

General Two Higgs Doublet Model with CPV
Lagrangian, Rotations and Interactions for eigenstates 'EWSB'
including Renormalization Group Equations
including one-loop Self-Energies

SARAH 4.6.0

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References: [arXiv: 1309.7223](#) , [Comput.Phys.Commun.184:1792-1809,2011 \(1207.0906\)](#) , [Comput.Phys.Commun.182:833,2011 \(1002.0840\)](#) , [Comput.Phys.Commun.181:1077-1086,2010 \(0909.2863\)](#) , [arXiv: 0806.0538](#)

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1 Fields

1.1 Gauge Fields

Name	$SU(N)$	Coupling	Name
B	$U(1)$	g_1	hypercharge
W	$SU(2)$	g_2	left
g	$SU(3)$	g_3	color

1.2 Matter Superfields

Name	Spin	Generations	$(U(1) \otimes SU(2) \otimes SU(3))$
H1	0	1	$(\frac{1}{2}, \mathbf{2}, \mathbf{1})$
H2	0	1	$(\frac{1}{2}, \mathbf{2}, \mathbf{1})$
q	$\frac{1}{2}$	3	$(\frac{1}{6}, \mathbf{2}, \mathbf{3})$
l	$\frac{1}{2}$	3	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
d	$\frac{1}{2}$	3	$(\frac{1}{3}, \mathbf{1}, \mathbf{\bar{3}})$
u	$\frac{1}{2}$	3	$(-\frac{2}{3}, \mathbf{1}, \mathbf{\bar{3}})$
e	$\frac{1}{2}$	3	$(1, \mathbf{1}, \mathbf{1})$

2 Lagrangian

2.1 Input Lagrangian for Eigenstates GaugeES

$$L = 0 \tag{1}$$

2.2 Gauge fixing terms

2.2.1 Gauge fixing terms for eigenstates 'GaugeES'

$$L_{GF} = -\frac{1}{2}|\partial_\mu B|^2 \xi_B^{-1} - \frac{1}{2}|\partial_\mu g|^2 \xi_g^{-1} - \frac{1}{2}|\partial_\mu W|^2 \xi_W^{-1} \tag{2}$$

2.2.2 Gauge fixing terms for eigenstates 'EWSB'

$$L_{GF} = -\frac{1}{2}|\partial_\mu g|^2 \xi_g^{-1} - \frac{1}{2}|\partial_\mu \gamma|^2 \xi_\gamma^{-1} - \frac{i}{2}g_2(v_1 H_1^{+,*} + v_2 H_2^{+,*})\xi_{W^-} + \partial_\mu W^-|^2 \xi_{W^-}^{-1} - \frac{1}{2}\frac{1}{2}\left(2\partial_\mu Z - (\sigma_1 v_1 + \sigma_2 v_2)\xi_Z\left(g_1 \sin \Theta_W + g_2 \cos \Theta_W\right)\right)^2 \xi_Z^{-1} \tag{3}$$

2.3 Fields integrated out

None

3 Renormalization Group Equations

3.1 Gauge Couplings

$$\beta_{g_1}^{(1)} = \frac{21}{5}g_1^3 \quad (4)$$

$$\begin{aligned} \beta_{g_1}^{(2)} = & \frac{1}{50}g_1^3 \left(208g_1^2 + 180g_2^2 + 440g_3^2 - 25\text{Tr}(\epsilon_d\epsilon_d^\dagger) - 75\text{Tr}(\epsilon_e\epsilon_e^\dagger) - 85\text{Tr}(\epsilon_u\epsilon_u^\dagger) - 25\text{Tr}(Y_dY_d^\dagger) \right. \\ & \left. - 75\text{Tr}(Y_eY_e^\dagger) - 85\text{Tr}(Y_uY_u^\dagger) \right) \end{aligned} \quad (5)$$

$$\beta_{g_2}^{(1)} = -3g_2^3 \quad (6)$$

$$\begin{aligned} \beta_{g_2}^{(2)} = & \frac{1}{10}g_2^3 \left(12g_1^2 + 80g_2^2 + 120g_3^2 - 15\text{Tr}(\epsilon_d\epsilon_d^\dagger) - 5\text{Tr}(\epsilon_e\epsilon_e^\dagger) - 15\text{Tr}(\epsilon_u\epsilon_u^\dagger) - 15\text{Tr}(Y_dY_d^\dagger) \right. \\ & \left. - 5\text{Tr}(Y_eY_e^\dagger) - 15\text{Tr}(Y_uY_u^\dagger) \right) \end{aligned} \quad (7)$$

$$\beta_{g_3}^{(1)} = -7g_3^3 \quad (8)$$

$$\beta_{g_3}^{(2)} = -\frac{1}{10}g_3^3 \left(-11g_1^2 + 20\text{Tr}(\epsilon_d\epsilon_d^\dagger) + 20\text{Tr}(\epsilon_u\epsilon_u^\dagger) + 20\text{Tr}(Y_dY_d^\dagger) + 20\text{Tr}(Y_uY_u^\dagger) + 260g_3^2 - 45g_2^2 \right) \quad (9)$$

3.2 Quartic scalar couplings

$$\begin{aligned} \beta_{\lambda_6}^{(1)} = & -\frac{9}{5}g_1^2\lambda_6 - 9g_2^2\lambda_6 + 24\lambda_1\lambda_6 + 6\lambda_3\lambda_6 + 8\lambda_4\lambda_6 + 10\lambda_5\lambda_6 + 6\lambda_3\lambda_7 + 4\lambda_4\lambda_7 + 2\lambda_5\lambda_7 + 3\lambda_6\text{Tr}(\epsilon_d\epsilon_d^\dagger) \\ & + 3(\lambda_3 + \lambda_4 + \lambda_5)\text{Tr}(\epsilon_dY_d^\dagger) + \lambda_6\text{Tr}(\epsilon_e\epsilon_e^\dagger) + \lambda_3\text{Tr}(\epsilon_eY_e^\dagger) + \lambda_4\text{Tr}(\epsilon_eY_e^\dagger) + \lambda_5\text{Tr}(\epsilon_eY_e^\dagger) \\ & + 9\lambda_6\text{Tr}(\epsilon_u\epsilon_u^\dagger) + 6\lambda_1\text{Tr}(\epsilon_uY_u^\dagger) + 6\lambda_1\text{Tr}(Y_d\epsilon_d^\dagger) + 9\lambda_6\text{Tr}(Y_dY_d^\dagger) + 2\lambda_1\text{Tr}(Y_e\epsilon_e^\dagger) \\ & + 3\lambda_6\text{Tr}(Y_eY_e^\dagger) + 3\lambda_3\text{Tr}(Y_u\epsilon_u^\dagger) + 3\lambda_4\text{Tr}(Y_u\epsilon_u^\dagger) + 3\lambda_5\text{Tr}(Y_u\epsilon_u^\dagger) + 3\lambda_6\text{Tr}(Y_uY_u^\dagger) \\ & - 6\text{Tr}(\epsilon_dY_d^\dagger Y_dY_d^\dagger) - 2\text{Tr}(\epsilon_eY_e^\dagger Y_eY_e^\dagger) - 3\text{Tr}(\epsilon_u\epsilon_u^\dagger\epsilon_uY_u^\dagger) - 6\text{Tr}(\epsilon_u\epsilon_u^\dagger Y_u\epsilon_u^\dagger) \\ & - 3\text{Tr}(\epsilon_uY_u^\dagger\epsilon_u\epsilon_u^\dagger) - 3\text{Tr}(Y_d\epsilon_d^\dagger Y_dY_d^\dagger) - 3\text{Tr}(Y_dY_d^\dagger Y_d\epsilon_d^\dagger) - \text{Tr}(Y_e\epsilon_e^\dagger Y_eY_e^\dagger) \\ & - \text{Tr}(Y_eY_e^\dagger Y_e\epsilon_e^\dagger) \end{aligned} \quad (10)$$

$$\begin{aligned} \beta_{\lambda_6}^{(2)} = & +\frac{1683}{200}g_1^4\lambda_6 + \frac{87}{20}g_1^2g_2^2\lambda_6 - \frac{141}{8}g_2^4\lambda_6 + \frac{108}{5}g_1^2\lambda_1\lambda_6 + 108g_2^2\lambda_1\lambda_6 - 318\lambda_1^2\lambda_6 + 6\lambda_2^2\lambda_6 \\ & + \frac{18}{5}g_1^2\lambda_3\lambda_6 + 18g_2^2\lambda_3\lambda_6 - 132\lambda_1\lambda_3\lambda_6 - 36\lambda_2\lambda_3\lambda_6 - 32\lambda_3^2\lambda_6 + 6g_1^2\lambda_4\lambda_6 + 36g_2^2\lambda_4\lambda_6 \\ & - 140\lambda_1\lambda_4\lambda_6 - 28\lambda_2\lambda_4\lambda_6 - 68\lambda_3\lambda_4\lambda_6 - 34\lambda_4^2\lambda_6 + 12g_1^2\lambda_5\lambda_6 + 54g_2^2\lambda_5\lambda_6 - 148\lambda_1\lambda_5\lambda_6 \end{aligned}$$

