Gamma_{\tilde{h}_i, \bar{d}_a, d_a}^R \right) \\
& + 2 \sum_{a=1}^3 A_0 \left(m_{e_a}^2 \right) m_{e_a} \left(\Gamma_{\tilde{h}_i, \bar{e}_a, e_a}^L + \Gamma_{\tilde{h}_i, \bar{e}_a, e_a}^R \right)
\end{aligned}$$

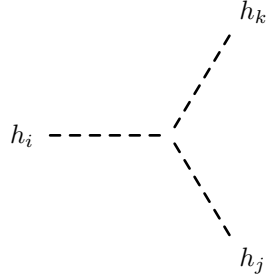
$$+ 6 \sum_{a=1}^3 A_0(m_{u_a}^2) m_{u_a} \left(\Gamma_{\tilde{h}_i, \bar{u}_a, u_a}^L + \Gamma_{\tilde{h}_i, \bar{u}_a, u_a}^R \right) + \sum_{a=1}^9 A_0(m_{\nu_a}^2) m_{\nu_a} \left(\Gamma_{\tilde{h}_i, \nu_a, \nu_a}^L + \Gamma_{\tilde{h}_i, \nu_a, \nu_a}^R \right) \quad (107)$$

9 Interactions for eigenstates 'EWSB'

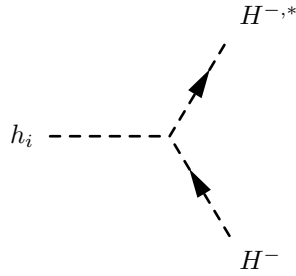
9.1 Three Scalar-Interaction



$$- \frac{i}{2} \left(Z_{i1}^A Z_{j1}^A \left(2\lambda_1 v Z_{k1}^H + \lambda_5 x Z_{k2}^H \right) + Z_{i2}^A Z_{j2}^A \left(4\lambda_3 x Z_{k2}^H + \lambda_5 v Z_{k1}^H \right) \right) \quad (108)$$

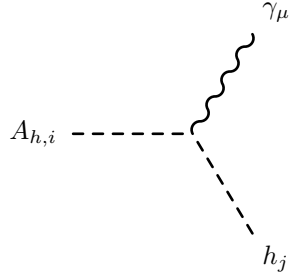


$$- \frac{i}{2} \left(Z_{i2}^H \left(\lambda_5 Z_{j1}^H \left(v Z_{k2}^H + x Z_{k1}^H \right) + Z_{j2}^H \left(12\lambda_3 x Z_{k2}^H + \lambda_5 v Z_{k1}^H \right) \right) + Z_{i1}^H \left(\lambda_5 Z_{j2}^H \left(v Z_{k2}^H + x Z_{k1}^H \right) + Z_{j1}^H \left(6\lambda_1 v Z_{k1}^H + \lambda_5 x Z_{k2}^H \right) \right) \right) \quad (109)$$

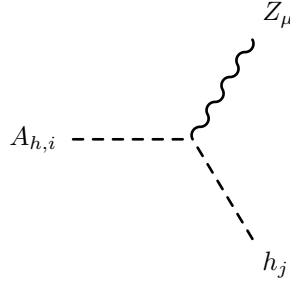


$$-\frac{i}{2} \left(2\lambda_1 v Z_{i1}^H + \lambda_5 x Z_{i2}^H \right) \quad (110)$$

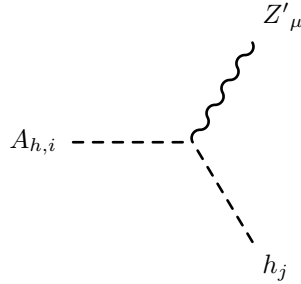
9.2 Two Scalar-One Vector Boson-Interaction



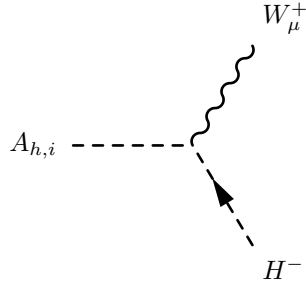
$$\frac{1}{2} \left(- \left(g_1 \cos \Theta_W - g_2 \sin \Theta_W \right) Z_{i1}^A Z_{j1}^H + g_{YB} \cos \Theta_W Z_{i2}^A Z_{j2}^H \right) \left(-p_\mu^{h_j} + p_\mu^{A_{h,i}} \right) \quad (111)$$



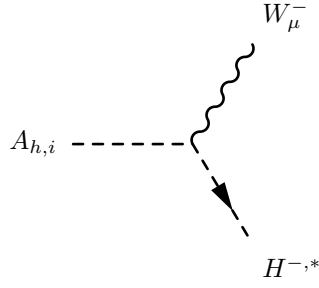
$$\begin{aligned} & \frac{1}{2} \left(\left(g_1 \cos \Theta'_W \sin \Theta_W + g_2 \cos \Theta_W \cos \Theta'_W - g_{BY} \sin \Theta'_W \right) Z_{i1}^A Z_{j1}^H \right. \\ & \left. + \left(g_B \sin \Theta'_W - g_{YB} \cos \Theta'_W \sin \Theta_W \right) Z_{i2}^A Z_{j2}^H \right) \left(-p_\mu^{h_j} + p_\mu^{A_{h,i}} \right) \end{aligned} \quad (112)$$



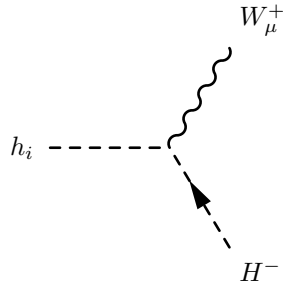
$$\frac{1}{2} \left(- \left((g_1 \sin \Theta_W + g_2 \cos \Theta_W) \sin \Theta'_W + g_{BY} \cos \Theta'_W \right) Z_{i1}^A Z_{j1}^H \right. \\ \left. + \left(g_B \cos \Theta'_W + g_{YB} \sin \Theta_W \sin \Theta'_W \right) Z_{i2}^A Z_{j2}^H \right) \left(- p_\mu^{h_j} + p_\mu^{A_{h,i}} \right) \quad (113)$$



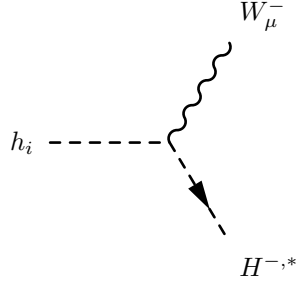
$$\frac{1}{2} g_2 Z_{i1}^A \left(- p_\mu^{H^-} + p_\mu^{A_{h,i}} \right) \quad (114)$$



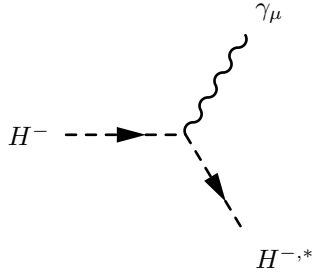
$$\frac{1}{2} g_2 Z_{i1}^A \left(- p_\mu^{H^{-,*}} + p_\mu^{A_{h,i}} \right) \quad (115)$$



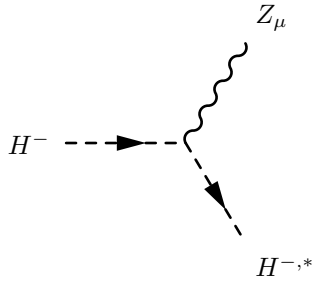
$$\frac{i}{2} g_2 Z_{i1}^H \left(- p_\mu^{H^-} + p_\mu^{h_i} \right) \quad (116)$$



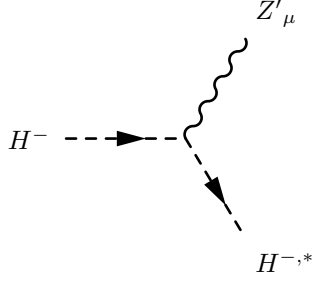
$$-\frac{i}{2}g_2 Z_{i1}^H \left(-p_\mu^{H^-,*} + p_\mu^{h_i} \right) \quad (117)$$



$$\frac{i}{2} \left(g_1 \cos \Theta_W + g_2 \sin \Theta_W \right) \left(-p_\mu^{H^-,*} + p_\mu^{H^-} \right) \quad (118)$$

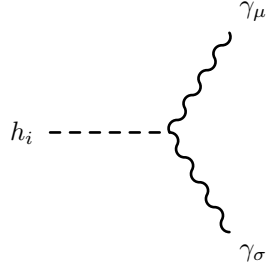


$$\frac{i}{2} \left(-g_1 \cos \Theta'_W \sin \Theta_W + g_2 \cos \Theta_W \cos \Theta'_W + g_{BY} \sin \Theta'_W \right) \left(-p_\mu^{H^-,*} + p_\mu^{H^-} \right) \quad (119)$$

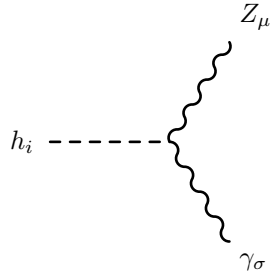


$$\frac{i}{2} \left((g_1 \sin \Theta_W - g_2 \cos \Theta_W) \sin \Theta'_W + g_{BY} \cos \Theta'_W \right) \left(-p_\mu^{H^{-,*}} + p_\mu^{H^-} \right) \quad (120)$$

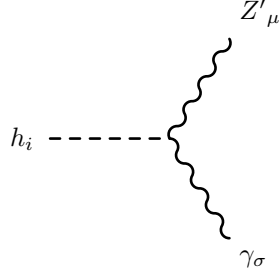
9.3 One Scalar-Two Vector Boson-Interaction



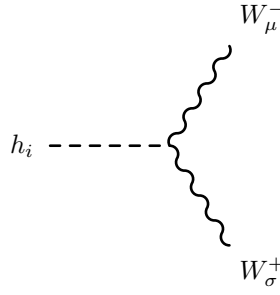
$$\frac{i}{2} \left(g_{YB}^2 x \cos \Theta_W^2 Z_{i2}^H + v \left(g_1 \cos \Theta_W - g_2 \sin \Theta_W \right)^2 Z_{i1}^H \right) \left(g_{\sigma\mu} \right) \quad (121)$$



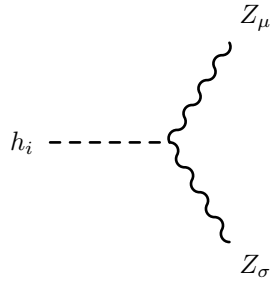
$$\begin{aligned} & -\frac{i}{2} \left(v \left(g_1 \cos \Theta_W - g_2 \sin \Theta_W \right) \left(g_1 \cos \Theta'_W \sin \Theta_W + g_2 \cos \Theta_W \cos \Theta'_W - g_{BY} \sin \Theta'_W \right) Z_{i1}^H \right. \\ & \left. + g_{YB} x \cos \Theta_W \left(-g_B \sin \Theta'_W + g_{YB} \cos \Theta'_W \sin \Theta_W \right) Z_{i2}^H \right) \left(g_{\sigma\mu} \right) \end{aligned} \quad (122)$$



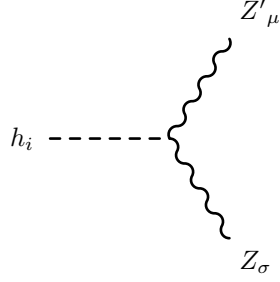
$$\frac{i}{2} \left(v (g_1 \cos \Theta_W - g_2 \sin \Theta_W) \left((g_1 \sin \Theta_W + g_2 \cos \Theta_W) \sin \Theta'_W + g_{BY} \cos \Theta'_W \right) Z_{i1}^H \right. \\ \left. + g_{YB} x \cos \Theta_W (g_B \cos \Theta'_W + g_{YB} \sin \Theta_W \sin \Theta'_W) Z_{i2}^H \right) (g_{\sigma\mu}) \quad (123)$$



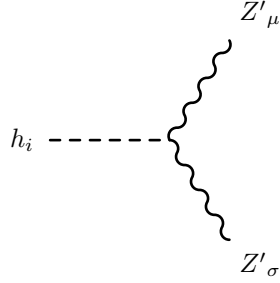
$$\frac{i}{2} g_2^2 v Z_{i1}^H (g_{\sigma\mu}) \quad (124)$$



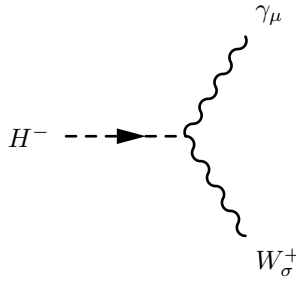
$$\frac{i}{2} \left(v (g_1 \cos \Theta'_W \sin \Theta_W + g_2 \cos \Theta_W \cos \Theta'_W - g_{BY} \sin \Theta'_W) \right)^2 Z_{i1}^H \\ + x \left(-g_B \sin \Theta'_W + g_{YB} \cos \Theta'_W \sin \Theta_W \right)^2 Z_{i2}^H (g_{\sigma\mu}) \quad (125)$$



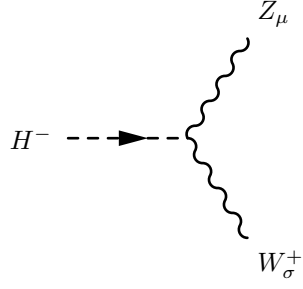
$$\begin{aligned}
& -\frac{i}{2} \left(v \left(g_1 g_{BY} \cos \Theta'_W \sin \Theta_W + g_2^2 \cos \Theta_W^2 \cos \Theta'_W \sin \Theta'_W \right. \right. \\
& + \cos \Theta'_W \left(g_1^2 \sin \Theta_W^2 - g_{BY}^2 \right) \sin \Theta'_W - g_1 g_{BY} \sin \Theta_W \sin \Theta'^2_W \\
& + g_2 \cos \Theta_W \left(g_1 \sin \Theta_W \sin 2\Theta'_W + g_{BY} \cos \Theta'^2_W - g_{BY} \sin \Theta'^2_W \right) \left. \right) Z_{i1}^H \\
& + x \left(g_B g_{YB} \cos \Theta'^2_W \sin \Theta_W + \cos \Theta'_W \left(-g_B^2 + g_{YB}^2 \sin \Theta_W^2 \right) \sin \Theta'_W \right. \\
& \left. - g_B g_{YB} \sin \Theta_W \sin \Theta'^2_W \right) Z_{i2}^H \left(g_{\sigma\mu} \right)
\end{aligned} \tag{126}$$



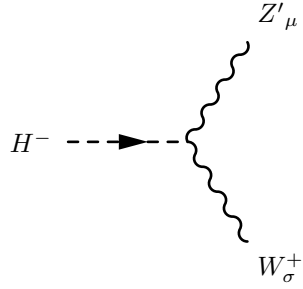
$$\begin{aligned}
& \frac{i}{2} \left(v \left(\left(g_1 \sin \Theta_W + g_2 \cos \Theta_W \right) \sin \Theta'_W + g_{BY} \cos \Theta'_W \right)^2 Z_{i1}^H \right. \\
& \left. + x \left(g_B \cos \Theta'_W + g_{YB} \sin \Theta_W \sin \Theta'_W \right)^2 Z_{i2}^H \right) \left(g_{\sigma\mu} \right)
\end{aligned} \tag{127}$$



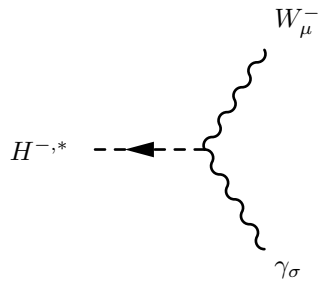
$$-\frac{i}{2}g_1g_2v \cos \Theta_W (g_{\sigma\mu}) \quad (128)$$



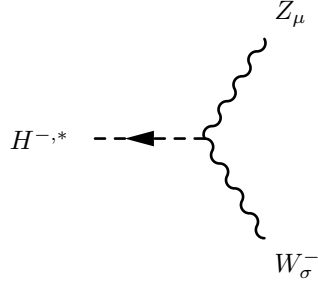
$$\frac{i}{2}g_2v (g_1 \cos \Theta'_W \sin \Theta_W - g_{BY} \sin \Theta'_W) (g_{\sigma\mu}) \quad (129)$$



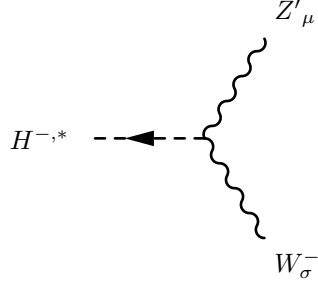
$$-\frac{i}{2}g_2v (g_1 \sin \Theta_W \sin \Theta'_W + g_{BY} \cos \Theta'_W) (g_{\sigma\mu}) \quad (130)$$



$$-\frac{i}{2}g_1g_2v \cos \Theta_W (g_{\sigma\mu}) \quad (131)$$

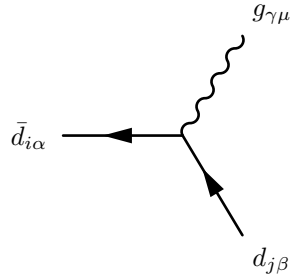


$$\frac{i}{2}g_2v\left(g_1\cos\Theta'_W\sin\Theta_W - g_{BY}\sin\Theta'_W\right)\left(g_{\sigma\mu}\right) \quad (132)$$



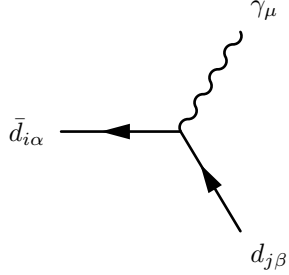
$$-\frac{i}{2}g_2v\left(g_1\sin\Theta_W\sin\Theta'_W + g_{BY}\cos\Theta'_W\right)\left(g_{\sigma\mu}\right) \quad (133)$$

9.4 Two Fermion-One Vector Boson-Interaction



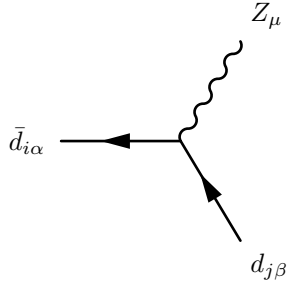
$$-\frac{i}{2}g_3\delta_{ij}\lambda_{\alpha,\beta}^{\gamma}\left(\gamma_{\mu}\cdot\frac{1-\gamma_5}{2}\right) \quad (134)$$

$$+\frac{i}{2}g_3\delta_{ij}\lambda_{\alpha,\beta}^{\gamma}\left(\gamma_{\mu}\cdot\frac{1+\gamma_5}{2}\right) \quad (135)$$



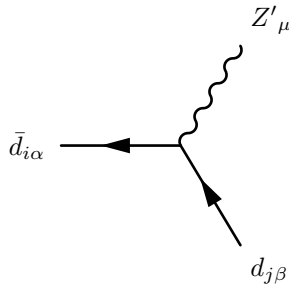
$$-\frac{i}{6}\delta_{\alpha\beta}\delta_{ij}\left(-3g_2\sin\Theta_W+(g_1+g_{YB})\cos\Theta_W\right)\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (136)$$

$$+\frac{i}{6}\left(2g_1-g_{YB}\right)\cos\Theta_W\delta_{\alpha\beta}\delta_{ij}\left(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\right) \quad (137)$$



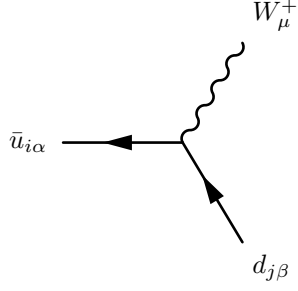
$$\frac{i}{6}\delta_{\alpha\beta}\delta_{ij}\left(3g_2\cos\Theta_W\cos\Theta'_W+(g_1+g_{YB})\cos\Theta'_W\sin\Theta_W-(g_{BY}+g_B)\sin\Theta'_W\right)\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (138)$$

$$+\frac{i}{6}\delta_{\alpha\beta}\delta_{ij}\left(\left(2g_1-g_{YB}\right)\cos\Theta'_W\sin\Theta_W+\left(-2g_{BY}+g_B\right)\sin\Theta'_W\right)\left(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\right) \quad (139)$$

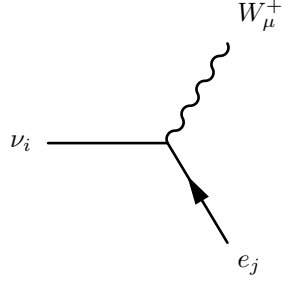


$$-\frac{i}{6}\delta_{\alpha\beta}\delta_{ij}\left(\left(3g_2\cos\Theta_W+(g_1+g_{YB})\sin\Theta_W\right)\sin\Theta'_W+(g_{BY}+g_B)\cos\Theta'_W\right)\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (140)$$

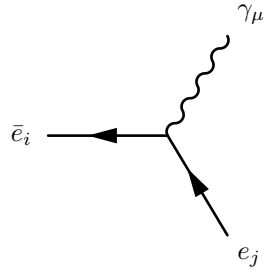
$$+\frac{i}{6}\delta_{\alpha\beta}\delta_{ij}\left(\left(2g_1-g_{YB}\right)\sin\Theta_W\sin\Theta'_W+\left(2g_{BY}-g_B\right)\cos\Theta'_W\right)\left(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\right) \quad (141)$$



$$-i \frac{1}{\sqrt{2}} g_2 \delta_{\alpha\beta} \sum_{a=1}^3 U_{L,ja}^{d,*} U_{L,ia}^u \left(\gamma_\mu \cdot \frac{1-\gamma_5}{2} \right) \quad (142)$$

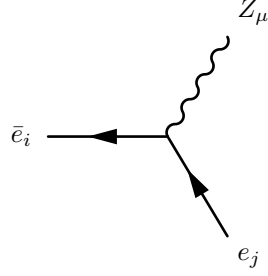


$$-i \frac{1}{\sqrt{2}} g_2 \sum_{a=1}^3 U_{L,ja}^{e,*} U_{ia}^V \left(\gamma_\mu \cdot \frac{1-\gamma_5}{2} \right) \quad (143)$$



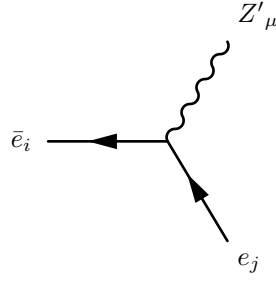
$$\frac{i}{2} \delta_{ij} \left((g_1 + g_{YB}) \cos \Theta_W + g_2 \sin \Theta_W \right) \left(\gamma_\mu \cdot \frac{1-\gamma_5}{2} \right) \quad (144)$$

$$+ \frac{i}{2} (2g_1 + g_{YB}) \cos \Theta_W \delta_{ij} \left(\gamma_\mu \cdot \frac{1+\gamma_5}{2} \right) \quad (145)$$



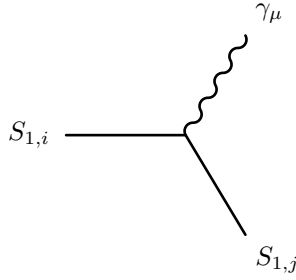
$$\frac{i}{2}\delta_{ij}\left(-\left(g_1 + g_{YB}\right)\cos\Theta'_W\sin\Theta_W + g_2\cos\Theta_W\cos\Theta'_W + \left(g_{BY} + g_B\right)\sin\Theta'_W\right)\left(\gamma_\mu \cdot \frac{1 - \gamma_5}{2}\right) \quad (146)$$

$$+ \frac{i}{2}\delta_{ij}\left(\left(2g_1 + g_{YB}\right)\cos\Theta'_W\sin\Theta_W - \left(2g_{BY} + g_B\right)\sin\Theta'_W\right)\left(\gamma_\mu \cdot \frac{1 + \gamma_5}{2}\right) \quad (147)$$



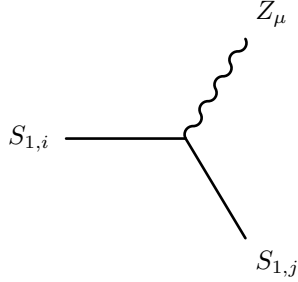
$$\frac{i}{2}\delta_{ij}\left(\left(\left(g_1 + g_{YB}\right)\sin\Theta_W - g_2\cos\Theta_W\right)\sin\Theta'_W + \left(g_{BY} + g_B\right)\cos\Theta'_W\right)\left(\gamma_\mu \cdot \frac{1 - \gamma_5}{2}\right) \quad (148)$$

$$+ \frac{i}{2}\delta_{ij}\left(\left(2g_1 + g_{YB}\right)\sin\Theta_W\sin\Theta'_W + \left(2g_{BY} + g_B\right)\cos\Theta'_W\right)\left(\gamma_\mu \cdot \frac{1 + \gamma_5}{2}\right) \quad (149)$$



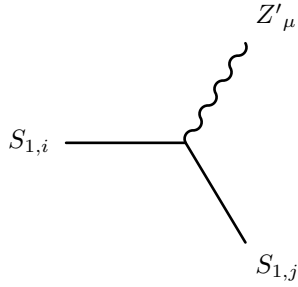
$$- ig_{YB}\cos\Theta_W\delta_{ij}\left(\gamma_\mu \cdot \frac{1 - \gamma_5}{2}\right) \quad (150)$$

$$+ ig_{YB}\cos\Theta_W\delta_{ij}\left(\gamma_\mu \cdot \frac{1 + \gamma_5}{2}\right) \quad (151)$$



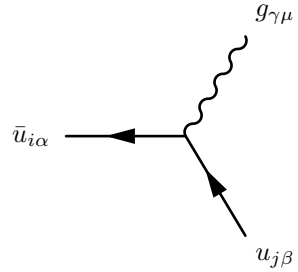
$$i\delta_{ij} \left(-g_B \sin \Theta'_W + g_{YB} \cos \Theta'_W \sin \Theta_W \right) \left(\gamma_{\mu} \cdot \frac{1 - \gamma_5}{2} \right) \quad (152)$$

$$+ -i\delta_{ij} \left(-g_B \sin \Theta'_W + g_{YB} \cos \Theta'_W \sin \Theta_W \right) \left(\gamma_{\mu} \cdot \frac{1 + \gamma_5}{2} \right) \quad (153)$$