$$\begin{aligned}
& -20\lambda_2\lambda_5\lambda_6 - 72\lambda_3\lambda_5\lambda_6 - 76\lambda_4\lambda_5\lambda_6 - 36\lambda_5^2\lambda_6 - 111\lambda_6^3 + \frac{27}{20}g_1^4\lambda_7 + \frac{3}{2}g_1^2g_2^2\lambda_7 + \frac{45}{4}g_2^4\lambda_7 \\
& + \frac{36}{5}g_1^2\lambda_3\lambda_7 + 36g_2^2\lambda_3\lambda_7 - 36\lambda_1\lambda_3\lambda_7 - 36\lambda_2\lambda_3\lambda_7 - 36\lambda_3^2\lambda_7 + \frac{24}{5}g_1^2\lambda_4\lambda_7 + 18g_2^2\lambda_4\lambda_7 \\
& - 28\lambda_1\lambda_4\lambda_7 - 28\lambda_2\lambda_4\lambda_7 - 56\lambda_3\lambda_4\lambda_7 - 34\lambda_4^2\lambda_7 - \frac{6}{5}g_1^2\lambda_5\lambda_7 - 20\lambda_1\lambda_5\lambda_7 - 20\lambda_2\lambda_5\lambda_7 \\
& - 40\lambda_3\lambda_5\lambda_7 - 44\lambda_4\lambda_5\lambda_7 - 42\lambda_5^2\lambda_7 - 126\lambda_6^2\lambda_7 - 33\lambda_6\lambda_7^2 - 42\lambda_7^3 \\
& + \frac{1}{8}\left(16\left(10g_3^2\lambda_6 - 3\left(3\lambda_3\left(2\lambda_7 + \lambda_6\right) + 4\lambda_4\left(\lambda_6 + \lambda_7\right) + \lambda_5\left(2\lambda_7 + 5\lambda_6\right)\right)\right) + 45g_2^2\lambda_6 + 5g_1^2\lambda_6\right)\text{Tr}\left(\epsilon_d\epsilon_d^\dagger\right) \\
& + \frac{3}{40}\left(3g_1^4 + 18g_1^2g_2^2\right. \\
& \left. - 5\left(3g_2^4 + 8\left(10\lambda_1\lambda_5 + 12\lambda_6^2 + 12\lambda_6\lambda_7 + 3\lambda_3\lambda_4 + 3\lambda_4\lambda_5 + 4\lambda_3\lambda_5 + 6\lambda_1\lambda_3 + 8\lambda_1\lambda_4 + \lambda_3^2 + \lambda_4^2\right)\right)\right)\text{Tr}\left(\epsilon_dY_d^\dagger\right) \\
& + \frac{15}{8}g_1^2\lambda_6\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) + \frac{15}{8}g_2^2\lambda_6\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - 6\lambda_3\lambda_6\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - 8\lambda_4\lambda_6\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) \\
& - 10\lambda_5\lambda_6\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - 12\lambda_3\lambda_7\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - 8\lambda_4\lambda_7\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - 4\lambda_5\lambda_7\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) \\
& - \frac{9}{8}g_1^4\text{Tr}\left(\epsilon_eY_e^\dagger\right) + \frac{33}{20}g_1^2g_2^2\text{Tr}\left(\epsilon_eY_e^\dagger\right) - \frac{3}{8}g_2^4\text{Tr}\left(\epsilon_eY_e^\dagger\right) - 6\lambda_1\lambda_3\text{Tr}\left(\epsilon_eY_e^\dagger\right) \\
& - \lambda_3^2\text{Tr}\left(\epsilon_eY_e^\dagger\right) - 8\lambda_1\lambda_4\text{Tr}\left(\epsilon_eY_e^\dagger\right) - 3\lambda_3\lambda_4\text{Tr}\left(\epsilon_eY_e^\dagger\right) - \lambda_4^2\text{Tr}\left(\epsilon_eY_e^\dagger\right) \\
& - 10\lambda_1\lambda_5\text{Tr}\left(\epsilon_eY_e^\dagger\right) - 4\lambda_3\lambda_5\text{Tr}\left(\epsilon_eY_e^\dagger\right) - 3\lambda_4\lambda_5\text{Tr}\left(\epsilon_eY_e^\dagger\right) - 12\lambda_6^2\text{Tr}\left(\epsilon_eY_e^\dagger\right) \\
& - 12\lambda_6\lambda_7\text{Tr}\left(\epsilon_eY_e^\dagger\right) + \frac{51}{8}g_1^2\lambda_6\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) + \frac{135}{8}g_2^2\lambda_6\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) + 60g_3^2\lambda_6\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) \\
& - 144\lambda_1\lambda_6\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) - 18\lambda_3\lambda_6\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) - 24\lambda_4\lambda_6\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) - 30\lambda_5\lambda_6\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) \\
& - \frac{171}{200}g_1^4\text{Tr}\left(\epsilon_uY_u^\dagger\right) + \frac{63}{20}g_1^2g_2^2\text{Tr}\left(\epsilon_uY_u^\dagger\right) - \frac{9}{8}g_2^4\text{Tr}\left(\epsilon_uY_u^\dagger\right) - 18\lambda_1\lambda_3\text{Tr}\left(\epsilon_uY_u^\dagger\right) \\
& - 3\lambda_3^2\text{Tr}\left(\epsilon_uY_u^\dagger\right) - 24\lambda_1\lambda_4\text{Tr}\left(\epsilon_uY_u^\dagger\right) - 9\lambda_3\lambda_4\text{Tr}\left(\epsilon_uY_u^\dagger\right) - 3\lambda_4^2\text{Tr}\left(\epsilon_uY_u^\dagger\right) \\
& - 30\lambda_1\lambda_5\text{Tr}\left(\epsilon_uY_u^\dagger\right) - 12\lambda_3\lambda_5\text{Tr}\left(\epsilon_uY_u^\dagger\right) - 9\lambda_4\lambda_5\text{Tr}\left(\epsilon_uY_u^\dagger\right) - 36\lambda_6^2\text{Tr}\left(\epsilon_uY_u^\dagger\right) \\
& - 36\lambda_6\lambda_7\text{Tr}\left(\epsilon_uY_u^\dagger\right) + \frac{9}{40}g_1^4\text{Tr}\left(Y_d\epsilon_d^\dagger\right) + \frac{27}{20}g_1^2g_2^2\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - \frac{9}{8}g_2^4\text{Tr}\left(Y_d\epsilon_d^\dagger\right) \\
& - 18\lambda_1\lambda_3\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - 3\lambda_3^2\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - 24\lambda_1\lambda_4\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - 9\lambda_3\lambda_4\text{Tr}\left(Y_d\epsilon_d^\dagger\right) \\
& - 3\lambda_4^2\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - 30\lambda_1\lambda_5\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - 12\lambda_3\lambda_5\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - 9\lambda_4\lambda_5\text{Tr}\left(Y_d\epsilon_d^\dagger\right) \\
& - 36\lambda_6^2\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - 36\lambda_6\lambda_7\text{Tr}\left(Y_d\epsilon_d^\dagger\right) + \frac{15}{8}g_1^2\lambda_6\text{Tr}\left(Y_dY_d^\dagger\right) + \frac{135}{8}g_2^2\lambda_6\text{Tr}\left(Y_dY_d^\dagger\right) \\
& + 60g_3^2\lambda_6\text{Tr}\left(Y_dY_d^\dagger\right) - 144\lambda_1\lambda_6\text{Tr}\left(Y_dY_d^\dagger\right) - 18\lambda_3\lambda_6\text{Tr}\left(Y_dY_d^\dagger\right) - 24\lambda_4\lambda_6\text{Tr}\left(Y_dY_d^\dagger\right) \\
& - 30\lambda_5\lambda_6\text{Tr}\left(Y_dY_d^\dagger\right) - \frac{9}{8}g_1^4\text{Tr}\left(Y_e\epsilon_e^\dagger\right) + \frac{33}{20}g_1^2g_2^2\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - \frac{3}{8}g_2^4\text{Tr}\left(Y_e\epsilon_e^\dagger\right) \\
& - 6\lambda_1\lambda_3\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - \lambda_3^2\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - 8\lambda_1\lambda_4\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - 3\lambda_3\lambda_4\text{Tr}\left(Y_e\epsilon_e^\dagger\right)
\end{aligned}$$

$$\begin{aligned}
& -\lambda_4^2 \text{Tr}(Y_e \epsilon_e^\dagger) - 10\lambda_1 \lambda_5 \text{Tr}(Y_e \epsilon_e^\dagger) - 4\lambda_3 \lambda_5 \text{Tr}(Y_e \epsilon_e^\dagger) - 3\lambda_4 \lambda_5 \text{Tr}(Y_e \epsilon_e^\dagger) \\
& - 12\lambda_6^2 \text{Tr}(Y_e \epsilon_e^\dagger) - 12\lambda_6 \lambda_7 \text{Tr}(Y_e \epsilon_e^\dagger) + \frac{45}{8} g_1^2 \lambda_6 \text{Tr}(Y_e Y_e^\dagger) + \frac{45}{8} g_2^2 \lambda_6 \text{Tr}(Y_e Y_e^\dagger) \\
& - 48\lambda_1 \lambda_6 \text{Tr}(Y_e Y_e^\dagger) - 6\lambda_3 \lambda_6 \text{Tr}(Y_e Y_e^\dagger) - 8\lambda_4 \lambda_6 \text{Tr}(Y_e Y_e^\dagger) - 10\lambda_5 \lambda_6 \text{Tr}(Y_e Y_e^\dagger) \\
& - \frac{171}{200} g_1^4 \text{Tr}(Y_u \epsilon_u^\dagger) + \frac{63}{20} g_1^2 g_2^2 \text{Tr}(Y_u \epsilon_u^\dagger) - \frac{9}{8} g_2^4 \text{Tr}(Y_u \epsilon_u^\dagger) - 18\lambda_1 \lambda_3 \text{Tr}(Y_u \epsilon_u^\dagger) \\
& - 3\lambda_3^2 \text{Tr}(Y_u \epsilon_u^\dagger) - 24\lambda_1 \lambda_4 \text{Tr}(Y_u \epsilon_u^\dagger) - 9\lambda_3 \lambda_4 \text{Tr}(Y_u \epsilon_u^\dagger) - 3\lambda_4^2 \text{Tr}(Y_u \epsilon_u^\dagger) \\
& - 30\lambda_1 \lambda_5 \text{Tr}(Y_u \epsilon_u^\dagger) - 12\lambda_3 \lambda_5 \text{Tr}(Y_u \epsilon_u^\dagger) - 9\lambda_4 \lambda_5 \text{Tr}(Y_u \epsilon_u^\dagger) - 36\lambda_6^2 \text{Tr}(Y_u \epsilon_u^\dagger) \\
& - 36\lambda_6 \lambda_7 \text{Tr}(Y_u \epsilon_u^\dagger) + \frac{17}{8} g_1^2 \lambda_6 \text{Tr}(Y_u Y_u^\dagger) + \frac{45}{8} g_2^2 \lambda_6 \text{Tr}(Y_u Y_u^\dagger) + 20g_3^2 \lambda_6 \text{Tr}(Y_u Y_u^\dagger) \\
& - 18\lambda_3 \lambda_6 \text{Tr}(Y_u Y_u^\dagger) - 24\lambda_4 \lambda_6 \text{Tr}(Y_u Y_u^\dagger) - 30\lambda_5 \lambda_6 \text{Tr}(Y_u Y_u^\dagger) - 36\lambda_3 \lambda_7 \text{Tr}(Y_u Y_u^\dagger) \\
& - 24\lambda_4 \lambda_7 \text{Tr}(Y_u Y_u^\dagger) - 12\lambda_5 \lambda_7 \text{Tr}(Y_u Y_u^\dagger) - \frac{27}{4} \lambda_6 \text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger) + 3\lambda_5 \text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d Y_d^\dagger) \\
& + 6\lambda_5 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger) - 6\lambda_6 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger) - 21\lambda_6 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) \\
& - 12\lambda_1 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_d^\dagger) - 6\lambda_4 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_d^\dagger) - 6\lambda_5 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_d^\dagger) \\
& - 6\lambda_3 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u \epsilon_d^\dagger) + 12\lambda_6 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) - 12\lambda_7 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) \\
& + 3\lambda_5 \text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger) + 6\lambda_7 \text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d Y_d^\dagger) + 3\lambda_6 \text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger) + \frac{4}{5} g_1^2 \text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger) \\
& - 32g_3^2 \text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger) + 12\lambda_1 \text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger) + 6\lambda_3 \text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger) + 6\lambda_4 \text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger) \\
& - 6\lambda_3 \text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) - 12\lambda_6 \text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u Y_d^\dagger) + \frac{3}{2} \lambda_6 \text{Tr}(\epsilon_d Y_u^\dagger Y_u \epsilon_d^\dagger) \\
& - 6\lambda_3 \text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger) - \frac{9}{4} \lambda_6 \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger) + \lambda_5 \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger) + 2\lambda_5 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) \\
& - 2\lambda_6 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger) + \lambda_5 \text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger) + 2\lambda_7 \text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e Y_e^\dagger) + \lambda_6 \text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger) \\
& - \frac{12}{5} g_1^2 \text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger) + 4\lambda_1 \text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger) + 2\lambda_3 \text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger) + 2\lambda_4 \text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger) \\
& - 12\lambda_1 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d \epsilon_u^\dagger) - 6\lambda_4 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d \epsilon_u^\dagger) - 6\lambda_5 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d \epsilon_u^\dagger) \\
& + 12\lambda_6 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) - 12\lambda_7 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) - \frac{33}{4} \lambda_6 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger) \\
& - \frac{4}{5} g_1^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger) - 16g_3^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger) + 6\lambda_1 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger) \\
& + 3\lambda_3 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger) + 3\lambda_4 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger) - \frac{8}{5} g_1^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) \\
& - 32g_3^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) + 12\lambda_1 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) + 6\lambda_3 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) + 6\lambda_4 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) \\
& - 6\lambda_6 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger) - \frac{39}{2} \lambda_6 \text{Tr}(\epsilon_u Y_d^\dagger Y_d \epsilon_u^\dagger) - 12\lambda_1 \text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger)
\end{aligned}$$