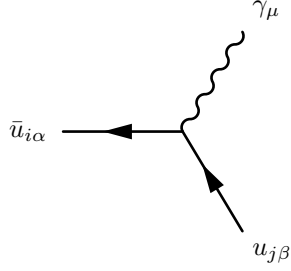
$$-i\delta_{ij} \left(g_B \cos \Theta'_W + g_{YB} \sin \Theta_W \sin \Theta'_W \right) \left(\gamma_{\mu} \cdot \frac{1 - \gamma_5}{2} \right) \quad (154)$$

$$+ i\delta_{ij} \left(g_B \cos \Theta'_W + g_{YB} \sin \Theta_W \sin \Theta'_W \right) \left(\gamma_{\mu} \cdot \frac{1 + \gamma_5}{2} \right) \quad (155)$$



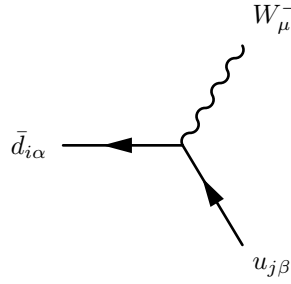
$$- \frac{i}{2} g_3 \delta_{ij} \lambda_{\alpha,\beta}^{\gamma} \left(\gamma_{\mu} \cdot \frac{1 - \gamma_5}{2} \right) \quad (156)$$

$$+ - \frac{i}{2} g_3 \delta_{ij} \lambda_{\alpha,\beta}^{\gamma} \left(\gamma_{\mu} \cdot \frac{1 + \gamma_5}{2} \right) \quad (157)$$

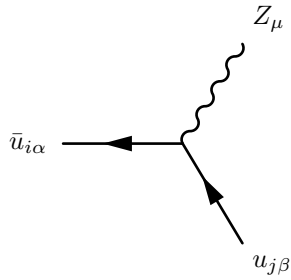


$$-\frac{i}{6}\delta_{\alpha\beta}\delta_{ij}\left(3g_2\sin\Theta_W + (g_1 + g_{YB})\cos\Theta_W\right)\left(\gamma_\mu \cdot \frac{1-\gamma_5}{2}\right) \quad (158)$$

$$+ -\frac{i}{6}\left(4g_1 + g_{YB}\right)\cos\Theta_W\delta_{\alpha\beta}\delta_{ij}\left(\gamma_\mu \cdot \frac{1+\gamma_5}{2}\right) \quad (159)$$

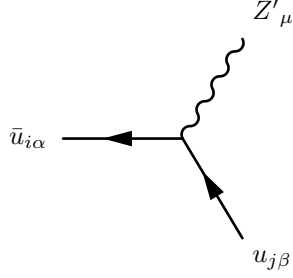


$$-i\frac{1}{\sqrt{2}}g_2\delta_{\alpha\beta}\sum_{a=1}^3U_{L,ja}^{u,*}U_{L,ia}^d\left(\gamma_\mu \cdot \frac{1-\gamma_5}{2}\right) \quad (160)$$



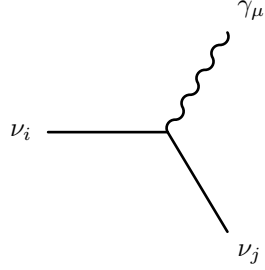
$$-\frac{i}{6}\delta_{\alpha\beta}\delta_{ij}\left(3g_2\cos\Theta_W\cos\Theta'_W - (g_1 + g_{YB})\cos\Theta'_W\sin\Theta_W + (g_{BY} + g_B)\sin\Theta'_W\right)\left(\gamma_\mu \cdot \frac{1-\gamma_5}{2}\right) \quad (161)$$

$$+\frac{i}{6}\delta_{\alpha\beta}\delta_{ij}\left((4g_1 + g_{YB})\cos\Theta'_W\sin\Theta_W - (4g_{BY} + g_B)\sin\Theta'_W\right)\left(\gamma_\mu \cdot \frac{1+\gamma_5}{2}\right) \quad (162)$$



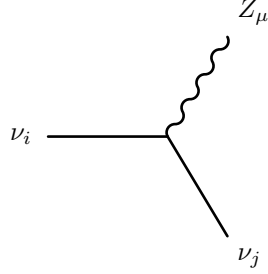
$$-\frac{i}{6}\delta_{\alpha\beta}\delta_{ij}\left(\left(-3g_2\cos\Theta_W+(g_1+g_{YB})\sin\Theta_W\right)\sin\Theta'_W+(g_{BY}+g_B)\cos\Theta'_W\right)\left(\gamma_{\mu}\cdot\frac{1-\gamma_5}{2}\right) \quad (163)$$

$$+\frac{i}{6}\delta_{\alpha\beta}\delta_{ij}\left(\left(4g_1+g_{YB}\right)\sin\Theta_W\sin\Theta'_W+(4g_{BY}+g_B)\cos\Theta'_W\right)\left(\gamma_{\mu}\cdot\frac{1+\gamma_5}{2}\right) \quad (164)$$



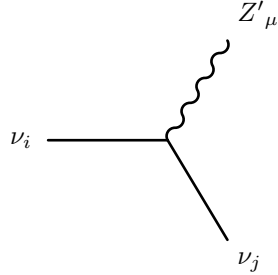
$$\begin{aligned} &\frac{i}{2}\left(\left((g_1+g_{YB})\cos\Theta_W-g_2\sin\Theta_W\right)\sum_{a=1}^3U_{ja}^{V,*}U_{ia}^V\right. \\ &\left.-g_{YB}\cos\Theta_W\left(-2\sum_{a=1}^3U_{j6+a}^{V,*}U_{i6+a}^V+\sum_{a=1}^3U_{j3+a}^{V,*}U_{i3+a}^V\right)\right)\left(\gamma_{\mu}\cdot\frac{1-\gamma_5}{2}\right) \quad (165) \end{aligned}$$

$$\begin{aligned} &+\frac{i}{2}\left(\left((g_1+g_{YB})\cos\Theta_W-g_2\sin\Theta_W\right)\sum_{a=1}^3U_{ia}^{V,*}U_{ja}^V\right. \\ &\left.-g_{YB}\cos\Theta_W\left(-2\sum_{a=1}^3U_{i6+a}^{V,*}U_{j6+a}^V+\sum_{a=1}^3U_{i3+a}^{V,*}U_{j3+a}^V\right)\right)\left(\gamma_{\mu}\cdot\frac{1+\gamma_5}{2}\right) \quad (166) \end{aligned}$$



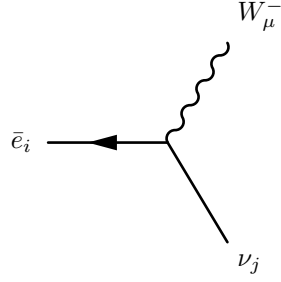
$$\begin{aligned}
& -\frac{i}{2} \left(\left((g_1 + g_{YB}) \cos \Theta'_W \sin \Theta_W + g_2 \cos \Theta_W \cos \Theta'_W - (g_{BY} + g_B) \sin \Theta'_W \right) \sum_{a=1}^3 U_{ja}^{V,*} U_{ia}^V \right. \\
& - \left. \left(-g_B \sin \Theta'_W + g_{YB} \cos \Theta'_W \sin \Theta_W \right) \left(-2 \sum_{a=1}^3 U_{j6+a}^{V,*} U_{i6+a}^V + \sum_{a=1}^3 U_{j3+a}^{V,*} U_{i3+a}^V \right) \right) \left(\gamma_\mu \cdot \frac{1 - \gamma_5}{2} \right) \quad (167)
\end{aligned}$$

$$\begin{aligned}
& + \frac{i}{2} \left(\left((g_1 + g_{YB}) \cos \Theta'_W \sin \Theta_W + g_2 \cos \Theta_W \cos \Theta'_W - (g_{BY} + g_B) \sin \Theta'_W \right) \sum_{a=1}^3 U_{ia}^{V,*} U_{ja}^V \right. \\
& - \left. \left(-g_B \sin \Theta'_W + g_{YB} \cos \Theta'_W \sin \Theta_W \right) \left(-2 \sum_{a=1}^3 U_{i6+a}^{V,*} U_{j6+a}^V + \sum_{a=1}^3 U_{i3+a}^{V,*} U_{j3+a}^V \right) \right) \left(\gamma_\mu \cdot \frac{1 + \gamma_5}{2} \right) \quad (168)
\end{aligned}$$



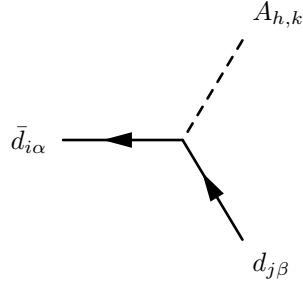
$$\begin{aligned}
& \frac{i}{2} \left(\left(\left((g_1 + g_{YB}) \sin \Theta_W + g_2 \cos \Theta_W \right) \sin \Theta'_W + (g_{BY} + g_B) \cos \Theta'_W \right) \sum_{a=1}^3 U_{ja}^{V,*} U_{ia}^V \right. \\
& - \left. \left(g_B \cos \Theta'_W + g_{YB} \sin \Theta_W \sin \Theta'_W \right) \left(-2 \sum_{a=1}^3 U_{j6+a}^{V,*} U_{i6+a}^V + \sum_{a=1}^3 U_{j3+a}^{V,*} U_{i3+a}^V \right) \right) \left(\gamma_\mu \cdot \frac{1 - \gamma_5}{2} \right) \quad (169)
\end{aligned}$$

$$\begin{aligned}
& + \frac{i}{2} \left(\left(\left((g_1 + g_{YB}) \sin \Theta_W + g_2 \cos \Theta_W \right) \sin \Theta'_W + (g_{BY} + g_B) \cos \Theta'_W \right) \sum_{a=1}^3 U_{ia}^{V,*} U_{ja}^V \right. \\
& - \left. \left(g_B \cos \Theta'_W + g_{YB} \sin \Theta_W \sin \Theta'_W \right) \left(-2 \sum_{a=1}^3 U_{i6+a}^{V,*} U_{j6+a}^V + \sum_{a=1}^3 U_{i3+a}^{V,*} U_{j3+a}^V \right) \right) \left(\gamma_\mu \cdot \frac{1 + \gamma_5}{2} \right) \quad (170)
\end{aligned}$$



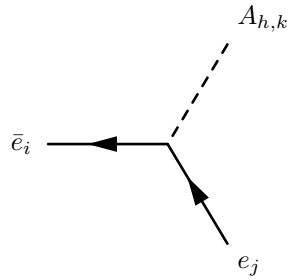
$$-i \frac{1}{\sqrt{2}} g_2 \sum_{a=1}^3 U_{ja}^{V,*} U_{L,ia}^e \left(\gamma_\mu \cdot \frac{1 - \gamma_5}{2} \right) \quad (171)$$

9.5 Two Fermion-One Scalar Boson-Interaction



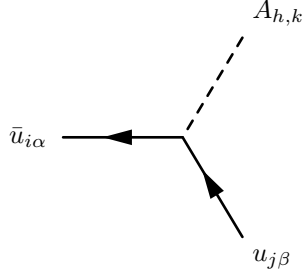
$$\frac{1}{\sqrt{2}} \delta_{\alpha\beta} \sum_{b=1}^3 U_{L,jb}^{d,*} \sum_{a=1}^3 U_{R,ia}^{d,*} Y_{d,ab} Z_{k1}^A \left(\frac{1 - \gamma_5}{2} \right) \quad (172)$$

$$+ -\frac{1}{\sqrt{2}} \delta_{\alpha\beta} \sum_{b=1}^3 \sum_{a=1}^3 Y_{d,ab}^* U_{R,ja}^d U_{L,ib}^d Z_{k1}^A \left(\frac{1 + \gamma_5}{2} \right) \quad (173)$$



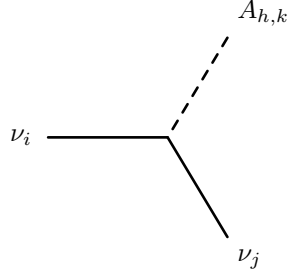
$$\frac{1}{\sqrt{2}} \sum_{b=1}^3 U_{L,jb}^{e,*} \sum_{a=1}^3 U_{R,ia}^{e,*} Y_{e,ab} Z_{k1}^A \left(\frac{1-\gamma_5}{2} \right) \quad (174)$$

$$+ -\frac{1}{\sqrt{2}} \sum_{b=1}^3 \sum_{a=1}^3 Y_{e,ab}^* U_{R,ja}^e U_{L,ib}^e Z_{k1}^A \left(\frac{1+\gamma_5}{2} \right) \quad (175)$$



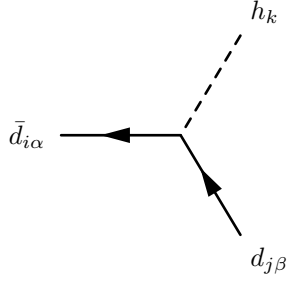
$$- \frac{1}{\sqrt{2}} \delta_{\alpha\beta} \sum_{b=1}^3 U_{L,jb}^{u,*} \sum_{a=1}^3 U_{R,ia}^{u,*} Y_{u,ab} Z_{k1}^A \left(\frac{1-\gamma_5}{2} \right) \quad (176)$$

$$+ \frac{1}{\sqrt{2}} \delta_{\alpha\beta} \sum_{b=1}^3 \sum_{a=1}^3 Y_{u,ab}^* U_{R,ja}^u U_{L,ib}^u Z_{k1}^A \left(\frac{1+\gamma_5}{2} \right) \quad (177)$$



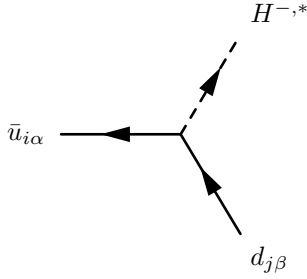
$$\begin{aligned} & \frac{1}{\sqrt{2}} \left(- \sum_{b=1}^3 U_{j3+b}^{V,*} \sum_{a=1}^3 U_{ia}^{V,*} Y_{\nu,ab} Z_{k1}^A - \sum_{b=1}^3 U_{i3+b}^{V,*} \sum_{a=1}^3 U_{ja}^{V,*} Y_{\nu,ab} Z_{k1}^A \right. \\ & + \left. \left(\sum_{b=1}^3 U_{j6+b}^{V,*} \sum_{a=1}^3 U_{i3+a}^{V,*} Y_{S,ab} + \sum_{b=1}^3 U_{i6+b}^{V,*} \sum_{a=1}^3 U_{j3+a}^{V,*} Y_{S,ab} \right) Z_{k2}^A \right) \left(\frac{1-\gamma_5}{2} \right) \quad (178) \\ & + \frac{1}{\sqrt{2}} \left(\sum_{b=1}^3 \sum_{a=1}^3 Y_{\nu,ab}^* U_{ja}^V U_{i3+b}^V Z_{k1}^A + \sum_{b=1}^3 \sum_{a=1}^3 Y_{\nu,ab}^* U_{ia}^V U_{j3+b}^V Z_{k1}^A \right) \end{aligned}$$

$$- \left(\sum_{b=1}^3 \sum_{a=1}^3 Y_{S,ab}^* U_{j3+a}^V U_{i6+b}^V + \sum_{b=1}^3 \sum_{a=1}^3 Y_{S,ab}^* U_{i3+a}^V U_{j6+b}^V \right) Z_{k2}^A \left(\frac{1+\gamma_5}{2} \right) \quad (179)$$



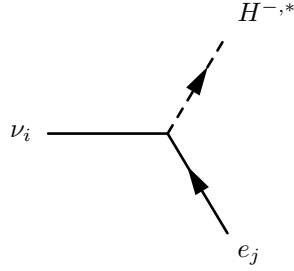
$$- i \frac{1}{\sqrt{2}} \delta_{\alpha\beta} \sum_{b=1}^3 U_{L,jb}^{d,*} \sum_{a=1}^3 U_{R,ia}^{d,*} Y_{d,ab} Z_{k1}^H \left(\frac{1-\gamma_5}{2} \right) \quad (180)$$

$$+ -i \frac{1}{\sqrt{2}} \delta_{\alpha\beta} \sum_{b=1}^3 \sum_{a=1}^3 Y_{d,ab}^* U_{R,ja}^d U_{L,ib}^d Z_{k1}^H \left(\frac{1+\gamma_5}{2} \right) \quad (181)$$



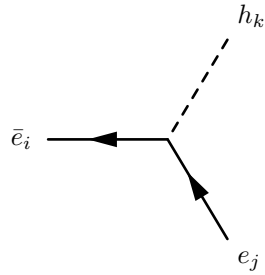
$$- i \delta_{\alpha\beta} \sum_{b=1}^3 U_{L,jb}^{d,*} \sum_{a=1}^3 U_{R,ia}^{u,*} Y_{u,ab} \left(\frac{1-\gamma_5}{2} \right) \quad (182)$$

$$+ i \delta_{\alpha\beta} \sum_{b=1}^3 \sum_{a=1}^3 Y_{d,ab}^* U_{R,ja}^d U_{L,ib}^u \left(\frac{1+\gamma_5}{2} \right) \quad (183)$$



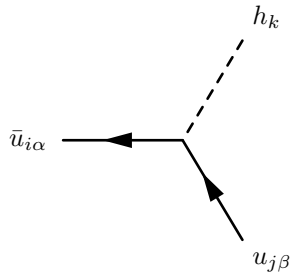
$$-i \sum_{b=1}^3 U_{i3+b}^{V,*} \sum_{a=1}^3 U_{L,ja}^{e,*} Y_{\nu,ab} \left(\frac{1-\gamma_5}{2} \right) \quad (184)$$

$$+ i \sum_{b=1}^3 \sum_{a=1}^3 Y_{e,ab}^* U_{R,ja}^e U_{ib}^V \left(\frac{1+\gamma_5}{2} \right) \quad (185)$$



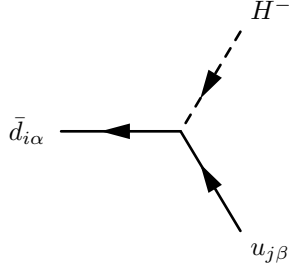
$$-i \frac{1}{\sqrt{2}} \sum_{b=1}^3 U_{L,jb}^{e,*} \sum_{a=1}^3 U_{R,ia}^{e,*} Y_{e,ab} Z_{k1}^H \left(\frac{1-\gamma_5}{2} \right) \quad (186)$$

$$+ -i \frac{1}{\sqrt{2}} \sum_{b=1}^3 \sum_{a=1}^3 Y_{e,ab}^* U_{R,ja}^e U_{L,ib}^e Z_{k1}^H \left(\frac{1+\gamma_5}{2} \right) \quad (187)$$



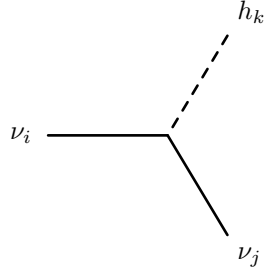
$$-i \frac{1}{\sqrt{2}} \delta_{\alpha\beta} \sum_{b=1}^3 U_{L,jb}^{u,*} \sum_{a=1}^3 U_{R,ia}^{u,*} Y_{u,ab} Z_{k1}^H \left(\frac{1-\gamma_5}{2} \right) \quad (188)$$

$$+ -i \frac{1}{\sqrt{2}} \delta_{\alpha\beta} \sum_{b=1}^3 \sum_{a=1}^3 Y_{u,ab}^* U_{R,ja}^u U_{L,ib}^u Z_{k1}^H \left(\frac{1+\gamma_5}{2} \right) \quad (189)$$



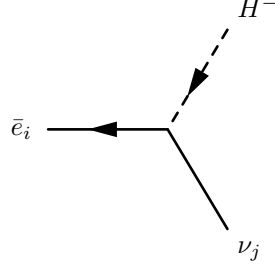
$$i \delta_{\alpha\beta} \sum_{b=1}^3 U_{L,jb}^{u,*} \sum_{a=1}^3 U_{R,ia}^{d,*} Y_{d,ab} \left(\frac{1-\gamma_5}{2} \right) \quad (190)$$

$$+ -i \delta_{\alpha\beta} \sum_{b=1}^3 \sum_{a=1}^3 Y_{u,ab}^* U_{R,ja}^u U_{L,ib}^d \left(\frac{1+\gamma_5}{2} \right) \quad (191)$$



$$-i \frac{1}{\sqrt{2}} \left(\sum_{b=1}^3 U_{j3+b}^{V,*} \sum_{a=1}^3 U_{ia}^{V,*} Y_{\nu,ab} Z_{k1}^H + \sum_{b=1}^3 U_{i3+b}^{V,*} \sum_{a=1}^3 U_{ja}^{V,*} Y_{\nu,ab} Z_{k1}^H \right. \\ \left. + \left(\sum_{b=1}^3 U_{j6+b}^{V,*} \sum_{a=1}^3 U_{i3+a}^{V,*} Y_{S,ab} + \sum_{b=1}^3 U_{i6+b}^{V,*} \sum_{a=1}^3 U_{j3+a}^{V,*} Y_{S,ab} \right) Z_{k2}^H \right) \left(\frac{1-\gamma_5}{2} \right) \quad (192) \\ + -i \frac{1}{\sqrt{2}} \left(\sum_{b=1}^3 \sum_{a=1}^3 Y_{\nu,ab}^* U_{ja}^V U_{i3+b}^V Z_{k1}^H + \sum_{b=1}^3 \sum_{a=1}^3 Y_{\nu,ab}^* U_{ia}^V U_{j3+b}^V Z_{k1}^H \right)$$

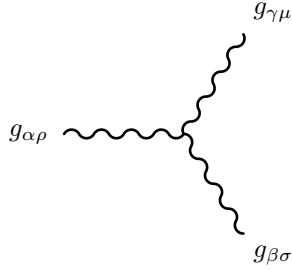
$$+ \left(\sum_{b=1}^3 \sum_{a=1}^3 Y_{S,ab}^* U_{j3+a}^V U_{i6+b}^V + \sum_{b=1}^3 \sum_{a=1}^3 Y_{S,ab}^* U_{i3+a}^V U_{j6+b}^V \right) Z_{k2}^H \left(\frac{1+\gamma_5}{2} \right) \quad (193)$$



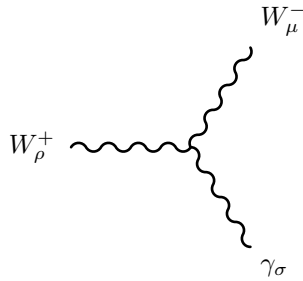
$$i \sum_{b=1}^3 U_{jb}^{V,*} \sum_{a=1}^3 U_{R,ia}^{e,*} Y_{e,ab} \left(\frac{1-\gamma_5}{2} \right) \quad (194)$$

$$+ -i \sum_{b=1}^3 \sum_{a=1}^3 Y_{\nu,ab}^* U_{L,ia}^e U_{j3+b}^V \left(\frac{1+\gamma_5}{2} \right) \quad (195)$$

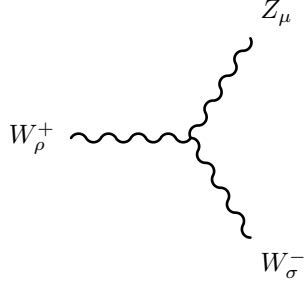
9.6 Three Vector Boson-Interaction



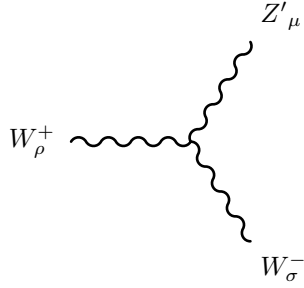
$$g_3 f_{\alpha,\beta,\gamma} \left(g_{\rho\mu} \left(-p_\sigma^{g\gamma\mu} + p_\sigma^{g\alpha\rho} \right) + g_{\rho\sigma} \left(-p_\mu^{g\alpha\rho} + p_\mu^{g\beta\sigma} \right) + g_{\sigma\mu} \left(-p_\rho^{g\beta\sigma} + p_\rho^{g\gamma\mu} \right) \right) \quad (196)$$



$$ig_2 \sin \Theta_W \left(g_{\rho\mu} \left(-p_\sigma^{W^-} + p_\sigma^{W^+} \right) + g_{\rho\sigma} \left(-p_\mu^{W^+} + p_\mu^{\gamma\sigma} \right) + g_{\sigma\mu} \left(-p_\rho^{\gamma\sigma} + p_\rho^{W^-} \right) \right) \quad (197)$$

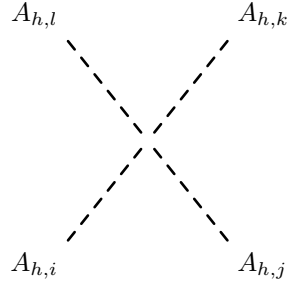


$$-ig_2 \cos \Theta_W \cos \Theta'_W \left(g_{\rho\mu} \left(-p_\sigma^{Z_\mu} + p_\sigma^{W^+} \right) + g_{\rho\sigma} \left(-p_\mu^{W^+} + p_\mu^{W^-} \right) + g_{\sigma\mu} \left(-p_\rho^{W^-} + p_\rho^{Z_\mu} \right) \right) \quad (198)$$

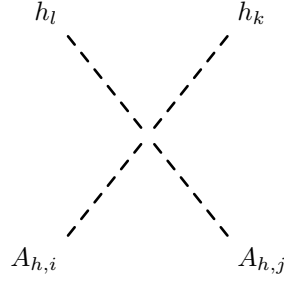


$$ig_2 \cos \Theta_W \sin \Theta'_W \left(g_{\rho\mu} \left(-p_\sigma^{Z'_\mu} + p_\sigma^{W^+} \right) + g_{\rho\sigma} \left(-p_\mu^{W^+} + p_\mu^{W^-} \right) + g_{\sigma\mu} \left(-p_\rho^{W^-} + p_\rho^{Z'_\mu} \right) \right) \quad (199)$$