$$\begin{aligned}
& -6\lambda_4 \text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger) - 6\lambda_5 \text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger) - \frac{4}{5} g_1^2 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger) \\
& - 16g_3^2 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger) + 6\lambda_1 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger) + 3\lambda_3 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger) \\
& + 3\lambda_4 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger) + 6\lambda_7 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u Y_u^\dagger) + 3\lambda_6 \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) + 6\lambda_5 \text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger) \\
& + 6\lambda_7 \text{Tr}(Y_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger) + \frac{2}{5} g_1^2 \text{Tr}(Y_d \epsilon_d^\dagger Y_d Y_d^\dagger) - 16g_3^2 \text{Tr}(Y_d \epsilon_d^\dagger Y_d Y_d^\dagger) \\
& + 6\lambda_1 \text{Tr}(Y_d \epsilon_d^\dagger Y_d Y_d^\dagger) + 3\lambda_3 \text{Tr}(Y_d \epsilon_d^\dagger Y_d Y_d^\dagger) + 3\lambda_4 \text{Tr}(Y_d \epsilon_d^\dagger Y_d Y_d^\dagger) \\
& - 12\lambda_6 \text{Tr}(Y_d \epsilon_u^\dagger Y_u \epsilon_u^\dagger) - 12\lambda_1 \text{Tr}(Y_d \epsilon_u^\dagger Y_u Y_d^\dagger) - 6\lambda_4 \text{Tr}(Y_d \epsilon_u^\dagger Y_u Y_d^\dagger) \\
& - 6\lambda_5 \text{Tr}(Y_d \epsilon_u^\dagger Y_u Y_d^\dagger) + \frac{2}{5} g_1^2 \text{Tr}(Y_d Y_d^\dagger Y_d \epsilon_d^\dagger) - 16g_3^2 \text{Tr}(Y_d Y_d^\dagger Y_d \epsilon_d^\dagger) \\
& + 6\lambda_1 \text{Tr}(Y_d Y_d^\dagger Y_d \epsilon_d^\dagger) + 3\lambda_3 \text{Tr}(Y_d Y_d^\dagger Y_d \epsilon_d^\dagger) + 3\lambda_4 \text{Tr}(Y_d Y_d^\dagger Y_d \epsilon_d^\dagger) - \frac{33}{4} \lambda_6 \text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) \\
& - 6\lambda_3 \text{Tr}(Y_d Y_u^\dagger Y_u \epsilon_d^\dagger) - 21\lambda_6 \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) + 2\lambda_7 \text{Tr}(Y_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) \\
& - \frac{6}{5} g_1^2 \text{Tr}(Y_e \epsilon_e^\dagger Y_e Y_e^\dagger) + 2\lambda_1 \text{Tr}(Y_e \epsilon_e^\dagger Y_e Y_e^\dagger) + \lambda_3 \text{Tr}(Y_e \epsilon_e^\dagger Y_e Y_e^\dagger) \\
& + \lambda_4 \text{Tr}(Y_e \epsilon_e^\dagger Y_e Y_e^\dagger) - \frac{6}{5} g_1^2 \text{Tr}(Y_e Y_e^\dagger Y_e \epsilon_e^\dagger) + 2\lambda_1 \text{Tr}(Y_e Y_e^\dagger Y_e \epsilon_e^\dagger) \\
& + \lambda_3 \text{Tr}(Y_e Y_e^\dagger Y_e \epsilon_e^\dagger) + \lambda_4 \text{Tr}(Y_e Y_e^\dagger Y_e \epsilon_e^\dagger) - \frac{11}{4} \lambda_6 \text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) + 6\lambda_7 \text{Tr}(Y_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) \\
& + 3\lambda_5 \text{Tr}(Y_u \epsilon_u^\dagger Y_u Y_u^\dagger) + 3\lambda_5 \text{Tr}(Y_u Y_u^\dagger Y_u \epsilon_u^\dagger) - \frac{27}{4} \lambda_6 \text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger) + 3\text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d Y_d^\dagger Y_d Y_d^\dagger) \\
& + 12\text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger Y_d Y_d^\dagger) + 9\text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger Y_d \epsilon_d^\dagger) + 3\text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger \epsilon_d Y_d^\dagger) \\
& + 6\text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger Y_d \epsilon_d^\dagger) - 3\text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger Y_u \epsilon_d^\dagger) - 3\text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_d^\dagger Y_d Y_d^\dagger) \\
& + 3\text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_u^\dagger \epsilon_u \epsilon_d^\dagger) - 6\text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_u^\dagger Y_u Y_d^\dagger) + 3\text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) \\
& - 6\text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u \epsilon_u^\dagger Y_u Y_d^\dagger) - 6\text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger \epsilon_d Y_d^\dagger) - 6\text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger Y_d \epsilon_d^\dagger) \\
& + 12\text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger) + 3\text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) + 6\text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger) \\
& + 3\text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger \epsilon_d Y_d^\dagger) + 9\text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger) - 3\text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_u^\dagger \epsilon_u Y_d^\dagger) \\
& + 6\text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger) + 30\text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger Y_d Y_d^\dagger) - 6\text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) \\
& + 3\text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_u^\dagger Y_u Y_d^\dagger) - 3\text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) - 3\text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger Y_d Y_d^\dagger) \\
& + \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger Y_e Y_e^\dagger) + 4\text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger Y_e Y_e^\dagger) + 3\text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger Y_e \epsilon_e^\dagger) \\
& + 4\text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger) + 2\text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger) + \text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger) \\
& + 3\text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) + 2\text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger) + 10\text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger Y_e Y_e^\dagger)
\end{aligned}$$

$$\begin{aligned}
& -6\text{Tr}\left(\epsilon_u\epsilon_d^\dagger Y_d\epsilon_d^\dagger Y_d Y_u^\dagger\right) - 3\text{Tr}\left(\epsilon_u\epsilon_d^\dagger Y_d\epsilon_u^\dagger\epsilon_u\epsilon_u^\dagger\right) - 3\text{Tr}\left(\epsilon_u\epsilon_d^\dagger Y_d Y_d^\dagger Y_d\epsilon_u^\dagger\right) \\
& -6\text{Tr}\left(\epsilon_u\epsilon_d^\dagger Y_d Y_u^\dagger\epsilon_u Y_u^\dagger\right) - 6\text{Tr}\left(\epsilon_u\epsilon_d^\dagger Y_d Y_u^\dagger Y_u\epsilon_u^\dagger\right) + 3\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger Y_d\epsilon_u^\dagger\right) \\
& +6\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\epsilon_u\epsilon_u^\dagger\epsilon_u Y_u^\dagger\right) + 21\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\epsilon_u\epsilon_u^\dagger Y_u\epsilon_u^\dagger\right) + 12\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\epsilon_u Y_u^\dagger\epsilon_u\epsilon_u^\dagger\right) + 6\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\epsilon_u Y_u^\dagger Y_u Y_u^\dagger\right) \\
& +9\text{Tr}\left(\epsilon_u\epsilon_u^\dagger Y_u\epsilon_u^\dagger\epsilon_u\epsilon_u^\dagger\right) + 12\text{Tr}\left(\epsilon_u\epsilon_u^\dagger Y_u\epsilon_u^\dagger Y_u Y_u^\dagger\right) - 3\text{Tr}\left(\epsilon_u\epsilon_u^\dagger Y_u Y_d^\dagger Y_d\epsilon_u^\dagger\right) \\
& +3\text{Tr}\left(\epsilon_u\epsilon_u^\dagger Y_u Y_u^\dagger\epsilon_u Y_u^\dagger\right) + 9\text{Tr}\left(\epsilon_u\epsilon_u^\dagger Y_u Y_u^\dagger Y_u\epsilon_u^\dagger\right) - 3\text{Tr}\left(\epsilon_u Y_d^\dagger Y_d\epsilon_d^\dagger Y_d\epsilon_u^\dagger\right) \\
& -6\text{Tr}\left(\epsilon_u Y_d^\dagger Y_d\epsilon_u^\dagger\epsilon_u Y_u^\dagger\right) - 3\text{Tr}\left(\epsilon_u Y_d^\dagger Y_d\epsilon_u^\dagger Y_u\epsilon_u^\dagger\right) - 3\text{Tr}\left(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger\epsilon_u\epsilon_u^\dagger\right) \\
& +12\text{Tr}\left(\epsilon_u Y_u^\dagger\epsilon_u\epsilon_u^\dagger\epsilon_u\epsilon_u^\dagger\right) + 9\text{Tr}\left(\epsilon_u Y_u^\dagger\epsilon_u\epsilon_u^\dagger Y_u Y_u^\dagger\right) + 3\text{Tr}\left(\epsilon_u Y_u^\dagger\epsilon_u Y_d^\dagger Y_d\epsilon_u^\dagger\right) \\
& +3\text{Tr}\left(\epsilon_u Y_u^\dagger\epsilon_u Y_u^\dagger Y_u\epsilon_u^\dagger\right) + 6\text{Tr}\left(\epsilon_u Y_u^\dagger Y_u\epsilon_u^\dagger\epsilon_u Y_u^\dagger\right) + 9\text{Tr}\left(\epsilon_u Y_u^\dagger Y_u\epsilon_u^\dagger Y_u\epsilon_u^\dagger\right) \\
& +6\text{Tr}\left(\epsilon_u Y_u^\dagger Y_u Y_d^\dagger Y_d Y_u^\dagger\right) + 3\text{Tr}\left(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger\epsilon_u\epsilon_u^\dagger\right) + 12\text{Tr}\left(Y_d\epsilon_d^\dagger Y_d Y_d^\dagger Y_d Y_d^\dagger\right) \\
& -3\text{Tr}\left(Y_d\epsilon_d^\dagger Y_u Y_d^\dagger Y_d Y_d^\dagger\right) + 12\text{Tr}\left(Y_d Y_d^\dagger Y_d\epsilon_d^\dagger Y_d Y_d^\dagger\right) + 3\text{Tr}\left(Y_d Y_d^\dagger Y_d\epsilon_u^\dagger Y_u Y_d^\dagger\right) \\
& +6\text{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger Y_d\epsilon_d^\dagger\right) - 3\text{Tr}\left(Y_d Y_u^\dagger Y_u\epsilon_d^\dagger Y_d Y_d^\dagger\right) + 6\text{Tr}\left(Y_d Y_u^\dagger Y_u\epsilon_u^\dagger Y_u Y_d^\dagger\right) \\
& +3\text{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger Y_d\epsilon_d^\dagger\right) + 4\text{Tr}\left(Y_e\epsilon_e^\dagger Y_e Y_e^\dagger Y_e Y_e^\dagger\right) + 4\text{Tr}\left(Y_e Y_e^\dagger Y_e\epsilon_e^\dagger Y_e Y_e^\dagger\right) \\
& +2\text{Tr}\left(Y_e Y_e^\dagger Y_e Y_e^\dagger Y_e\epsilon_e^\dagger\right)
\end{aligned} \tag{11}$$

$$\begin{aligned}
\beta_{\lambda_5}^{(1)} &= -\frac{9}{5}g_1^2\lambda_5 - 9g_2^2\lambda_5 + 4\lambda_1\lambda_5 + 4\lambda_2\lambda_5 + 8\lambda_3\lambda_5 + 12\lambda_4\lambda_5 + 10\lambda_6^2 + 4\lambda_6\lambda_7 + 10\lambda_7^2 + 6\lambda_5\text{Tr}\left(\epsilon_d\epsilon_d^\dagger\right) \\
& +6\lambda_7\text{Tr}\left(\epsilon_d Y_d^\dagger\right) + 2\lambda_5\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) + 2\lambda_7\text{Tr}\left(\epsilon_e Y_e^\dagger\right) + 6\lambda_5\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) + 6\lambda_6\text{Tr}\left(\epsilon_u Y_u^\dagger\right) \\
& +6\lambda_6\text{Tr}\left(Y_d\epsilon_d^\dagger\right) + 6\lambda_5\text{Tr}\left(Y_d Y_d^\dagger\right) + 2\lambda_6\text{Tr}\left(Y_e\epsilon_e^\dagger\right) + 2\lambda_5\text{Tr}\left(Y_e Y_e^\dagger\right) + 6\lambda_7\text{Tr}\left(Y_u\epsilon_u^\dagger\right) \\
& +6\lambda_5\text{Tr}\left(Y_u Y_u^\dagger\right) - 6\text{Tr}\left(\epsilon_d Y_d^\dagger\epsilon_d Y_d^\dagger\right) - 2\text{Tr}\left(\epsilon_e Y_e^\dagger\epsilon_e Y_e^\dagger\right) - 6\text{Tr}\left(\epsilon_u Y_u^\dagger\epsilon_u Y_u^\dagger\right) - 6\text{Tr}\left(Y_d\epsilon_d^\dagger Y_d\epsilon_d^\dagger\right) \\
& -2\text{Tr}\left(Y_e\epsilon_e^\dagger Y_e\epsilon_e^\dagger\right) - 6\text{Tr}\left(Y_u\epsilon_u^\dagger Y_u\epsilon_u^\dagger\right)
\end{aligned} \tag{12}$$

$$\begin{aligned}
\beta_{\lambda_5}^{(2)} &= +\frac{1413}{200}g_1^4\lambda_5 + \frac{57}{20}g_1^2g_2^2\lambda_5 - \frac{231}{8}g_2^4\lambda_5 - \frac{12}{5}g_1^2\lambda_1\lambda_5 - 28\lambda_1^2\lambda_5 - \frac{12}{5}g_1^2\lambda_2\lambda_5 - 28\lambda_2^2\lambda_5 \\
& +\frac{48}{5}g_1^2\lambda_3\lambda_5 + 36g_2^2\lambda_3\lambda_5 - 80\lambda_1\lambda_3\lambda_5 - 80\lambda_2\lambda_3\lambda_5 - 28\lambda_3^2\lambda_5 + \frac{72}{5}g_1^2\lambda_4\lambda_5 + 72g_2^2\lambda_4\lambda_5 \\
& -88\lambda_1\lambda_4\lambda_5 - 88\lambda_2\lambda_4\lambda_5 - 76\lambda_3\lambda_4\lambda_5 - 32\lambda_4^2\lambda_5 + 6\lambda_5^3 + 12g_1^2\lambda_6^2 + 54g_2^2\lambda_6^2 - 148\lambda_1\lambda_6^2 \\
& -20\lambda_2\lambda_6^2 - 72\lambda_3\lambda_6^2 - 76\lambda_4\lambda_6^2 - 72\lambda_5\lambda_6^2 - \frac{12}{5}g_1^2\lambda_6\lambda_7 - 40\lambda_1\lambda_6\lambda_7 - 40\lambda_2\lambda_6\lambda_7 \\
& -80\lambda_3\lambda_6\lambda_7 - 88\lambda_4\lambda_6\lambda_7 - 168\lambda_5\lambda_6\lambda_7 + 12g_1^2\lambda_7^2 + 54g_2^2\lambda_7^2 - 20\lambda_1\lambda_7^2 - 148\lambda_2\lambda_7^2 \\
& -72\lambda_3\lambda_7^2 - 76\lambda_4\lambda_7^2 - 72\lambda_5\lambda_7^2 \\
& +\frac{1}{4}\left(16\left(10g_3^2\lambda_5 - 3\left(2\lambda_2\lambda_5 + 2\lambda_3\lambda_5 + 3\lambda_4\lambda_5 + 5\lambda_7^2 + \lambda_6\lambda_7\right)\right) + 45g_2^2\lambda_5 + 5g_1^2\lambda_5\right)\text{Tr}\left(\epsilon_d\epsilon_d^\dagger\right)
\end{aligned}$$