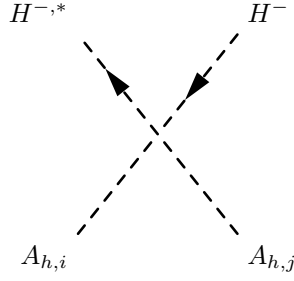
9.7 Four Scalar-Interaction



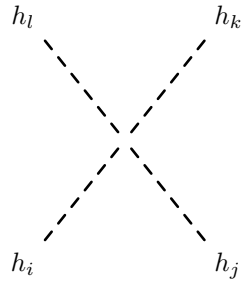
$$\begin{aligned}
& -\frac{i}{2} \left(Z_{i2}^A \left(\lambda_5 Z_{j1}^A \left(Z_{k1}^A Z_{l2}^A + Z_{k2}^A Z_{l1}^A \right) \right) + Z_{j2}^A \left(12\lambda_3 Z_{k2}^A Z_{l2}^A + \lambda_5 Z_{k1}^A Z_{l1}^A \right) \right) \\
& + Z_{i1}^A \left(\lambda_5 Z_{j2}^A \left(Z_{k1}^A Z_{l2}^A + Z_{k2}^A Z_{l1}^A \right) \right) + Z_{j1}^A \left(6\lambda_1 Z_{k1}^A Z_{l1}^A + \lambda_5 Z_{k2}^A Z_{l2}^A \right) \Big) \quad (200)
\end{aligned}$$



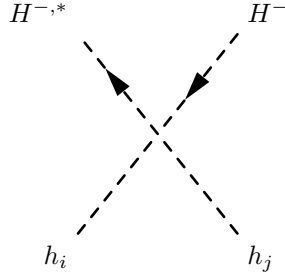
$$-\frac{i}{2} \left(Z_{i1}^A Z_{j1}^A \left(2\lambda_1 Z_{k1}^H Z_{l1}^H + \lambda_5 Z_{k2}^H Z_{l2}^H \right) + Z_{i2}^A Z_{j2}^A \left(4\lambda_3 Z_{k2}^H Z_{l2}^H + \lambda_5 Z_{k1}^H Z_{l1}^H \right) \right) \quad (201)$$



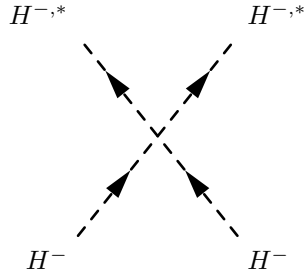
$$-\frac{i}{2} \left(2\lambda_1 Z_{i1}^A Z_{j1}^A + \lambda_5 Z_{i2}^A Z_{j2}^A \right) \quad (202)$$



$$\begin{aligned}
& -\frac{i}{2} \left(Z_{i2}^H \left(\lambda_5 Z_{j1}^H \left(Z_{k1}^H Z_{l2}^H + Z_{k2}^H Z_{l1}^H \right) + Z_{j2}^H \left(12\lambda_3 Z_{k2}^H Z_{l2}^H + \lambda_5 Z_{k1}^H Z_{l1}^H \right) \right) \right. \\
& \left. + Z_{i1}^H \left(\lambda_5 Z_{j2}^H \left(Z_{k1}^H Z_{l2}^H + Z_{k2}^H Z_{l1}^H \right) + Z_{j1}^H \left(6\lambda_1 Z_{k1}^H Z_{l1}^H + \lambda_5 Z_{k2}^H Z_{l2}^H \right) \right) \right)
\end{aligned} \tag{203}$$

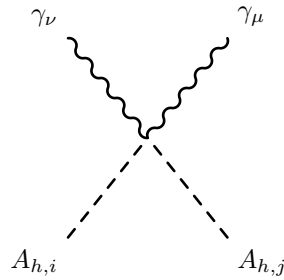


$$-\frac{i}{2} \left(2\lambda_1 Z_{i1}^H Z_{j1}^H + \lambda_5 Z_{i2}^H Z_{j2}^H \right) \tag{204}$$

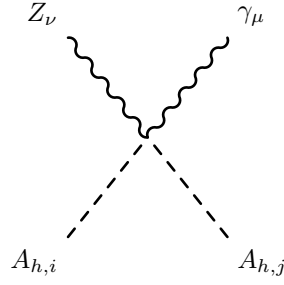


$$-2i\lambda_1 \tag{205}$$

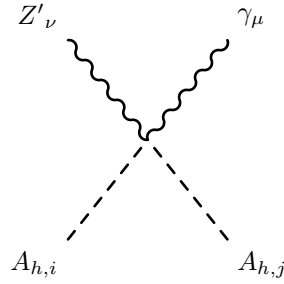
9.8 Two Scalar-Two Vector Boson-Interaction



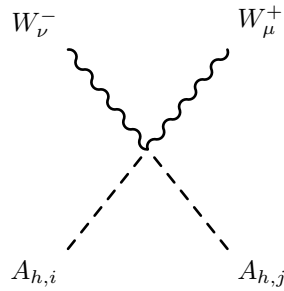
$$\frac{i}{2} \left((g_1 \cos \Theta_W - g_2 \sin \Theta_W)^2 Z_{i1}^A Z_{j1}^A + g_{YB}^2 \cos^2 \Theta_W Z_{i2}^A Z_{j2}^A \right) (g_{\mu\nu}) \quad (206)$$



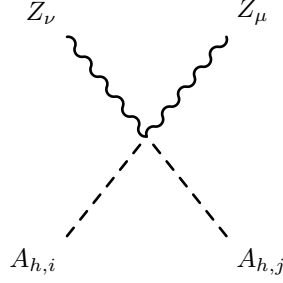
$$-\frac{i}{2} \left((g_1 \cos \Theta_W - g_2 \sin \Theta_W) (g_1 \cos \Theta'_W \sin \Theta_W + g_2 \cos \Theta_W \cos \Theta'_W - g_{BY} \sin \Theta'_W) Z_{i1}^A Z_{j1}^A + g_{YB} \cos \Theta_W (-g_B \sin \Theta'_W + g_{YB} \cos \Theta'_W \sin \Theta_W) Z_{i2}^A Z_{j2}^A \right) (g_{\mu\nu}) \quad (207)$$



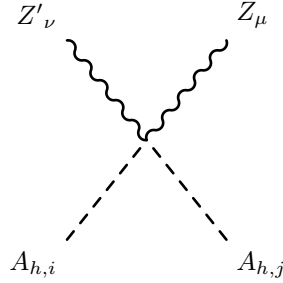
$$\frac{i}{2} \left((g_1 \cos \Theta_W - g_2 \sin \Theta_W) \left((g_1 \sin \Theta_W + g_2 \cos \Theta_W) \sin \Theta'_W + g_{BY} \cos \Theta'_W \right) Z_{i1}^A Z_{j1}^A + g_{YB} \cos \Theta_W (g_B \cos \Theta'_W + g_{YB} \sin \Theta_W \sin \Theta'_W) Z_{i2}^A Z_{j2}^A \right) (g_{\mu\nu}) \quad (208)$$



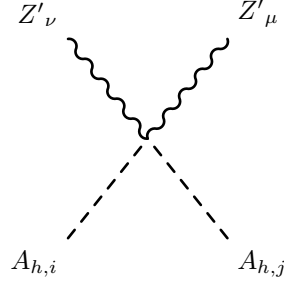
$$\frac{i}{2} g_2^2 Z_{i1}^A Z_{j1}^A (g_{\mu\nu}) \quad (209)$$



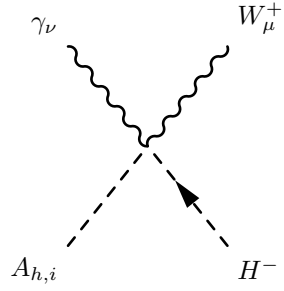
$$\begin{aligned} & \frac{i}{2} \left(\left(g_1 \cos \Theta'_W \sin \Theta_W + g_2 \cos \Theta_W \cos \Theta'_W - g_{BY} \sin \Theta'_W \right)^2 Z_{i1}^A Z_{j1}^A \right. \\ & \left. + \left(-g_B \sin \Theta'_W + g_{YB} \cos \Theta'_W \sin \Theta_W \right)^2 Z_{i2}^A Z_{j2}^A \right) (g_{\mu\nu}) \end{aligned} \quad (210)$$



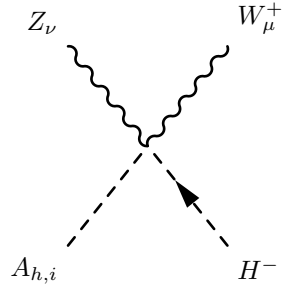
$$\begin{aligned} & -\frac{i}{2} \left(\left(g_1 g_{BY} \cos \Theta'^2_W \sin \Theta_W + g_2^2 \cos \Theta_W^2 \cos \Theta'_W \sin \Theta'_W \right. \right. \\ & \left. \left. + \cos \Theta'_W \left(g_1^2 \sin \Theta_W^2 - g_{BY}^2 \right) \sin \Theta'_W - g_1 g_{BY} \sin \Theta_W \sin \Theta'^2_W \right. \right. \\ & \left. \left. + g_2 \cos \Theta_W \left(g_1 \sin \Theta_W \sin 2\Theta'_W + g_{BY} \cos \Theta'^2_W - g_{BY} \sin \Theta'^2_W \right) \right) Z_{i1}^A Z_{j1}^A \right. \\ & \left. + \left(g_B g_{YB} \cos \Theta'^2_W \sin \Theta_W + \cos \Theta'_W \left(-g_B^2 + g_{YB}^2 \sin \Theta_W^2 \right) \sin \Theta'_W \right. \right. \\ & \left. \left. - g_B g_{YB} \sin \Theta_W \sin \Theta'^2_W \right) Z_{i2}^A Z_{j2}^A \right) (g_{\mu\nu}) \end{aligned} \quad (211)$$



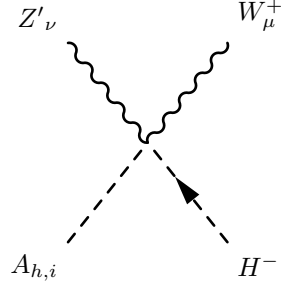
$$\frac{i}{2} \left(\left((g_1 \sin \Theta_W + g_2 \cos \Theta_W) \sin \Theta'_W + g_{BY} \cos \Theta'_W \right)^2 Z_{i1}^A Z_{j1}^A + \left(g_B \cos \Theta'_W + g_{YB} \sin \Theta_W \sin \Theta'_W \right)^2 Z_{i2}^A Z_{j2}^A \right) (g_{\mu\nu}) \quad (212)$$



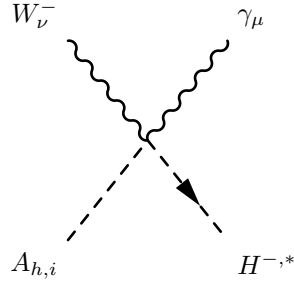
$$-\frac{1}{2} g_1 g_2 \cos \Theta_W Z_{i1}^A (g_{\mu\nu}) \quad (213)$$



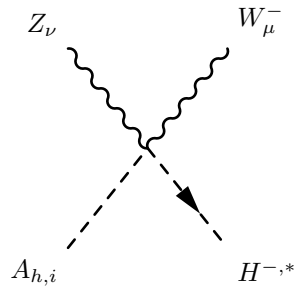
$$\frac{1}{2} g_2 \left(g_1 \cos \Theta'_W \sin \Theta_W - g_{BY} \sin \Theta'_W \right) Z_{i1}^A (g_{\mu\nu}) \quad (214)$$



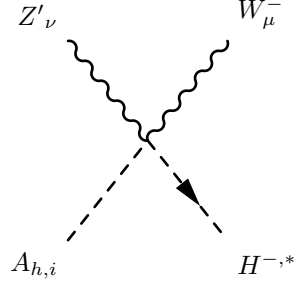
$$-\frac{1}{2}g_2(g_1 \sin \Theta_W \sin \Theta'_W + g_{BY} \cos \Theta'_W)Z_{i1}^A(g_{\mu\nu}) \quad (215)$$



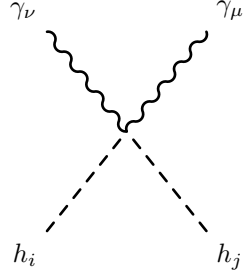
$$\frac{1}{2}g_1g_2 \cos \Theta_W Z_{i1}^A(g_{\mu\nu}) \quad (216)$$



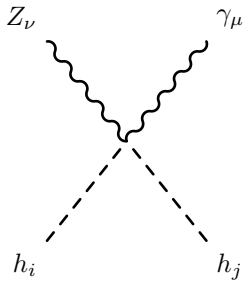
$$\frac{1}{2}g_2(-g_1 \cos \Theta'_W \sin \Theta_W + g_{BY} \sin \Theta'_W)Z_{i1}^A(g_{\mu\nu}) \quad (217)$$



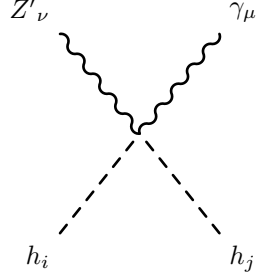
$$\frac{1}{2}g_2(g_1 \sin \Theta_W \sin \Theta'_W + g_{BY} \cos \Theta'_W)Z_{i1}^A(g_{\mu\nu}) \quad (218)$$