$$\begin{aligned}
& -6\left(2\lambda_1\lambda_7 + 2\lambda_2\lambda_6 + 2\lambda_3(\lambda_6 + \lambda_7) + 3\lambda_4\lambda_6 + 3\lambda_4\lambda_7 + 6\lambda_5\lambda_6 + 6\lambda_5\lambda_7\right)\text{Tr}\left(\epsilon_d Y_d^\dagger\right) + \frac{15}{4}g_1^2\lambda_5\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) \\
& + \frac{15}{4}g_2^2\lambda_5\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - 8\lambda_2\lambda_5\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - 8\lambda_3\lambda_5\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - 12\lambda_4\lambda_5\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) \\
& - 4\lambda_6\lambda_7\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - 20\lambda_7^2\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - 4\lambda_2\lambda_6\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 4\lambda_3\lambda_6\text{Tr}\left(\epsilon_e Y_e^\dagger\right) \\
& - 6\lambda_4\lambda_6\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 12\lambda_5\lambda_6\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 4\lambda_1\lambda_7\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 4\lambda_3\lambda_7\text{Tr}\left(\epsilon_e Y_e^\dagger\right) \\
& - 6\lambda_4\lambda_7\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 12\lambda_5\lambda_7\text{Tr}\left(\epsilon_e Y_e^\dagger\right) + \frac{17}{4}g_1^2\lambda_5\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) + \frac{45}{4}g_2^2\lambda_5\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) \\
& + 40g_3^2\lambda_5\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) - 24\lambda_1\lambda_5\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) - 24\lambda_3\lambda_5\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) - 36\lambda_4\lambda_5\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) \\
& - 60\lambda_6^2\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) - 12\lambda_6\lambda_7\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) - 12\lambda_2\lambda_6\text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 12\lambda_3\lambda_6\text{Tr}\left(\epsilon_u Y_u^\dagger\right) \\
& - 18\lambda_4\lambda_6\text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 36\lambda_5\lambda_6\text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 12\lambda_1\lambda_7\text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 12\lambda_3\lambda_7\text{Tr}\left(\epsilon_u Y_u^\dagger\right) \\
& - 18\lambda_4\lambda_7\text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 36\lambda_5\lambda_7\text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 12\lambda_2\lambda_6\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - 12\lambda_3\lambda_6\text{Tr}\left(Y_d\epsilon_d^\dagger\right) \\
& - 18\lambda_4\lambda_6\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - 36\lambda_5\lambda_6\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - 12\lambda_1\lambda_7\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - 12\lambda_3\lambda_7\text{Tr}\left(Y_d\epsilon_d^\dagger\right) \\
& - 18\lambda_4\lambda_7\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - 36\lambda_5\lambda_7\text{Tr}\left(Y_d\epsilon_d^\dagger\right) + \frac{5}{4}g_1^2\lambda_5\text{Tr}\left(Y_d Y_d^\dagger\right) + \frac{45}{4}g_2^2\lambda_5\text{Tr}\left(Y_d Y_d^\dagger\right) \\
& + 40g_3^2\lambda_5\text{Tr}\left(Y_d Y_d^\dagger\right) - 24\lambda_1\lambda_5\text{Tr}\left(Y_d Y_d^\dagger\right) - 24\lambda_3\lambda_5\text{Tr}\left(Y_d Y_d^\dagger\right) - 36\lambda_4\lambda_5\text{Tr}\left(Y_d Y_d^\dagger\right) \\
& - 60\lambda_6^2\text{Tr}\left(Y_d Y_d^\dagger\right) - 12\lambda_6\lambda_7\text{Tr}\left(Y_d Y_d^\dagger\right) - 4\lambda_2\lambda_6\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - 4\lambda_3\lambda_6\text{Tr}\left(Y_e\epsilon_e^\dagger\right) \\
& - 6\lambda_4\lambda_6\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - 12\lambda_5\lambda_6\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - 4\lambda_1\lambda_7\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - 4\lambda_3\lambda_7\text{Tr}\left(Y_e\epsilon_e^\dagger\right) \\
& - 6\lambda_4\lambda_7\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - 12\lambda_5\lambda_7\text{Tr}\left(Y_e\epsilon_e^\dagger\right) + \frac{15}{4}g_1^2\lambda_5\text{Tr}\left(Y_e Y_e^\dagger\right) + \frac{15}{4}g_2^2\lambda_5\text{Tr}\left(Y_e Y_e^\dagger\right) \\
& - 8\lambda_1\lambda_5\text{Tr}\left(Y_e Y_e^\dagger\right) - 8\lambda_3\lambda_5\text{Tr}\left(Y_e Y_e^\dagger\right) - 12\lambda_4\lambda_5\text{Tr}\left(Y_e Y_e^\dagger\right) - 20\lambda_6^2\text{Tr}\left(Y_e Y_e^\dagger\right) \\
& - 4\lambda_6\lambda_7\text{Tr}\left(Y_e Y_e^\dagger\right) - 12\lambda_2\lambda_6\text{Tr}\left(Y_u\epsilon_u^\dagger\right) - 12\lambda_3\lambda_6\text{Tr}\left(Y_u\epsilon_u^\dagger\right) - 18\lambda_4\lambda_6\text{Tr}\left(Y_u\epsilon_u^\dagger\right) \\
& - 36\lambda_5\lambda_6\text{Tr}\left(Y_u\epsilon_u^\dagger\right) - 12\lambda_1\lambda_7\text{Tr}\left(Y_u\epsilon_u^\dagger\right) - 12\lambda_3\lambda_7\text{Tr}\left(Y_u\epsilon_u^\dagger\right) - 18\lambda_4\lambda_7\text{Tr}\left(Y_u\epsilon_u^\dagger\right) \\
& - 36\lambda_5\lambda_7\text{Tr}\left(Y_u\epsilon_u^\dagger\right) + \frac{17}{4}g_1^2\lambda_5\text{Tr}\left(Y_u Y_u^\dagger\right) + \frac{45}{4}g_2^2\lambda_5\text{Tr}\left(Y_u Y_u^\dagger\right) + 40g_3^2\lambda_5\text{Tr}\left(Y_u Y_u^\dagger\right) \\
& - 24\lambda_2\lambda_5\text{Tr}\left(Y_u Y_u^\dagger\right) - 24\lambda_3\lambda_5\text{Tr}\left(Y_u Y_u^\dagger\right) - 36\lambda_4\lambda_5\text{Tr}\left(Y_u Y_u^\dagger\right) - 12\lambda_6\lambda_7\text{Tr}\left(Y_u Y_u^\dagger\right) \\
& - 60\lambda_7^2\text{Tr}\left(Y_u Y_u^\dagger\right) - \frac{3}{2}\lambda_5\text{Tr}\left(\epsilon_d\epsilon_d^\dagger\epsilon_d\epsilon_d^\dagger\right) + 6\lambda_6\text{Tr}\left(\epsilon_d\epsilon_d^\dagger\epsilon_d Y_d^\dagger\right) + 12\lambda_6\text{Tr}\left(\epsilon_d\epsilon_d^\dagger Y_d\epsilon_d^\dagger\right) \\
& - 18\lambda_5\text{Tr}\left(\epsilon_d\epsilon_d^\dagger Y_d Y_d^\dagger\right) - 33\lambda_5\text{Tr}\left(\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger\right) - 12\lambda_6\text{Tr}\left(\epsilon_d\epsilon_u^\dagger\epsilon_u Y_d^\dagger\right) \\
& - 12\lambda_7\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_u\epsilon_d^\dagger\right) + 12\lambda_5\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_u Y_d^\dagger\right) + 6\lambda_6\text{Tr}\left(\epsilon_d Y_d^\dagger\epsilon_d\epsilon_d^\dagger\right) \\
& + \frac{4}{5}g_1^2\text{Tr}\left(\epsilon_d Y_d^\dagger\epsilon_d Y_d^\dagger\right) - 32g_3^2\text{Tr}\left(\epsilon_d Y_d^\dagger\epsilon_d Y_d^\dagger\right) + 12\lambda_1\text{Tr}\left(\epsilon_d Y_d^\dagger\epsilon_d Y_d^\dagger\right) + 12\lambda_2\text{Tr}\left(\epsilon_d Y_d^\dagger\epsilon_d Y_d^\dagger\right)
\end{aligned}$$

$$\begin{aligned}
& -9\lambda_5 \text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger) + 12\lambda_7 \text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger) - 12\lambda_7 \text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) \\
& -24\lambda_4 \text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u Y_d^\dagger) + 3\lambda_5 \text{Tr}(\epsilon_d Y_u^\dagger Y_u \epsilon_d^\dagger) - 12\lambda_7 \text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger) \\
& -\frac{1}{2}\lambda_5 \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger) + 2\lambda_6 \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger) + 4\lambda_6 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) - 6\lambda_5 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger) \\
& + 2\lambda_6 \text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger) - \frac{12}{5}g_1^2 \text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e Y_e^\dagger) + 4\lambda_1 \text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e Y_e^\dagger) + 4\lambda_2 \text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e Y_e^\dagger) \\
& - 3\lambda_5 \text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger) + 4\lambda_7 \text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger) - 12\lambda_6 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_d \epsilon_u^\dagger) \\
& + 12\lambda_5 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_d Y_u^\dagger) - \frac{3}{2}\lambda_5 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger) + 6\lambda_7 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger) + 12\lambda_7 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) \\
& - 18\lambda_5 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger) + 3\lambda_5 \text{Tr}(\epsilon_u Y_d^\dagger Y_d \epsilon_u^\dagger) - 12\lambda_6 \text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger) \\
& + 6\lambda_7 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger) - \frac{8}{5}g_1^2 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u Y_u^\dagger) - 32g_3^2 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u Y_u^\dagger) + 12\lambda_1 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u Y_u^\dagger) \\
& + 12\lambda_2 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u Y_u^\dagger) - 9\lambda_5 \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) + 12\lambda_6 \text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger) + \frac{4}{5}g_1^2 \text{Tr}(Y_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger) \\
& - 32g_3^2 \text{Tr}(Y_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger) + 12\lambda_1 \text{Tr}(Y_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger) + 12\lambda_2 \text{Tr}(Y_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger) + 6\lambda_7 \text{Tr}(Y_d \epsilon_d^\dagger Y_d Y_d^\dagger) \\
& - 24\lambda_4 \text{Tr}(Y_d \epsilon_u^\dagger Y_u \epsilon_d^\dagger) - 12\lambda_6 \text{Tr}(Y_d \epsilon_u^\dagger Y_u Y_d^\dagger) + 6\lambda_7 \text{Tr}(Y_d Y_d^\dagger Y_d \epsilon_d^\dagger) \\
& - \frac{3}{2}\lambda_5 \text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) - 12\lambda_7 \text{Tr}(Y_d Y_u^\dagger Y_u \epsilon_d^\dagger) - 33\lambda_5 \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) \\
& - \frac{12}{5}g_1^2 \text{Tr}(Y_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) + 4\lambda_1 \text{Tr}(Y_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) + 4\lambda_2 \text{Tr}(Y_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) + 2\lambda_7 \text{Tr}(Y_e \epsilon_e^\dagger Y_e Y_e^\dagger) \\
& + 2\lambda_7 \text{Tr}(Y_e Y_e^\dagger Y_e \epsilon_e^\dagger) - \frac{1}{2}\lambda_5 \text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) - \frac{8}{5}g_1^2 \text{Tr}(Y_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) - 32g_3^2 \text{Tr}(Y_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) \\
& + 12\lambda_1 \text{Tr}(Y_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) + 12\lambda_2 \text{Tr}(Y_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) + 6\lambda_6 \text{Tr}(Y_u \epsilon_u^\dagger Y_u Y_u^\dagger) + 6\lambda_6 \text{Tr}(Y_u Y_u^\dagger Y_u \epsilon_u^\dagger) \\
& - \frac{3}{2}\lambda_5 \text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger) + 6\text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d Y_d^\dagger \epsilon_d Y_d^\dagger) + 30\text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger) - 6\text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_d^\dagger \epsilon_d Y_d^\dagger) \\
& - 6\text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u \epsilon_u^\dagger Y_u \epsilon_d^\dagger) + 6\text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger \epsilon_d Y_d^\dagger) + 18\text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger) + 18\text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d Y_d^\dagger Y_d Y_d^\dagger) \\
& + 6\text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) + 12\text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger \epsilon_d Y_d^\dagger) - 6\text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger \epsilon_d Y_d^\dagger) \\
& - 6\text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u Y_u^\dagger \epsilon_u \epsilon_d^\dagger) - 6\text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger \epsilon_d Y_d^\dagger) + 2\text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger \epsilon_e Y_e^\dagger) + 10\text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) \\
& + 2\text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger) + 6\text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger) + 6\text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e Y_e^\dagger Y_e Y_e^\dagger) + 4\text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger \epsilon_e Y_e^\dagger) \\
& - 6\text{Tr}(\epsilon_u \epsilon_u^\dagger Y_d \epsilon_d^\dagger Y_d \epsilon_u^\dagger) + 6\text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger \epsilon_u Y_u^\dagger) + 30\text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) - 6\text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger \epsilon_u Y_u^\dagger) \\
& + 6\text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger) + 18\text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger) + 18\text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u Y_u^\dagger Y_u Y_u^\dagger) + 12\text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger \epsilon_u Y_u^\dagger) \\
& + 18\text{Tr}(Y_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger Y_d Y_d^\dagger) + 6\text{Tr}(Y_d \epsilon_d^\dagger Y_d \epsilon_u^\dagger Y_u Y_d^\dagger) + 6\text{Tr}(Y_d \epsilon_d^\dagger Y_d Y_d^\dagger Y_d \epsilon_d^\dagger) \\
& - 6\text{Tr}(Y_d \epsilon_u^\dagger Y_u \epsilon_u^\dagger Y_u Y_d^\dagger) - 6\text{Tr}(Y_d \epsilon_u^\dagger Y_u Y_d^\dagger Y_d \epsilon_d^\dagger) + 6\text{Tr}(Y_d Y_d^\dagger Y_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger)
\end{aligned}$$