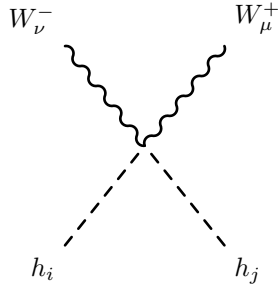
$$\frac{i}{2}\left(\left(g_1 \cos \Theta_W - g_2 \sin \Theta_W\right)^2 Z_{i1}^H Z_{j1}^H + g_{YB}^2 \cos^2 \Theta_W Z_{i2}^H Z_{j2}^H\right)(g_{\mu\nu}) \quad (219)$$



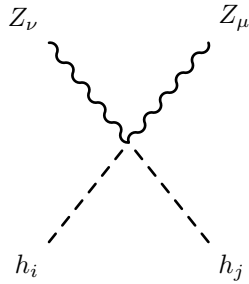
$$\begin{aligned} & -\frac{i}{2}\left(\left(g_1 \cos \Theta_W - g_2 \sin \Theta_W\right)\left(g_1 \cos \Theta'_W \sin \Theta_W + g_2 \cos \Theta_W \cos \Theta'_W - g_{BY} \sin \Theta'_W\right)Z_{i1}^H Z_{j1}^H \right. \\ & \left. + g_{YB} \cos \Theta_W\left(-g_B \sin \Theta'_W + g_{YB} \cos \Theta'_W \sin \Theta_W\right)Z_{i2}^H Z_{j2}^H\right)(g_{\mu\nu}) \quad (220) \end{aligned}$$



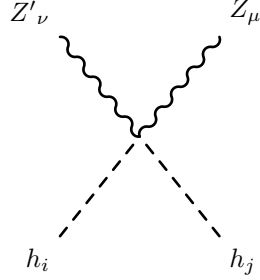
$$\begin{aligned} & \frac{i}{2} \left((g_1 \cos \Theta_W - g_2 \sin \Theta_W) \left((g_1 \sin \Theta_W + g_2 \cos \Theta_W) \sin \Theta'_W + g_{BY} \cos \Theta'_W \right) Z_{i1}^H Z_{j1}^H \right. \\ & \left. + g_{YB} \cos \Theta_W \left(g_B \cos \Theta'_W + g_{YB} \sin \Theta_W \sin \Theta'_W \right) Z_{i2}^H Z_{j2}^H \right) (g_{\mu\nu}) \end{aligned} \quad (221)$$



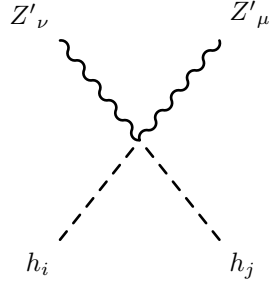
$$\frac{i}{2} g_2^2 Z_{i1}^H Z_{j1}^H (g_{\mu\nu}) \quad (222)$$



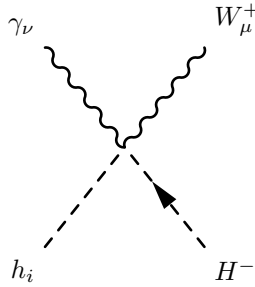
$$\begin{aligned} & \frac{i}{2} \left((g_1 \cos \Theta'_W \sin \Theta_W + g_2 \cos \Theta_W \cos \Theta'_W - g_{BY} \sin \Theta'_W) \right)^2 Z_{i1}^H Z_{j1}^H \\ & + \left(-g_B \sin \Theta'_W + g_{YB} \cos \Theta'_W \sin \Theta_W \right)^2 Z_{i2}^H Z_{j2}^H (g_{\mu\nu}) \end{aligned} \quad (223)$$



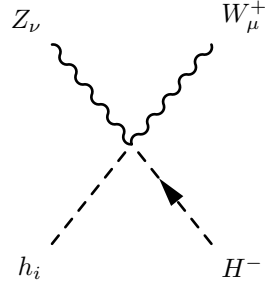
$$\begin{aligned}
& -\frac{i}{2} \left(\left(g_1 g_{BY} \cos \Theta'_W \sin \Theta_W + g_2^2 \cos \Theta_W^2 \cos \Theta'_W \sin \Theta'_W \right. \right. \\
& + \cos \Theta'_W \left(g_1^2 \sin \Theta_W^2 - g_{BY}^2 \right) \sin \Theta'_W - g_1 g_{BY} \sin \Theta_W \sin \Theta'^2_W \\
& + g_2 \cos \Theta_W \left(g_1 \sin \Theta_W \sin 2\Theta'_W + g_{BY} \cos \Theta'^2_W - g_{BY} \sin \Theta'^2_W \right) \left. \right) Z_{i1}^H Z_{j1}^H \\
& + \left(g_B g_{YB} \cos \Theta'^2_W \sin \Theta_W + \cos \Theta'_W \left(-g_B^2 + g_{YB}^2 \sin \Theta_W^2 \right) \sin \Theta'_W \right. \\
& \left. - g_B g_{YB} \sin \Theta_W \sin \Theta'^2_W \right) Z_{i2}^H Z_{j2}^H \left(g_{\mu\nu} \right)
\end{aligned} \tag{224}$$



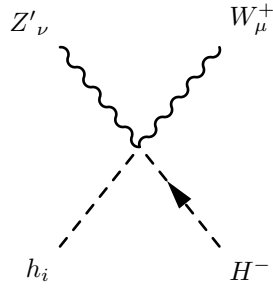
$$\begin{aligned}
& \frac{i}{2} \left(\left(\left(g_1 \sin \Theta_W + g_2 \cos \Theta_W \right) \sin \Theta'_W + g_{BY} \cos \Theta'_W \right)^2 Z_{i1}^H Z_{j1}^H \right. \\
& \left. + \left(g_B \cos \Theta'_W + g_{YB} \sin \Theta_W \sin \Theta'_W \right)^2 Z_{i2}^H Z_{j2}^H \right) \left(g_{\mu\nu} \right)
\end{aligned} \tag{225}$$



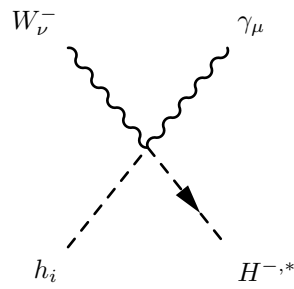
$$-\frac{i}{2}g_1g_2\cos\Theta_W Z_{i1}^H(g_{\mu\nu}) \quad (226)$$



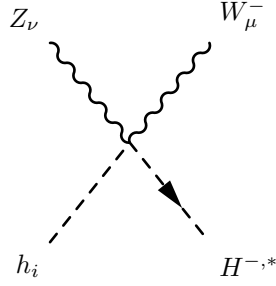
$$\frac{i}{2}g_2(g_1\cos\Theta'_W\sin\Theta_W - g_{BY}\sin\Theta'_W)Z_{i1}^H(g_{\mu\nu}) \quad (227)$$



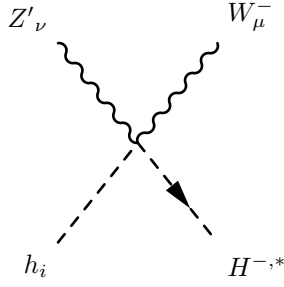
$$-\frac{i}{2}g_2(g_1\sin\Theta_W\sin\Theta'_W + g_{BY}\cos\Theta'_W)Z_{i1}^H(g_{\mu\nu}) \quad (228)$$



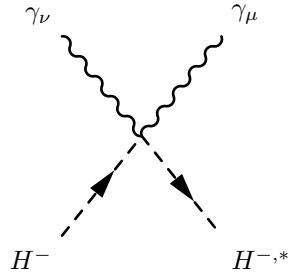
$$-\frac{i}{2}g_1g_2\cos\Theta_W Z_{i1}^H(g_{\mu\nu}) \quad (229)$$



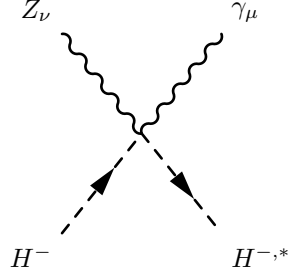
$$\frac{i}{2}g_2\left(g_1\cos\Theta'_W\sin\Theta_W-g_{BY}\sin\Theta'_W\right)Z_{i1}^H\left(g_{\mu\nu}\right) \quad (230)$$



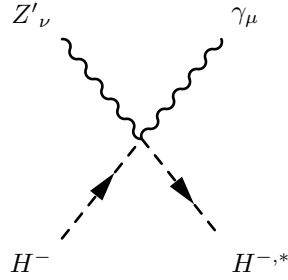
$$-\frac{i}{2}g_2\left(g_1\sin\Theta_W\sin\Theta'_W+g_{BY}\cos\Theta'_W\right)Z_{i1}^H\left(g_{\mu\nu}\right) \quad (231)$$



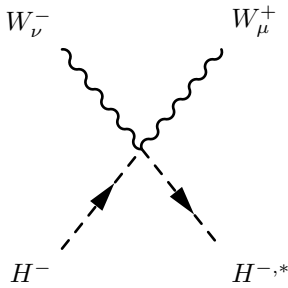
$$\frac{i}{2}\left(g_1\cos\Theta_W+g_2\sin\Theta_W\right)^2\left(g_{\mu\nu}\right) \quad (232)$$



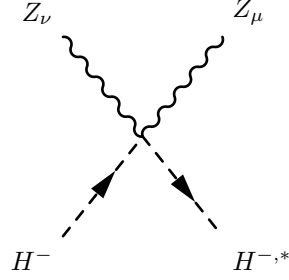
$$\frac{i}{2} (g_1 \cos \Theta_W + g_2 \sin \Theta_W) \left(-g_1 \cos \Theta'_W \sin \Theta_W + g_2 \cos \Theta_W \cos \Theta'_W + g_{BY} \sin \Theta'_W \right) (g_{\mu\nu}) \quad (233)$$



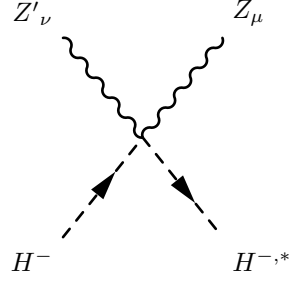
$$\frac{i}{2} (g_1 \cos \Theta_W + g_2 \sin \Theta_W) \left((g_1 \sin \Theta_W - g_2 \cos \Theta_W) \sin \Theta'_W + g_{BY} \cos \Theta'_W \right) (g_{\mu\nu}) \quad (234)$$



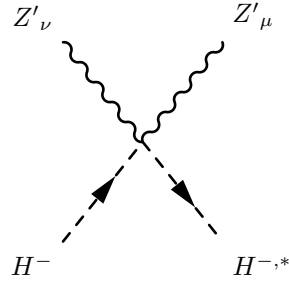
$$\frac{i}{2} g_2^2 (g_{\mu\nu}) \quad (235)$$



$$\frac{i}{2} \left(-g_1 \cos \Theta'_W \sin \Theta_W + g_2 \cos \Theta_W \cos \Theta'_W + g_{BY} \sin \Theta'_W \right)^2 (g_{\mu\nu}) \quad (236)$$

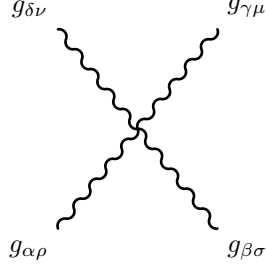


$$\begin{aligned} & \frac{i}{2} \left(-g_1 g_{BY} \cos \Theta'^2_W \sin \Theta_W - g_2^2 \cos \Theta^2_W \cos \Theta'_W \sin \Theta'_W \right. \\ & + \cos \Theta'_W \left(-g_1^2 \sin \Theta^2_W + g_{BY}^2 \right) \sin \Theta'_W + g_1 g_{BY} \sin \Theta_W \sin \Theta'^2_W \\ & \left. + g_2 \cos \Theta_W \left(g_1 \sin \Theta_W \sin 2\Theta'_W + g_{BY} \cos \Theta'^2_W - g_{BY} \sin \Theta'^2_W \right) \right) (g_{\mu\nu}) \end{aligned} \quad (237)$$



$$\frac{i}{2} \left(\left(g_1 \sin \Theta_W - g_2 \cos \Theta_W \right) \sin \Theta'_W + g_{BY} \cos \Theta'_W \right)^2 (g_{\mu\nu}) \quad (238)$$

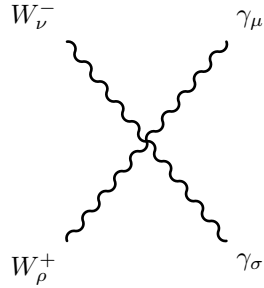
9.9 Four Vector Boson-Interaction



$$-ig_3^2 \left(\sum_{a=1}^8 f_{\alpha,\delta,a} f_{\beta,\gamma,a} + \sum_{a=1}^8 f_{\alpha,\gamma,a} f_{\beta,\delta,a} \right) (g_{\rho\sigma} g_{\mu\nu}) \quad (239)$$

$$+ ig_3^2 \left(- \sum_{a=1}^8 f_{\alpha,\beta,a} f_{\gamma,\delta,a} + \sum_{a=1}^8 f_{\alpha,\delta,a} f_{\beta,\gamma,a} \right) (g_{\rho\mu} g_{\sigma\nu}) \quad (240)$$