$$\begin{aligned}
& -6\text{Tr}\left(Y_d Y_u^\dagger Y_u \epsilon_d^\dagger Y_d \epsilon_d^\dagger\right) + 6\text{Tr}\left(Y_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger Y_e Y_e^\dagger\right) + 2\text{Tr}\left(Y_e \epsilon_e^\dagger Y_e Y_e^\dagger Y_e \epsilon_e^\dagger\right) + 2\text{Tr}\left(Y_e Y_e^\dagger Y_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger\right) \\
& + 18\text{Tr}\left(Y_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger Y_u Y_u^\dagger\right) + 6\text{Tr}\left(Y_u \epsilon_u^\dagger Y_u Y_u^\dagger Y_u \epsilon_u^\dagger\right) + 6\text{Tr}\left(Y_u Y_u^\dagger Y_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger\right)
\end{aligned} \tag{13}$$

$$\begin{aligned}
\beta_{\lambda_7}^{(1)} = & +6\lambda_3\lambda_6 + 4\lambda_4\lambda_6 + 2\lambda_5\lambda_6 - \frac{9}{5}g_1^2\lambda_7 - 9g_2^2\lambda_7 + 24\lambda_2\lambda_7 + 6\lambda_3\lambda_7 + 8\lambda_4\lambda_7 + 10\lambda_5\lambda_7 + 9\lambda_7\text{Tr}\left(\epsilon_d \epsilon_d^\dagger\right) \\
& + 6\lambda_2\text{Tr}\left(\epsilon_d Y_d^\dagger\right) + 3\lambda_7\text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) + 2\lambda_2\text{Tr}\left(\epsilon_e Y_e^\dagger\right) + 3\lambda_7\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) + 3\lambda_3\text{Tr}\left(\epsilon_u Y_u^\dagger\right) \\
& + 3\lambda_4\text{Tr}\left(\epsilon_u Y_u^\dagger\right) + 3\lambda_5\text{Tr}\left(\epsilon_u Y_u^\dagger\right) + 3\lambda_3\text{Tr}\left(Y_d \epsilon_d^\dagger\right) + 3\lambda_4\text{Tr}\left(Y_d \epsilon_d^\dagger\right) + 3\lambda_5\text{Tr}\left(Y_d \epsilon_d^\dagger\right) \\
& + 3\lambda_7\text{Tr}\left(Y_d Y_d^\dagger\right) + \lambda_3\text{Tr}\left(Y_e \epsilon_e^\dagger\right) + \lambda_4\text{Tr}\left(Y_e \epsilon_e^\dagger\right) + \lambda_5\text{Tr}\left(Y_e \epsilon_e^\dagger\right) + \lambda_7\text{Tr}\left(Y_e Y_e^\dagger\right) \\
& + 6\lambda_2\text{Tr}\left(Y_u \epsilon_u^\dagger\right) + 9\lambda_7\text{Tr}\left(Y_u Y_u^\dagger\right) - 3\text{Tr}\left(\epsilon_d \epsilon_d^\dagger \epsilon_d Y_d^\dagger\right) - 6\text{Tr}\left(\epsilon_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger\right) - 3\text{Tr}\left(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger\right) \\
& - \text{Tr}\left(\epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger\right) - 2\text{Tr}\left(\epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger\right) - \text{Tr}\left(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger\right) - 6\text{Tr}\left(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger\right) \\
& - 3\text{Tr}\left(Y_u \epsilon_u^\dagger Y_u Y_u^\dagger\right) - 3\text{Tr}\left(Y_u Y_u^\dagger Y_u \epsilon_u^\dagger\right)
\end{aligned} \tag{14}$$

$$\begin{aligned}
\beta_{\lambda_7}^{(2)} = & +\frac{27}{20}g_1^4\lambda_6 + \frac{3}{2}g_1^2g_2^2\lambda_6 + \frac{45}{4}g_2^4\lambda_6 + \frac{36}{5}g_1^2\lambda_3\lambda_6 + 36g_2^2\lambda_3\lambda_6 - 36\lambda_1\lambda_3\lambda_6 - 36\lambda_2\lambda_3\lambda_6 \\
& - 36\lambda_3^2\lambda_6 + \frac{24}{5}g_1^2\lambda_4\lambda_6 + 18g_2^2\lambda_4\lambda_6 - 28\lambda_1\lambda_4\lambda_6 - 28\lambda_2\lambda_4\lambda_6 - 56\lambda_3\lambda_4\lambda_6 - 34\lambda_4^2\lambda_6 \\
& - \frac{6}{5}g_1^2\lambda_5\lambda_6 - 20\lambda_1\lambda_5\lambda_6 - 20\lambda_2\lambda_5\lambda_6 - 40\lambda_3\lambda_5\lambda_6 - 44\lambda_4\lambda_5\lambda_6 - 42\lambda_5^2\lambda_6 - 42\lambda_6^3 + \frac{1683}{200}g_1^4\lambda_7 \\
& + \frac{87}{20}g_1^2g_2^2\lambda_7 - \frac{141}{8}g_2^4\lambda_7 + 6\lambda_1^2\lambda_7 + \frac{108}{5}g_1^2\lambda_2\lambda_7 + 108g_2^2\lambda_2\lambda_7 - 318\lambda_2^2\lambda_7 + \frac{18}{5}g_1^2\lambda_3\lambda_7 \\
& + 18g_2^2\lambda_3\lambda_7 - 36\lambda_1\lambda_3\lambda_7 - 132\lambda_2\lambda_3\lambda_7 - 32\lambda_3^2\lambda_7 + 6g_1^2\lambda_4\lambda_7 + 36g_2^2\lambda_4\lambda_7 - 28\lambda_1\lambda_4\lambda_7 \\
& - 140\lambda_2\lambda_4\lambda_7 - 68\lambda_3\lambda_4\lambda_7 - 34\lambda_4^2\lambda_7 + 12g_1^2\lambda_5\lambda_7 + 54g_2^2\lambda_5\lambda_7 - 20\lambda_1\lambda_5\lambda_7 - 148\lambda_2\lambda_5\lambda_7 \\
& - 72\lambda_3\lambda_5\lambda_7 - 76\lambda_4\lambda_5\lambda_7 - 36\lambda_5^2\lambda_7 - 33\lambda_6^2\lambda_7 - 126\lambda_6\lambda_7^2 - 111\lambda_7^3 \\
& + \frac{3}{8}\left(16\left(10g_3^2 - 24\lambda_2 - 3\lambda_3 - 4\lambda_4 - 5\lambda_5\right) + 45g_2^2 + 5g_1^2\right)\lambda_7\text{Tr}\left(\epsilon_d \epsilon_d^\dagger\right) \\
& + \frac{3}{40}\left(3g_1^4 + 18g_1^2g_2^2\right. \\
& \left. - 5\left(3g_2^4 + 8\left(10\lambda_2\lambda_5 + 12\lambda_6\lambda_7 + 12\lambda_7^2 + 3\lambda_3\lambda_4 + 3\lambda_4\lambda_5 + 4\lambda_3\lambda_5 + 6\lambda_2\lambda_3 + 8\lambda_2\lambda_4 + \lambda_3^2 + \lambda_4^2\right)\right)\right)\text{Tr}\left(\epsilon_d Y_d^\dagger\right) \\
& + \frac{45}{8}g_1^2\lambda_7\text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) + \frac{45}{8}g_2^2\lambda_7\text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) - 48\lambda_2\lambda_7\text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) - 6\lambda_3\lambda_7\text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) \\
& - 8\lambda_4\lambda_7\text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) - 10\lambda_5\lambda_7\text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) - \frac{9}{8}g_1^4\text{Tr}\left(\epsilon_e Y_e^\dagger\right) + \frac{33}{20}g_1^2g_2^2\text{Tr}\left(\epsilon_e Y_e^\dagger\right) \\
& - \frac{3}{8}g_2^4\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 6\lambda_2\lambda_3\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - \lambda_3^2\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 8\lambda_2\lambda_4\text{Tr}\left(\epsilon_e Y_e^\dagger\right) \\
& - 3\lambda_3\lambda_4\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - \lambda_4^2\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 10\lambda_2\lambda_5\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 4\lambda_3\lambda_5\text{Tr}\left(\epsilon_e Y_e^\dagger\right) \\
& - 3\lambda_4\lambda_5\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 12\lambda_6\lambda_7\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 12\lambda_7^2\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 36\lambda_3\lambda_6\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) \\
& - 24\lambda_4\lambda_6\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) - 12\lambda_5\lambda_6\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) + \frac{17}{8}g_1^2\lambda_7\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) + \frac{45}{8}g_2^2\lambda_7\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right)
\end{aligned}$$

$$\begin{aligned}
& + 20g_3^2\lambda_7\text{Tr}(\epsilon_u\epsilon_u^\dagger) - 18\lambda_3\lambda_7\text{Tr}(\epsilon_u\epsilon_u^\dagger) - 24\lambda_4\lambda_7\text{Tr}(\epsilon_u\epsilon_u^\dagger) - 30\lambda_5\lambda_7\text{Tr}(\epsilon_u\epsilon_u^\dagger) \\
& - \frac{171}{200}g_1^4\text{Tr}(\epsilon_u Y_u^\dagger) + \frac{63}{20}g_1^2g_2^2\text{Tr}(\epsilon_u Y_u^\dagger) - \frac{9}{8}g_2^4\text{Tr}(\epsilon_u Y_u^\dagger) - 18\lambda_2\lambda_3\text{Tr}(\epsilon_u Y_u^\dagger) \\
& - 3\lambda_3^2\text{Tr}(\epsilon_u Y_u^\dagger) - 24\lambda_2\lambda_4\text{Tr}(\epsilon_u Y_u^\dagger) - 9\lambda_3\lambda_4\text{Tr}(\epsilon_u Y_u^\dagger) - 3\lambda_4^2\text{Tr}(\epsilon_u Y_u^\dagger) \\
& - 30\lambda_2\lambda_5\text{Tr}(\epsilon_u Y_u^\dagger) - 12\lambda_3\lambda_5\text{Tr}(\epsilon_u Y_u^\dagger) - 9\lambda_4\lambda_5\text{Tr}(\epsilon_u Y_u^\dagger) - 36\lambda_6\lambda_7\text{Tr}(\epsilon_u Y_u^\dagger) \\
& - 36\lambda_7^2\text{Tr}(\epsilon_u Y_u^\dagger) + \frac{9}{40}g_1^4\text{Tr}(Y_d\epsilon_d^\dagger) + \frac{27}{20}g_1^2g_2^2\text{Tr}(Y_d\epsilon_d^\dagger) - \frac{9}{8}g_2^4\text{Tr}(Y_d\epsilon_d^\dagger) \\
& - 18\lambda_2\lambda_3\text{Tr}(Y_d\epsilon_d^\dagger) - 3\lambda_3^2\text{Tr}(Y_d\epsilon_d^\dagger) - 24\lambda_2\lambda_4\text{Tr}(Y_d\epsilon_d^\dagger) - 9\lambda_3\lambda_4\text{Tr}(Y_d\epsilon_d^\dagger) \\
& - 3\lambda_4^2\text{Tr}(Y_d\epsilon_d^\dagger) - 30\lambda_2\lambda_5\text{Tr}(Y_d\epsilon_d^\dagger) - 12\lambda_3\lambda_5\text{Tr}(Y_d\epsilon_d^\dagger) - 9\lambda_4\lambda_5\text{Tr}(Y_d\epsilon_d^\dagger) \\
& - 36\lambda_6\lambda_7\text{Tr}(Y_d\epsilon_d^\dagger) - 36\lambda_7^2\text{Tr}(Y_d\epsilon_d^\dagger) - 36\lambda_3\lambda_6\text{Tr}(Y_d Y_d^\dagger) - 24\lambda_4\lambda_6\text{Tr}(Y_d Y_d^\dagger) \\
& - 12\lambda_5\lambda_6\text{Tr}(Y_d Y_d^\dagger) + \frac{5}{8}g_1^2\lambda_7\text{Tr}(Y_d Y_d^\dagger) + \frac{45}{8}g_2^2\lambda_7\text{Tr}(Y_d Y_d^\dagger) + 20g_3^2\lambda_7\text{Tr}(Y_d Y_d^\dagger) \\
& - 18\lambda_3\lambda_7\text{Tr}(Y_d Y_d^\dagger) - 24\lambda_4\lambda_7\text{Tr}(Y_d Y_d^\dagger) - 30\lambda_5\lambda_7\text{Tr}(Y_d Y_d^\dagger) - \frac{9}{8}g_1^4\text{Tr}(Y_e\epsilon_e^\dagger) \\
& + \frac{33}{20}g_1^2g_2^2\text{Tr}(Y_e\epsilon_e^\dagger) - \frac{3}{8}g_2^4\text{Tr}(Y_e\epsilon_e^\dagger) - 6\lambda_2\lambda_3\text{Tr}(Y_e\epsilon_e^\dagger) - \lambda_3^2\text{Tr}(Y_e\epsilon_e^\dagger) \\
& - 8\lambda_2\lambda_4\text{Tr}(Y_e\epsilon_e^\dagger) - 3\lambda_3\lambda_4\text{Tr}(Y_e\epsilon_e^\dagger) - \lambda_4^2\text{Tr}(Y_e\epsilon_e^\dagger) - 10\lambda_2\lambda_5\text{Tr}(Y_e\epsilon_e^\dagger) \\
& - 4\lambda_3\lambda_5\text{Tr}(Y_e\epsilon_e^\dagger) - 3\lambda_4\lambda_5\text{Tr}(Y_e\epsilon_e^\dagger) - 12\lambda_6\lambda_7\text{Tr}(Y_e\epsilon_e^\dagger) - 12\lambda_7^2\text{Tr}(Y_e\epsilon_e^\dagger) \\
& - 12\lambda_3\lambda_6\text{Tr}(Y_e Y_e^\dagger) - 8\lambda_4\lambda_6\text{Tr}(Y_e Y_e^\dagger) - 4\lambda_5\lambda_6\text{Tr}(Y_e Y_e^\dagger) + \frac{15}{8}g_1^2\lambda_7\text{Tr}(Y_e Y_e^\dagger) \\
& + \frac{15}{8}g_2^2\lambda_7\text{Tr}(Y_e Y_e^\dagger) - 6\lambda_3\lambda_7\text{Tr}(Y_e Y_e^\dagger) - 8\lambda_4\lambda_7\text{Tr}(Y_e Y_e^\dagger) - 10\lambda_5\lambda_7\text{Tr}(Y_e Y_e^\dagger) \\
& - \frac{171}{200}g_1^4\text{Tr}(Y_u\epsilon_u^\dagger) + \frac{63}{20}g_1^2g_2^2\text{Tr}(Y_u\epsilon_u^\dagger) - \frac{9}{8}g_2^4\text{Tr}(Y_u\epsilon_u^\dagger) - 18\lambda_2\lambda_3\text{Tr}(Y_u\epsilon_u^\dagger) \\
& - 3\lambda_3^2\text{Tr}(Y_u\epsilon_u^\dagger) - 24\lambda_2\lambda_4\text{Tr}(Y_u\epsilon_u^\dagger) - 9\lambda_3\lambda_4\text{Tr}(Y_u\epsilon_u^\dagger) - 3\lambda_4^2\text{Tr}(Y_u\epsilon_u^\dagger) \\
& - 30\lambda_2\lambda_5\text{Tr}(Y_u\epsilon_u^\dagger) - 12\lambda_3\lambda_5\text{Tr}(Y_u\epsilon_u^\dagger) - 9\lambda_4\lambda_5\text{Tr}(Y_u\epsilon_u^\dagger) - 36\lambda_6\lambda_7\text{Tr}(Y_u\epsilon_u^\dagger) \\
& - 36\lambda_7^2\text{Tr}(Y_u\epsilon_u^\dagger) + \frac{51}{8}g_1^2\lambda_7\text{Tr}(Y_u Y_u^\dagger) + \frac{135}{8}g_2^2\lambda_7\text{Tr}(Y_u Y_u^\dagger) + 60g_3^2\lambda_7\text{Tr}(Y_u Y_u^\dagger) \\
& - 144\lambda_2\lambda_7\text{Tr}(Y_u Y_u^\dagger) - 18\lambda_3\lambda_7\text{Tr}(Y_u Y_u^\dagger) - 24\lambda_4\lambda_7\text{Tr}(Y_u Y_u^\dagger) - 30\lambda_5\lambda_7\text{Tr}(Y_u Y_u^\dagger) \\
& - \frac{33}{4}\lambda_7\text{Tr}(\epsilon_d\epsilon_d^\dagger\epsilon_d\epsilon_d^\dagger) + \frac{2}{5}g_1^2\text{Tr}(\epsilon_d\epsilon_d^\dagger\epsilon_d Y_d^\dagger) - 16g_3^2\text{Tr}(\epsilon_d\epsilon_d^\dagger\epsilon_d Y_d^\dagger) \\
& + 6\lambda_2\text{Tr}(\epsilon_d\epsilon_d^\dagger\epsilon_d Y_d^\dagger) + 3\lambda_3\text{Tr}(\epsilon_d\epsilon_d^\dagger\epsilon_d Y_d^\dagger) + 3\lambda_4\text{Tr}(\epsilon_d\epsilon_d^\dagger\epsilon_d Y_d^\dagger) + \frac{4}{5}g_1^2\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d\epsilon_d^\dagger) \\
& - 32g_3^2\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d\epsilon_d^\dagger) + 12\lambda_2\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d\epsilon_d^\dagger) + 6\lambda_3\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d\epsilon_d^\dagger) + 6\lambda_4\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d\epsilon_d^\dagger) \\
& - 6\lambda_7\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d Y_d^\dagger) - 21\lambda_7\text{Tr}(\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger) - 6\lambda_3\text{Tr}(\epsilon_d\epsilon_u^\dagger\epsilon_u Y_d^\dagger)
\end{aligned}$$

$$\begin{aligned}
& -12\lambda_2 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u \epsilon_d^\dagger) - 6\lambda_4 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u \epsilon_d^\dagger) - 6\lambda_5 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u \epsilon_d^\dagger) \\
& -12\lambda_6 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) + 12\lambda_7 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) + \frac{2}{5} g_1^2 \text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger) \\
& -16g_3^2 \text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger) + 6\lambda_2 \text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger) + 3\lambda_3 \text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger) \\
& + 3\lambda_4 \text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger) + 6\lambda_6 \text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d Y_d^\dagger) + 3\lambda_7 \text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger) + 6\lambda_5 \text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger) \\
& -12\lambda_2 \text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) - 6\lambda_4 \text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) - 6\lambda_5 \text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) \\
& -12\lambda_7 \text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u Y_d^\dagger) - \frac{39}{2} \lambda_7 \text{Tr}(\epsilon_d Y_u^\dagger Y_u \epsilon_d^\dagger) - 12\lambda_2 \text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger) \\
& -6\lambda_4 \text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger) - 6\lambda_5 \text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger) - \frac{11}{4} \lambda_7 \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger) \\
& -\frac{6}{5} g_1^2 \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger) + 2\lambda_2 \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger) + \lambda_3 \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger) \\
& + \lambda_4 \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger) - \frac{12}{5} g_1^2 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) + 4\lambda_2 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) + 2\lambda_3 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) \\
& + 2\lambda_4 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) - 2\lambda_7 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger) - \frac{6}{5} g_1^2 \text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger) \\
& + 2\lambda_2 \text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger) + \lambda_3 \text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger) + \lambda_4 \text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger) + 2\lambda_6 \text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e Y_e^\dagger) \\
& + \lambda_7 \text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger) + 2\lambda_5 \text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger) - 6\lambda_3 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_d \epsilon_u^\dagger) \\
& -12\lambda_6 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_d Y_u^\dagger) + 12\lambda_7 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_d Y_u^\dagger) - \frac{27}{4} \lambda_7 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger) \\
& + 3\lambda_5 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger) + 6\lambda_5 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) - 6\lambda_7 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger) \\
& + \frac{3}{2} \lambda_7 \text{Tr}(\epsilon_u Y_d^\dagger Y_d \epsilon_u^\dagger) - 6\lambda_3 \text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger) + 3\lambda_5 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger) \\
& + 6\lambda_6 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u Y_u^\dagger) + 3\lambda_7 \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) - \frac{8}{5} g_1^2 \text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger) - 32g_3^2 \text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger) \\
& + 12\lambda_2 \text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger) + 6\lambda_3 \text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger) + 6\lambda_4 \text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger) + 6\lambda_6 \text{Tr}(Y_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger) \\
& + 3\lambda_5 \text{Tr}(Y_d \epsilon_d^\dagger Y_d Y_d^\dagger) - 12\lambda_7 \text{Tr}(Y_d \epsilon_u^\dagger Y_u \epsilon_d^\dagger) - 6\lambda_3 \text{Tr}(Y_d \epsilon_u^\dagger Y_u Y_d^\dagger) \\
& + 3\lambda_5 \text{Tr}(Y_d Y_d^\dagger Y_d \epsilon_d^\dagger) - \frac{27}{4} \lambda_7 \text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) - 12\lambda_2 \text{Tr}(Y_d Y_u^\dagger Y_u \epsilon_d^\dagger) \\
& -6\lambda_4 \text{Tr}(Y_d Y_u^\dagger Y_u \epsilon_d^\dagger) - 6\lambda_5 \text{Tr}(Y_d Y_u^\dagger Y_u \epsilon_d^\dagger) - 21\lambda_7 \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) \\
& + 2\lambda_6 \text{Tr}(Y_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) + \lambda_5 \text{Tr}(Y_e \epsilon_e^\dagger Y_e Y_e^\dagger) + \lambda_5 \text{Tr}(Y_e Y_e^\dagger Y_e \epsilon_e^\dagger) - \frac{9}{4} \lambda_7 \text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) \\
& + 6\lambda_6 \text{Tr}(Y_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) - \frac{4}{5} g_1^2 \text{Tr}(Y_u \epsilon_u^\dagger Y_u Y_u^\dagger) - 16g_3^2 \text{Tr}(Y_u \epsilon_u^\dagger Y_u Y_u^\dagger) \\
& + 6\lambda_2 \text{Tr}(Y_u \epsilon_u^\dagger Y_u Y_u^\dagger) + 3\lambda_3 \text{Tr}(Y_u \epsilon_u^\dagger Y_u Y_u^\dagger) + 3\lambda_4 \text{Tr}(Y_u \epsilon_u^\dagger Y_u Y_u^\dagger)
\end{aligned}$$

$$+ 6\text{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger Y_u \epsilon_u^\dagger\right) \quad (15)$$

$$\begin{aligned} \beta_{\lambda_1}^{(1)} = & + \frac{27}{200}g_1^4 + \frac{9}{20}g_1^2g_2^2 + \frac{9}{8}g_2^4 - \frac{9}{5}g_1^2\lambda_1 - 9g_2^2\lambda_1 + 24\lambda_1^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + \lambda_5^2 + 12\lambda_6^2 \\ & + 6\lambda_6\text{Tr}\left(\epsilon_d Y_d^\dagger\right) + 2\lambda_6\text{Tr}\left(\epsilon_e Y_e^\dagger\right) + 12\lambda_1\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) + 12\lambda_1\text{Tr}\left(Y_d Y_d^\dagger\right) + 4\lambda_1\text{Tr}\left(Y_e Y_e^\dagger\right) \\ & + 6\lambda_6\text{Tr}\left(Y_u \epsilon_u^\dagger\right) - 6\text{Tr}\left(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger\right) - 6\text{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger\right) - 2\text{Tr}\left(Y_e Y_e^\dagger Y_e Y_e^\dagger\right) \end{aligned} \quad (16)$$