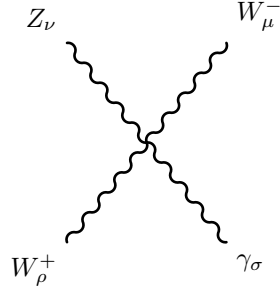
$$+ ig_3^2 \left(\sum_{a=1}^8 f_{\alpha,\gamma,a} f_{\beta,\delta,a} + \sum_{a=1}^8 f_{\alpha,\beta,a} f_{\gamma,\delta,a} \right) (g_{\rho\nu} g_{\sigma\mu}) \quad (241)$$



$$ig_2^2 \sin^2 \Theta_W (g_{\rho\sigma} g_{\mu\nu}) \quad (242)$$

$$+ ig_2^2 \sin^2 \Theta_W (g_{\rho\mu} g_{\sigma\nu}) \quad (243)$$

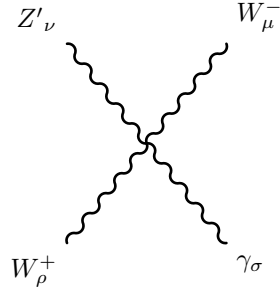
$$+ -2ig_2^2 \sin^2 \Theta_W (g_{\rho\nu} g_{\sigma\mu}) \quad (244)$$



$$ig_2^2 \cos \Theta_W \cos \Theta'_W \sin \Theta_W (g_{\rho\sigma} g_{\mu\nu}) \quad (245)$$

$$+ -ig_2^2 \cos \Theta'_W \sin 2\Theta_W (g_{\rho\mu} g_{\sigma\nu}) \quad (246)$$

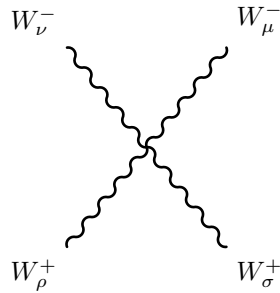
$$+ ig_2^2 \cos \Theta_W \cos \Theta'_W \sin \Theta_W (g_{\rho\nu} g_{\sigma\mu}) \quad (247)$$



$$- ig_2^2 \cos \Theta_W \sin \Theta_W \sin \Theta'_W (g_{\rho\sigma} g_{\mu\nu}) \quad (248)$$

$$+ ig_2^2 \sin 2\Theta_W \sin \Theta'_W (g_{\rho\mu} g_{\sigma\nu}) \quad (249)$$

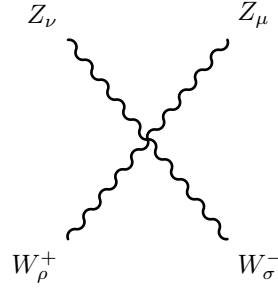
$$+ -ig_2^2 \cos \Theta_W \sin \Theta_W \sin \Theta'_W (g_{\rho\nu} g_{\sigma\mu}) \quad (250)$$



$$2ig_2^2(g_{\rho\sigma}g_{\mu\nu}) \quad (251)$$

$$+ -ig_2^2(g_{\rho\mu}g_{\sigma\nu}) \quad (252)$$

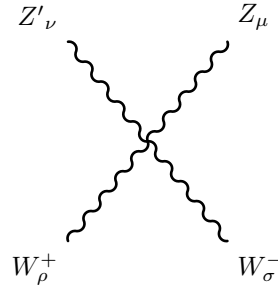
$$+ -ig_2^2(g_{\rho\nu}g_{\sigma\mu}) \quad (253)$$



$$- 2ig_2^2 \cos^2 \Theta_W \cos^2 \Theta'_W (g_{\rho\sigma}g_{\mu\nu}) \quad (254)$$

$$+ ig_2^2 \cos^2 \Theta_W \cos^2 \Theta'_W (g_{\rho\mu}g_{\sigma\nu}) \quad (255)$$

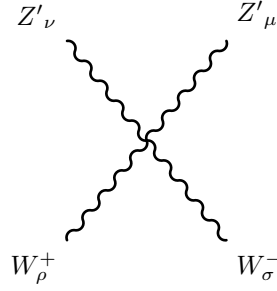
$$+ ig_2^2 \cos^2 \Theta_W \cos^2 \Theta'_W (g_{\rho\nu}g_{\sigma\mu}) \quad (256)$$



$$ig_2^2 \cos^2 \Theta_W \sin 2\Theta'_W (g_{\rho\sigma}g_{\mu\nu}) \quad (257)$$

$$+ -ig_2^2 \cos^2 \Theta_W \cos \Theta'_W \sin \Theta'_W (g_{\rho\mu}g_{\sigma\nu}) \quad (258)$$

$$+ -ig_2^2 \cos^2 \Theta_W \cos \Theta'_W \sin \Theta'_W (g_{\rho\nu}g_{\sigma\mu}) \quad (259)$$

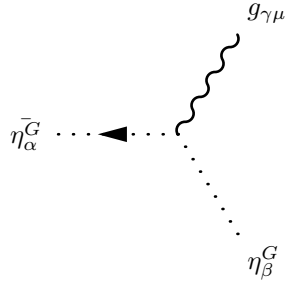


$$- 2ig_2^2 \cos \Theta_W^2 \sin \Theta_W'^2 (g_{\rho\sigma} g_{\mu\nu}) \quad (260)$$

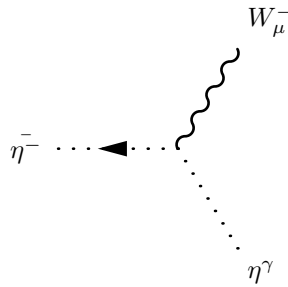
$$+ ig_2^2 \cos \Theta_W^2 \sin \Theta_W'^2 (g_{\rho\mu} g_{\sigma\nu}) \quad (261)$$

$$+ ig_2^2 \cos \Theta_W^2 \sin \Theta_W'^2 (g_{\rho\nu} g_{\sigma\mu}) \quad (262)$$

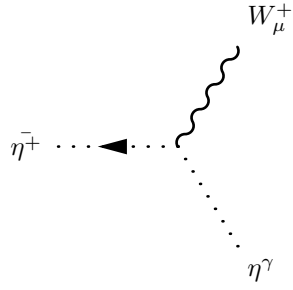
9.10 Two Ghosts-One Vector Boson-Interaction



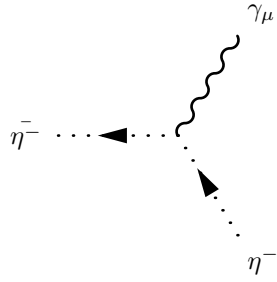
$$g_3 f_{\alpha,\beta,\gamma} (p_\mu^{\eta_\beta^G}) \quad (263)$$



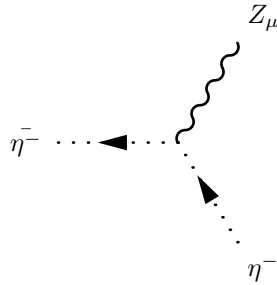
$$ig_2 \sin \Theta_W (p_\mu^{\eta^\gamma}) \quad (264)$$



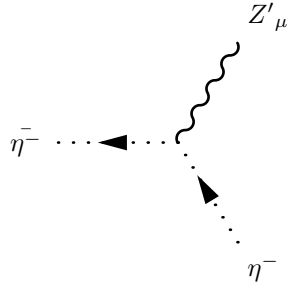
$$-ig_2 \sin \Theta_W (p_\mu^{\eta^+}) \quad (265)$$



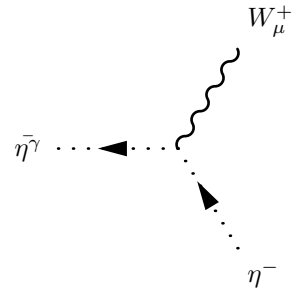
$$-ig_2 \sin \Theta_W (p_\mu^{\eta^-}) \quad (266)$$



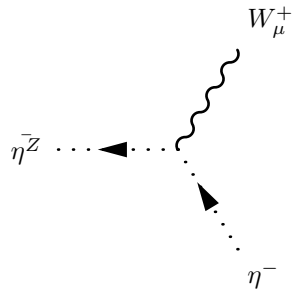
$$-ig_2 \cos \Theta_W \cos \Theta'_W (p_\mu^{\eta^-}) \quad (267)$$



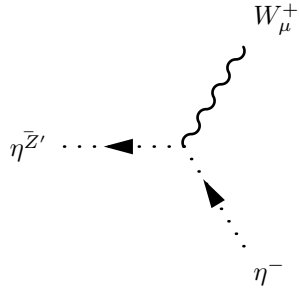
$$ig_2 \cos \Theta_W \sin \Theta'_W (p_\mu^{\eta^-}) \quad (268)$$



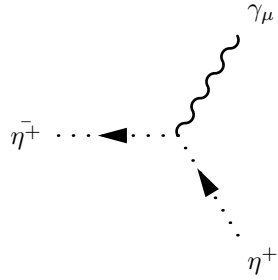
$$ig_2 \sin \Theta_W (p_\mu^{\eta^-}) \quad (269)$$



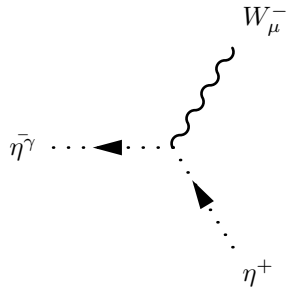
$$ig_2 \cos \Theta_W \cos \Theta'_W (p_\mu^{\eta^-}) \quad (270)$$



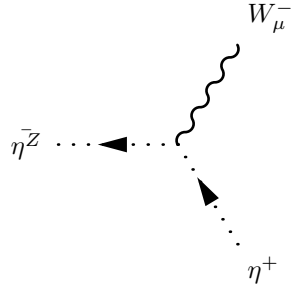
$$-ig_2 \cos \Theta_W \sin \Theta'_W (p_\mu^{\eta^-}) \quad (271)$$



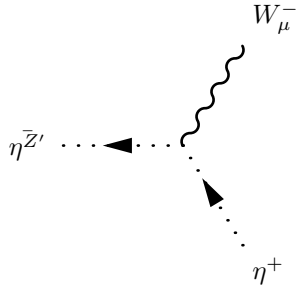
$$ig_2 \sin \Theta_W (p_\mu^{\eta^+}) \quad (272)$$



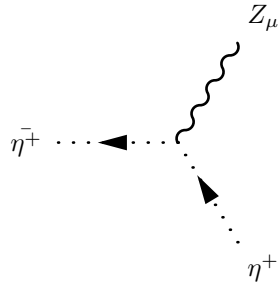
$$-ig_2 \sin \Theta_W (p_\mu^{\eta^+}) \quad (273)$$



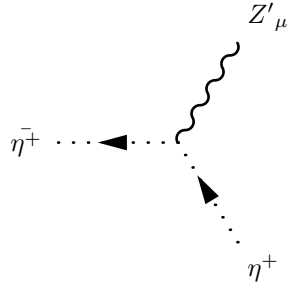
$$-ig_2 \cos \Theta_W \cos \Theta'_W (p_\mu^{\eta^+}) \quad (274)$$



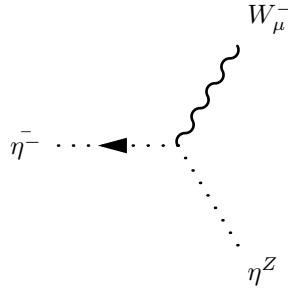
$$ig_2 \cos \Theta_W \sin \Theta'_W (p_\mu^{\eta^+}) \quad (275)$$



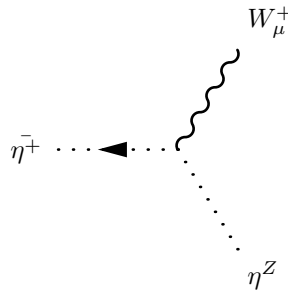
$$ig_2 \cos \Theta_W \cos \Theta'_W (p_\mu^{\eta^+}) \quad (276)$$



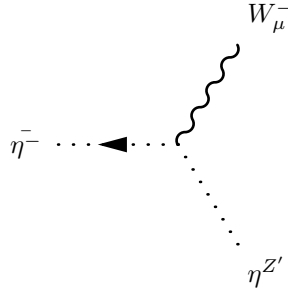
$$-ig_2 \cos \Theta_W \sin \Theta'_W (p_\mu^{\eta^+}) \quad (277)$$