$$\begin{aligned} \beta_{\lambda_1}^{(2)} = & - \frac{3537}{2000}g_1^6 - \frac{1719}{400}g_1^4g_2^2 - \frac{303}{80}g_1^2g_2^4 + \frac{291}{16}g_2^6 + \frac{1953}{200}g_1^4\lambda_1 + \frac{117}{20}g_1^2g_2^2\lambda_1 - \frac{51}{8}g_2^4\lambda_1 + \frac{108}{5}g_1^2\lambda_1^2 \\ & + 108g_2^2\lambda_1^2 - 312\lambda_1^3 + \frac{9}{10}g_1^4\lambda_3 + \frac{15}{2}g_2^4\lambda_3 + \frac{12}{5}g_1^2\lambda_3^2 + 12g_2^2\lambda_3^2 - 20\lambda_1\lambda_3^2 - 8\lambda_3^3 + \frac{9}{20}g_1^4\lambda_4 \\ & + \frac{3}{2}g_1^2g_2^2\lambda_4 + \frac{15}{4}g_2^4\lambda_4 + \frac{12}{5}g_1^2\lambda_3\lambda_4 + 12g_2^2\lambda_3\lambda_4 - 20\lambda_1\lambda_3\lambda_4 - 12\lambda_3^2\lambda_4 + \frac{6}{5}g_1^2\lambda_4^2 \\ & + 3g_2^2\lambda_4^2 - 12\lambda_1\lambda_4^2 - 16\lambda_3\lambda_4^2 - 6\lambda_4^3 - \frac{3}{5}g_1^2\lambda_5^2 - 14\lambda_1\lambda_5^2 - 20\lambda_3\lambda_5^2 - 22\lambda_4\lambda_5^2 \\ & + \frac{54}{5}g_1^2\lambda_6^2 + 54g_2^2\lambda_6^2 - 318\lambda_1\lambda_6^2 - 66\lambda_3\lambda_6^2 - 70\lambda_4\lambda_6^2 - 74\lambda_5\lambda_6^2 - 36\lambda_3\lambda_6\lambda_7 - 28\lambda_4\lambda_6\lambda_7 \\ & - 20\lambda_5\lambda_6\lambda_7 + 6\lambda_1\lambda_7^2 - 18\lambda_3\lambda_7^2 - 14\lambda_4\lambda_7^2 - 10\lambda_5\lambda_7^2 \\ & - 6\left(2\lambda_3^2 + 2\lambda_3\lambda_4 + 6\lambda_6^2 + \lambda_4^2 + \lambda_5^2\right)\text{Tr}\left(\epsilon_d \epsilon_d^\dagger\right) - 6\left(12\lambda_1 + 2\lambda_4 + 3\lambda_3 + \lambda_5\right)\lambda_6\text{Tr}\left(\epsilon_d Y_d^\dagger\right) \\ & - 4\lambda_3^2\text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) - 4\lambda_3\lambda_4\text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) - 2\lambda_4^2\text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) - 2\lambda_5^2\text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) \\ & - 12\lambda_6^2\text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) - 24\lambda_1\lambda_6\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 6\lambda_3\lambda_6\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 4\lambda_4\lambda_6\text{Tr}\left(\epsilon_e Y_e^\dagger\right) \\ & - 2\lambda_5\lambda_6\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - \frac{171}{100}g_1^4\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) + \frac{63}{10}g_1^2g_2^2\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) - \frac{9}{4}g_2^4\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) \\ & + \frac{17}{2}g_1^2\lambda_1\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) + \frac{45}{2}g_2^2\lambda_1\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) + 80g_3^2\lambda_1\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) - 144\lambda_1^2\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) \\ & - 36\lambda_6^2\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) - 72\lambda_1\lambda_6\text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 18\lambda_3\lambda_6\text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 12\lambda_4\lambda_6\text{Tr}\left(\epsilon_u Y_u^\dagger\right) \\ & - 6\lambda_5\lambda_6\text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 72\lambda_1\lambda_6\text{Tr}\left(Y_d \epsilon_d^\dagger\right) - 18\lambda_3\lambda_6\text{Tr}\left(Y_d \epsilon_d^\dagger\right) - 12\lambda_4\lambda_6\text{Tr}\left(Y_d \epsilon_d^\dagger\right) \\ & - 6\lambda_5\lambda_6\text{Tr}\left(Y_d \epsilon_d^\dagger\right) + \frac{9}{20}g_1^4\text{Tr}\left(Y_d Y_d^\dagger\right) + \frac{27}{10}g_1^2g_2^2\text{Tr}\left(Y_d Y_d^\dagger\right) - \frac{9}{4}g_2^4\text{Tr}\left(Y_d Y_d^\dagger\right) \\ & + \frac{5}{2}g_1^2\lambda_1\text{Tr}\left(Y_d Y_d^\dagger\right) + \frac{45}{2}g_2^2\lambda_1\text{Tr}\left(Y_d Y_d^\dagger\right) + 80g_3^2\lambda_1\text{Tr}\left(Y_d Y_d^\dagger\right) - 144\lambda_1^2\text{Tr}\left(Y_d Y_d^\dagger\right) \\ & - 36\lambda_6^2\text{Tr}\left(Y_d Y_d^\dagger\right) - 24\lambda_1\lambda_6\text{Tr}\left(Y_e \epsilon_e^\dagger\right) - 6\lambda_3\lambda_6\text{Tr}\left(Y_e \epsilon_e^\dagger\right) - 4\lambda_4\lambda_6\text{Tr}\left(Y_e \epsilon_e^\dagger\right) \\ & - 2\lambda_5\lambda_6\text{Tr}\left(Y_e \epsilon_e^\dagger\right) - \frac{9}{4}g_1^4\text{Tr}\left(Y_e Y_e^\dagger\right) + \frac{33}{10}g_1^2g_2^2\text{Tr}\left(Y_e Y_e^\dagger\right) - \frac{3}{4}g_2^4\text{Tr}\left(Y_e Y_e^\dagger\right) \\ & + \frac{15}{2}g_1^2\lambda_1\text{Tr}\left(Y_e Y_e^\dagger\right) + \frac{15}{2}g_2^2\lambda_1\text{Tr}\left(Y_e Y_e^\dagger\right) - 48\lambda_1^2\text{Tr}\left(Y_e Y_e^\dagger\right) - 12\lambda_6^2\text{Tr}\left(Y_e Y_e^\dagger\right) \\ & - 72\lambda_1\lambda_6\text{Tr}\left(Y_u \epsilon_u^\dagger\right) - 18\lambda_3\lambda_6\text{Tr}\left(Y_u \epsilon_u^\dagger\right) - 12\lambda_4\lambda_6\text{Tr}\left(Y_u \epsilon_u^\dagger\right) - 6\lambda_5\lambda_6\text{Tr}\left(Y_u \epsilon_u^\dagger\right) \\ & - 12\lambda_3^2\text{Tr}\left(Y_u Y_u^\dagger\right) - 12\lambda_3\lambda_4\text{Tr}\left(Y_u Y_u^\dagger\right) - 6\lambda_4^2\text{Tr}\left(Y_u Y_u^\dagger\right) - 6\lambda_5^2\text{Tr}\left(Y_u Y_u^\dagger\right) \end{aligned}$$

$$\begin{aligned}
& -36\lambda_6^2 \text{Tr}\left(Y_u Y_u^\dagger\right) - 18\lambda_1 \text{Tr}\left(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger\right) - 9\lambda_1 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger\right) \\
& - 12\lambda_6 \text{Tr}\left(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_d^\dagger\right) + 12\lambda_1 \text{Tr}\left(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger\right) - 12\lambda_3 \text{Tr}\left(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger\right) \\
& + 6\lambda_5 \text{Tr}\left(\epsilon_d Y_d^\dagger \epsilon_d Y_d^\dagger\right) - 9\lambda_1 \text{Tr}\left(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger\right) + 12\lambda_6 \text{Tr}\left(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger\right) - 6\lambda_1 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger\right) \\
& + 2\lambda_5 \text{Tr}\left(\epsilon_e Y_e^\dagger \epsilon_e Y_e^\dagger\right) - 3\lambda_1 \text{Tr}\left(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger\right) + 4\lambda_6 \text{Tr}\left(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger\right) - 12\lambda_6 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger Y_d \epsilon_u^\dagger\right) \\
& + 12\lambda_1 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger Y_d Y_u^\dagger\right) - 12\lambda_3 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger Y_d Y_u^\dagger\right) - \frac{8}{5} g_1^2 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger\right) \\
& - 32g_3^2 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger\right) - 3\lambda_1 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger\right) + 6\lambda_6 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger\right) + 12\lambda_6 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger\right) \\
& - 18\lambda_1 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger\right) - 42\lambda_1 \text{Tr}\left(\epsilon_u Y_d^\dagger Y_d \epsilon_u^\dagger\right) - 12\lambda_6 \text{Tr}\left(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger\right) \\
& + 6\lambda_6 \text{Tr}\left(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger\right) + 6\lambda_5 \text{Tr}\left(\epsilon_u Y_u^\dagger \epsilon_u Y_u^\dagger\right) - 9\lambda_1 \text{Tr}\left(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger\right) + 6\lambda_5 \text{Tr}\left(Y_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger\right) \\
& + 6\lambda_6 \text{Tr}\left(Y_d \epsilon_d^\dagger Y_d Y_d^\dagger\right) - 12\lambda_6 \text{Tr}\left(Y_d \epsilon_u^\dagger Y_u Y_d^\dagger\right) + 6\lambda_6 \text{Tr}\left(Y_d Y_d^\dagger Y_d \epsilon_d^\dagger\right) \\
& + \frac{4}{5} g_1^2 \text{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger\right) - 32g_3^2 \text{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger\right) - 3\lambda_1 \text{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger\right) - 9\lambda_1 \text{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger\right) \\
& + 2\lambda_5 \text{Tr}\left(Y_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger\right) + 2\lambda_6 \text{Tr}\left(Y_e \epsilon_e^\dagger Y_e Y_e^\dagger\right) + 2\lambda_6 \text{Tr}\left(Y_e Y_e^\dagger Y_e \epsilon_e^\dagger\right) - \frac{12}{5} g_1^2 \text{Tr}\left(Y_e Y_e^\dagger Y_e Y_e^\dagger\right) \\
& - \lambda_1 \text{Tr}\left(Y_e Y_e^\dagger Y_e Y_e^\dagger\right) + 6\lambda_5 \text{Tr}\left(Y_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger\right) + 12 \text{Tr}\left(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger Y_d Y_d^\dagger\right) \\
& + 12 \text{Tr}\left(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger Y_d Y_d^\dagger\right) + 6 \text{Tr}\left(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger\right) - 12 \text{Tr}\left(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger Y_u Y_d^\dagger\right) \\
& - 12 \text{Tr}\left(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger Y_d Y_d^\dagger\right) + 12 \text{Tr}\left(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger Y_d Y_d^\dagger\right) + 6 \text{Tr}\left(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger Y_d \epsilon_d^\dagger\right) \\
& + 4 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger Y_e Y_e^\dagger\right) + 4 \text{Tr}\left(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger Y_e Y_e^\dagger\right) + 2 \text{Tr}\left(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger Y_e \epsilon_e^\dagger\right) \\
& - 12 \text{Tr}\left(\epsilon_u \epsilon_d^\dagger Y_d Y_d^\dagger Y_d Y_u^\dagger\right) - 12 \text{Tr}\left(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger \epsilon_u \epsilon_u^\dagger\right) + 30 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger\right) \\
& + 12 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger\right) + 6 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger \epsilon_u Y_d^\dagger Y_d \epsilon_u^\dagger\right) + 6 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger\right) \\
& + 12 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger Y_u Y_d^\dagger Y_d Y_u^\dagger\right) - 12 \text{Tr}\left(\epsilon_u Y_d^\dagger Y_d \epsilon_d^\dagger \epsilon_u \epsilon_u^\dagger\right) - 6 \text{Tr}\left(\epsilon_u Y_d^\dagger Y_d Y_d^\dagger Y_d \epsilon_u^\dagger\right) \\
& + 6 \text{Tr}\left(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger\right) + 6 \text{Tr}\left(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger\right) + 30 \text{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger Y_d Y_d^\dagger\right) + 6 \text{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger Y_d Y_d^\dagger\right) \\
& + 10 \text{Tr}\left(Y_e Y_e^\dagger Y_e Y_e^\dagger Y_e Y_e^\dagger\right)
\end{aligned} \tag{17}$$

$$\begin{aligned}
\beta_{\lambda_4}^{(1)} &= \frac{9}{5} g_1^2 g_2^2 - \frac{9}{5} g_1^2 \lambda_4 - 9g_2^2 \lambda_4 + 4\lambda_1 \lambda_4 + 4\lambda_2 \lambda_4 + 8\lambda_3 \lambda_4 + 4\lambda_4^2 + 8\lambda_5^2 + 10\lambda_6^2 + 4\lambda_6 \lambda_7 + 10\lambda_7^2 \\
& + 6\lambda_4 \text{Tr}\left(\epsilon_d \epsilon_d^\dagger\right) + 6\lambda_7 \text{Tr}\left(\epsilon_d Y_d^\dagger\right) + 2\lambda_4 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) + 2\lambda_7 \text{Tr}\left(\epsilon_e Y_e^\dagger\right) + 6\lambda_4 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) \\
& + 6\lambda_6 \text{Tr}\left(\epsilon_u Y_u^\dagger\right) + 6\lambda_6 \text{Tr}\left(Y_d \epsilon_d^\dagger\right) + 6\lambda_4 \text{Tr}\left(Y_d Y_d^\dagger\right) + 2\lambda_6 \text{Tr}\left(Y_e \epsilon_e^\dagger\right) + 2\lambda_4 \text{Tr}\left(Y_e Y_e^\dagger\right) \\
& + 6\lambda_7 \text{Tr}\left(Y_u \epsilon_u^\dagger\right) + 6\lambda_4 \text{Tr}\left(Y_u Y_u^\dagger\right) + 12 \text{Tr}\left(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger\right) - 12 \text{Tr}\left(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger\right) \\
& - 12 \text{Tr}\left(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger\right) - 4 \text{Tr}\left(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger\right) - 12 \text{Tr}\left(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger\right) - 12 \text{Tr}\left(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger\right)
\end{aligned}$$