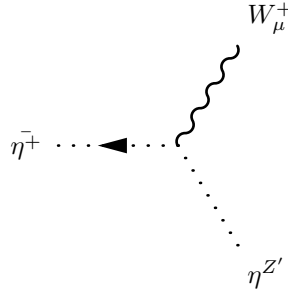
$$ig_2 \cos \Theta_W \cos \Theta'_W (p_\mu^{\eta^Z}) \quad (278)$$



$$-ig_2 \cos \Theta_W \cos \Theta'_W (p_\mu^{\eta^Z}) \quad (279)$$

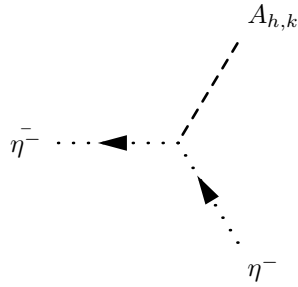


$$-ig_2 \cos \Theta_W \sin \Theta'_W (p_\mu^{\eta^{Z'}}) \quad (280)$$

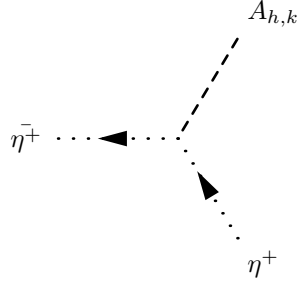


$$ig_2 \cos \Theta_W \sin \Theta'_W (p_\mu^{\eta^{Z'}}) \quad (281)$$

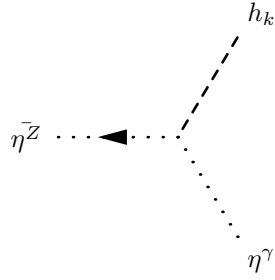
9.11 Two Ghosts-One Scalar-Interaction



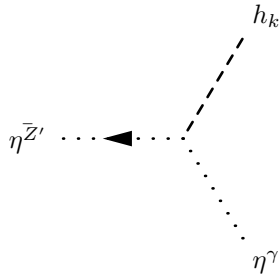
$$\frac{1}{4} g_2^2 v \xi_{W-Z_{k1}}^A \quad (282)$$



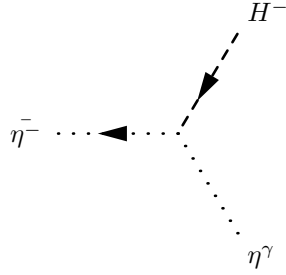
$$-\frac{1}{4}g_2^2 v \xi_{W^-} Z_{k1}^A \quad (283)$$



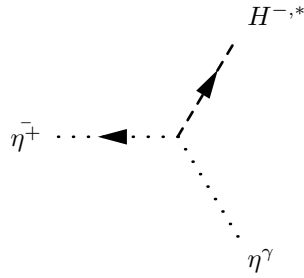
$$\begin{aligned} & \frac{i}{4} \xi_Z \left(v \left(g_1 \cos \Theta_W - g_2 \sin \Theta_W \right) \left(g_1 \cos \Theta'_W \sin \Theta_W + g_2 \cos \Theta_W \cos \Theta'_W - g_{BY} \sin \Theta'_W \right) Z_{k1}^H \right. \\ & \left. + g_{YB} x \cos \Theta_W \left(-g_B \sin \Theta'_W + g_{YB} \cos \Theta'_W \sin \Theta_W \right) Z_{k2}^H \right) \quad (284) \end{aligned}$$



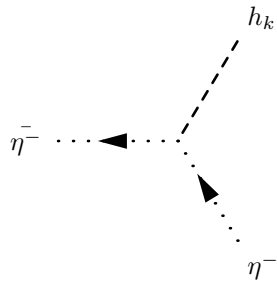
$$\begin{aligned} & -\frac{i}{4} \xi_{Z'} \left(v \left(g_1 \cos \Theta_W - g_2 \sin \Theta_W \right) \left(\left(g_1 \sin \Theta_W + g_2 \cos \Theta_W \right) \sin \Theta'_W + g_{BY} \cos \Theta'_W \right) Z_{k1}^H \right. \\ & \left. + g_{YB} x \cos \Theta_W \left(g_B \cos \Theta'_W + g_{YB} \sin \Theta_W \sin \Theta'_W \right) Z_{k2}^H \right) \quad (285) \end{aligned}$$



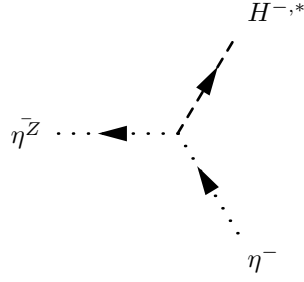
$$\frac{i}{4}g_2v\xi_{W^-}\left(g_1\cos\Theta_W+g_2\sin\Theta_W\right) \quad (286)$$



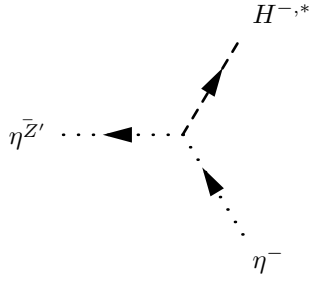
$$\frac{i}{4}g_2v\xi_{W^-}\left(g_1\cos\Theta_W+g_2\sin\Theta_W\right) \quad (287)$$



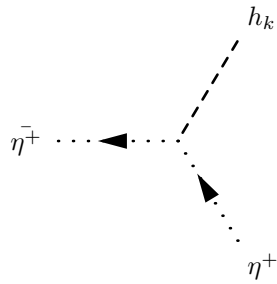
$$-\frac{i}{4}g_2^2v\xi_{W^-}Z_{k1}^H \quad (288)$$



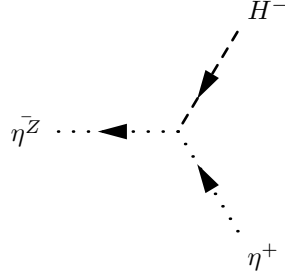
$$-\frac{i}{4}g_2v\xi_Z\left(g_1\cos\Theta'_W\sin\Theta_W+g_2\cos\Theta_W\cos\Theta'_W-g_{BY}\sin\Theta'_W\right) \quad (289)$$



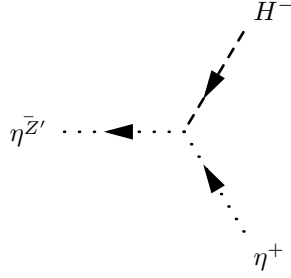
$$\frac{i}{4}g_2v\xi_{Z'}\left(\left(g_1\sin\Theta_W+g_2\cos\Theta_W\right)\sin\Theta'_W+g_{BY}\cos\Theta'_W\right) \quad (290)$$



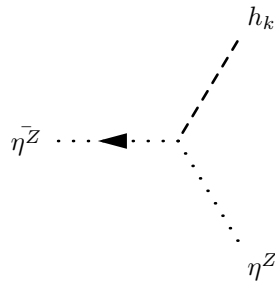
$$-\frac{i}{4}g_2^2v\xi_{W-Z_{k1}^H} \quad (291)$$



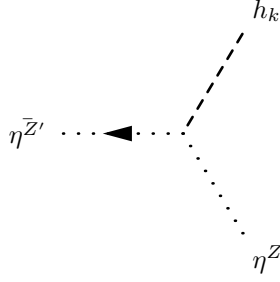
$$-\frac{i}{4}g_2v\xi_Z\left(g_1\cos\Theta'_W\sin\Theta_W+g_2\cos\Theta_W\cos\Theta'_W-g_{BY}\sin\Theta'_W\right) \quad (292)$$



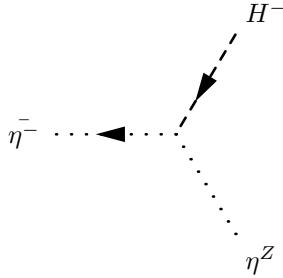
$$\frac{i}{4}g_2v\xi_{Z'}\left(\left(g_1\sin\Theta_W+g_2\cos\Theta_W\right)\sin\Theta'_W+g_{BY}\cos\Theta'_W\right) \quad (293)$$



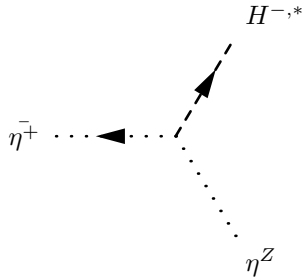
$$-\frac{i}{4}\xi_Z\left(v\left(g_1\cos\Theta'_W\sin\Theta_W+g_2\cos\Theta_W\cos\Theta'_W-g_{BY}\sin\Theta'_W\right)^2Z_{k1}^H\right. \\ \left.+x\left(-g_B\sin\Theta'_W+g_{YB}\cos\Theta'_W\sin\Theta_W\right)^2Z_{k2}^H\right) \quad (294)$$



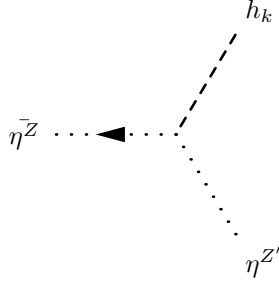
$$\begin{aligned}
& \frac{i}{4} \xi_{Z'} \left(v \left(g_1 g_{BY} \cos \Theta_W'^2 \sin \Theta_W + g_2^2 \cos \Theta_W'^2 \cos \Theta_W' \sin \Theta_W' \right. \right. \\
& + \cos \Theta_W' \left(g_1^2 \sin \Theta_W'^2 - g_{BY}^2 \right) \sin \Theta_W' - g_1 g_{BY} \sin \Theta_W \sin \Theta_W'^2 \\
& + g_2 \cos \Theta_W \left(g_1 \sin \Theta_W \sin 2\Theta_W' + g_{BY} \cos \Theta_W'^2 - g_{BY} \sin \Theta_W'^2 \right) \left. \right) Z_{k1}^H \\
& + x \left(g_B g_{YB} \cos \Theta_W'^2 \sin \Theta_W + \cos \Theta_W' \left(-g_B^2 + g_{YB}^2 \sin \Theta_W'^2 \right) \sin \Theta_W' \right. \\
& \left. - g_B g_{YB} \sin \Theta_W \sin \Theta_W'^2 \right) Z_{k2}^H \tag{295}
\end{aligned}$$



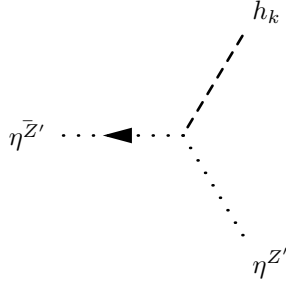
$$\frac{i}{4} g_2 v \xi_{W^-} \left(-g_1 \cos \Theta_W' \sin \Theta_W + g_2 \cos \Theta_W \cos \Theta_W' + g_{BY} \sin \Theta_W' \right) \tag{296}$$



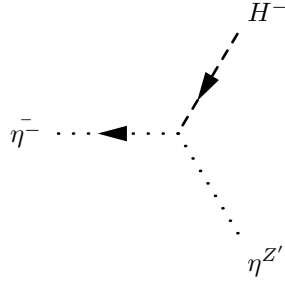
$$\frac{i}{4}g_2v\xi_{W^-} \left(-g_1 \cos \Theta'_W \sin \Theta_W + g_2 \cos \Theta_W \cos \Theta'_W + g_{BY} \sin \Theta'_W \right) \quad (297)$$



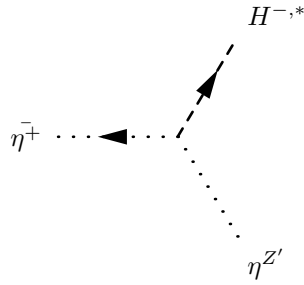
$$\begin{aligned} & \frac{i}{4}\xi_Z \left(v \left(g_1 g_{BY} \cos \Theta'^2_W \sin \Theta_W + g_2^2 \cos \Theta_W^2 \cos \Theta'_W \sin \Theta'_W \right. \right. \\ & + \cos \Theta'_W \left(g_1^2 \sin \Theta_W^2 - g_{BY}^2 \right) \sin \Theta'_W - g_1 g_{BY} \sin \Theta_W \sin \Theta'^2_W \\ & + g_2 \cos \Theta_W \left(g_1 \sin \Theta_W \sin 2\Theta'_W + g_{BY} \cos \Theta'^2_W - g_{BY} \sin \Theta'^2_W \right) \Big) Z_{k1}^H \\ & + x \left(g_B g_{YB} \cos \Theta'^2_W \sin \Theta_W + \cos \Theta'_W \left(-g_B^2 + g_{YB}^2 \sin \Theta_W^2 \right) \sin \Theta'_W \right. \\ & \left. - g_B g_{YB} \sin \Theta_W \sin \Theta'^2_W \right) Z_{k2}^H \end{aligned} \quad (298)$$



$$\begin{aligned} & -\frac{i}{4}\xi_{Z'} \left(v \left(\left(g_1 \sin \Theta_W + g_2 \cos \Theta_W \right) \sin \Theta'_W + g_{BY} \cos \Theta'_W \right)^2 Z_{k1}^H \right. \\ & \left. + x \left(g_B \cos \Theta'_W + g_{YB} \sin \Theta_W \sin \Theta'_W \right)^2 Z_{k2}^H \right) \end{aligned} \quad (299)$$



$$\frac{i}{4}g_2v\xi_{W^-} \left((g_1 \sin \Theta_W - g_2 \cos \Theta_W) \sin \Theta'_W + g_{BY} \cos \Theta'_W \right) \quad (300)$$



$$\frac{i}{4}g_2v\xi_{W^-} \left((g_1 \sin \Theta_W - g_2 \cos \Theta_W) \sin \Theta'_W + g_{BY} \cos \Theta'_W \right) \quad (301)$$

10 Clebsch-Gordan Coefficients