(18)

$$\begin{aligned}
& + 12\text{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger\right) \\
\beta_{\lambda_4}^{(2)} = & -\frac{657}{50}g_1^4 g_2^2 - \frac{42}{5}g_1^2 g_2^4 + 6g_1^2 g_2^2 \lambda_1 + 6g_1^2 g_2^2 \lambda_2 + \frac{6}{5}g_1^2 g_2^2 \lambda_3 + \frac{1413}{200}g_1^4 \lambda_4 + \frac{153}{20}g_1^2 g_2^2 \lambda_4 \\
& - \frac{231}{8}g_2^4 \lambda_4 + \frac{24}{5}g_1^2 \lambda_1 \lambda_4 - 28\lambda_1^2 \lambda_4 + \frac{24}{5}g_1^2 \lambda_2 \lambda_4 - 28\lambda_2^2 \lambda_4 + \frac{12}{5}g_1^2 \lambda_3 \lambda_4 + 36g_2^2 \lambda_3 \lambda_4 \\
& - 80\lambda_1 \lambda_3 \lambda_4 - 80\lambda_2 \lambda_3 \lambda_4 - 28\lambda_3^2 \lambda_4 + \frac{24}{5}g_1^2 \lambda_4^2 + 18g_2^2 \lambda_4^2 - 40\lambda_1 \lambda_4^2 - 40\lambda_2 \lambda_4^2 - 28\lambda_3 \lambda_4^2 \\
& + \frac{48}{5}g_1^2 \lambda_5^2 + 54g_2^2 \lambda_5^2 - 48\lambda_1 \lambda_5^2 - 48\lambda_2 \lambda_5^2 - 48\lambda_3 \lambda_5^2 - 26\lambda_4 \lambda_5^2 + \frac{42}{5}g_1^2 \lambda_6^2 + 54g_2^2 \lambda_6^2 \\
& - 148\lambda_1 \lambda_6^2 - 20\lambda_2 \lambda_6^2 - 72\lambda_3 \lambda_6^2 - 68\lambda_4 \lambda_6^2 - 80\lambda_5 \lambda_6^2 + \frac{24}{5}g_1^2 \lambda_6 \lambda_7 - 40\lambda_1 \lambda_6 \lambda_7 - 40\lambda_2 \lambda_6 \lambda_7 \\
& - 80\lambda_3 \lambda_6 \lambda_7 - 160\lambda_4 \lambda_6 \lambda_7 - 96\lambda_5 \lambda_6 \lambda_7 + \frac{42}{5}g_1^2 \lambda_7^2 + 54g_2^2 \lambda_7^2 - 20\lambda_1 \lambda_7^2 - 148\lambda_2 \lambda_7^2 \\
& - 72\lambda_3 \lambda_7^2 - 68\lambda_4 \lambda_7^2 - 80\lambda_5 \lambda_7^2 \\
& + \frac{1}{20}\left(5\left(16\left(10g_3^2 \lambda_4 - 3\left(2\lambda_2 \lambda_4 + 2\lambda_3 \lambda_4 + 2\lambda_5^2 + 5\lambda_7^2 + \lambda_6 \lambda_7 + \lambda_4^2\right)\right) + 45g_2^2 \lambda_4\right) + g_1^2\left(108g_2^2 + 25\lambda_4\right)\right)\text{Tr}\left(\epsilon_d \epsilon_d^\dagger\right) \\
& - 6\left(2\lambda_1 \lambda_7 + 2\lambda_2 \lambda_6 + 2\lambda_3\left(\lambda_6 + \lambda_7\right) + 4\lambda_5 \lambda_6 + 4\lambda_5 \lambda_7 + 5\lambda_4 \lambda_6 + 5\lambda_4 \lambda_7\right)\text{Tr}\left(\epsilon_d Y_d^\dagger\right) \\
& + \frac{33}{5}g_1^2 g_2^2 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) + \frac{15}{4}g_1^2 \lambda_4 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) + \frac{15}{4}g_2^2 \lambda_4 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) - 8\lambda_2 \lambda_4 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) \\
& - 8\lambda_3 \lambda_4 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) - 4\lambda_4^2 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) - 8\lambda_5^2 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) - 4\lambda_6 \lambda_7 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) \\
& - 20\lambda_7^2 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) - 4\lambda_2 \lambda_6 \text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 4\lambda_3 \lambda_6 \text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 10\lambda_4 \lambda_6 \text{Tr}\left(\epsilon_e Y_e^\dagger\right) \\
& - 8\lambda_5 \lambda_6 \text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 4\lambda_1 \lambda_7 \text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 4\lambda_3 \lambda_7 \text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 10\lambda_4 \lambda_7 \text{Tr}\left(\epsilon_e Y_e^\dagger\right) \\
& - 8\lambda_5 \lambda_7 \text{Tr}\left(\epsilon_e Y_e^\dagger\right) + \frac{63}{5}g_1^2 g_2^2 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) + \frac{17}{4}g_1^2 \lambda_4 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) + \frac{45}{4}g_2^2 \lambda_4 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) \\
& + 40g_3^2 \lambda_4 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) - 24\lambda_1 \lambda_4 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) - 24\lambda_3 \lambda_4 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) - 12\lambda_4^2 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) \\
& - 24\lambda_5^2 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) - 60\lambda_6^2 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) - 12\lambda_6 \lambda_7 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) - 12\lambda_2 \lambda_6 \text{Tr}\left(\epsilon_u Y_u^\dagger\right) \\
& - 12\lambda_3 \lambda_6 \text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 30\lambda_4 \lambda_6 \text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 24\lambda_5 \lambda_6 \text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 12\lambda_1 \lambda_7 \text{Tr}\left(\epsilon_u Y_u^\dagger\right) \\
& - 12\lambda_3 \lambda_7 \text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 30\lambda_4 \lambda_7 \text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 24\lambda_5 \lambda_7 \text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 12\lambda_2 \lambda_6 \text{Tr}\left(Y_d \epsilon_d^\dagger\right) \\
& - 12\lambda_3 \lambda_6 \text{Tr}\left(Y_d \epsilon_d^\dagger\right) - 30\lambda_4 \lambda_6 \text{Tr}\left(Y_d \epsilon_d^\dagger\right) - 24\lambda_5 \lambda_6 \text{Tr}\left(Y_d \epsilon_d^\dagger\right) - 12\lambda_1 \lambda_7 \text{Tr}\left(Y_d \epsilon_d^\dagger\right) \\
& - 12\lambda_3 \lambda_7 \text{Tr}\left(Y_d \epsilon_d^\dagger\right) - 30\lambda_4 \lambda_7 \text{Tr}\left(Y_d \epsilon_d^\dagger\right) - 24\lambda_5 \lambda_7 \text{Tr}\left(Y_d \epsilon_d^\dagger\right) + \frac{27}{5}g_1^2 g_2^2 \text{Tr}\left(Y_d Y_d^\dagger\right) \\
& + \frac{5}{4}g_1^2 \lambda_4 \text{Tr}\left(Y_d Y_d^\dagger\right) + \frac{45}{4}g_2^2 \lambda_4 \text{Tr}\left(Y_d Y_d^\dagger\right) + 40g_3^2 \lambda_4 \text{Tr}\left(Y_d Y_d^\dagger\right) - 24\lambda_1 \lambda_4 \text{Tr}\left(Y_d Y_d^\dagger\right) \\
& - 24\lambda_3 \lambda_4 \text{Tr}\left(Y_d Y_d^\dagger\right) - 12\lambda_4^2 \text{Tr}\left(Y_d Y_d^\dagger\right) - 24\lambda_5^2 \text{Tr}\left(Y_d Y_d^\dagger\right) - 60\lambda_6^2 \text{Tr}\left(Y_d Y_d^\dagger\right) \\
& - 12\lambda_6 \lambda_7 \text{Tr}\left(Y_d Y_d^\dagger\right) - 4\lambda_2 \lambda_6 \text{Tr}\left(Y_e \epsilon_e^\dagger\right) - 4\lambda_3 \lambda_6 \text{Tr}\left(Y_e \epsilon_e^\dagger\right) - 10\lambda_4 \lambda_6 \text{Tr}\left(Y_e \epsilon_e^\dagger\right)
\end{aligned}$$

$$\begin{aligned}
& - 8\lambda_5\lambda_6\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - 4\lambda_1\lambda_7\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - 4\lambda_3\lambda_7\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - 10\lambda_4\lambda_7\text{Tr}\left(Y_e\epsilon_e^\dagger\right) \\
& - 8\lambda_5\lambda_7\text{Tr}\left(Y_e\epsilon_e^\dagger\right) + \frac{33}{5}g_1^2g_2^2\text{Tr}\left(Y_eY_e^\dagger\right) + \frac{15}{4}g_1^2\lambda_4\text{Tr}\left(Y_eY_e^\dagger\right) + \frac{15}{4}g_2^2\lambda_4\text{Tr}\left(Y_eY_e^\dagger\right) \\
& - 8\lambda_1\lambda_4\text{Tr}\left(Y_eY_e^\dagger\right) - 8\lambda_3\lambda_4\text{Tr}\left(Y_eY_e^\dagger\right) - 4\lambda_4^2\text{Tr}\left(Y_eY_e^\dagger\right) - 8\lambda_5^2\text{Tr}\left(Y_eY_e^\dagger\right) \\
& - 20\lambda_6^2\text{Tr}\left(Y_eY_e^\dagger\right) - 4\lambda_6\lambda_7\text{Tr}\left(Y_eY_e^\dagger\right) - 12\lambda_2\lambda_6\text{Tr}\left(Y_u\epsilon_u^\dagger\right) - 12\lambda_3\lambda_6\text{Tr}\left(Y_u\epsilon_u^\dagger\right) \\
& - 30\lambda_4\lambda_6\text{Tr}\left(Y_u\epsilon_u^\dagger\right) - 24\lambda_5\lambda_6\text{Tr}\left(Y_u\epsilon_u^\dagger\right) - 12\lambda_1\lambda_7\text{Tr}\left(Y_u\epsilon_u^\dagger\right) - 12\lambda_3\lambda_7\text{Tr}\left(Y_u\epsilon_u^\dagger\right) \\
& - 30\lambda_4\lambda_7\text{Tr}\left(Y_u\epsilon_u^\dagger\right) - 24\lambda_5\lambda_7\text{Tr}\left(Y_u\epsilon_u^\dagger\right) + \frac{63}{5}g_1^2g_2^2\text{Tr}\left(Y_uY_u^\dagger\right) + \frac{17}{4}g_1^2\lambda_4\text{Tr}\left(Y_uY_u^\dagger\right) \\
& + \frac{45}{4}g_2^2\lambda_4\text{Tr}\left(Y_uY_u^\dagger\right) + 40g_3^2\lambda_4\text{Tr}\left(Y_uY_u^\dagger\right) - 24\lambda_2\lambda_4\text{Tr}\left(Y_uY_u^\dagger\right) - 24\lambda_3\lambda_4\text{Tr}\left(Y_uY_u^\dagger\right) \\
& - 12\lambda_4^2\text{Tr}\left(Y_uY_u^\dagger\right) - 24\lambda_5^2\text{Tr}\left(Y_uY_u^\dagger\right) - 12\lambda_6\lambda_7\text{Tr}\left(Y_uY_u^\dagger\right) - 60\lambda_7^2\text{Tr}\left(Y_uY_u^\dagger\right) \\
& - \frac{27}{2}\lambda_4\text{Tr}\left(\epsilon_d\epsilon_d^\dagger\epsilon_d\epsilon_d^\dagger\right) + 6\lambda_7\text{Tr}\left(\epsilon_d\epsilon_d^\dagger\epsilon_dY_d^\dagger\right) + 12\lambda_7\text{Tr}\left(\epsilon_d\epsilon_d^\dagger Y_d\epsilon_d^\dagger\right) + 6\lambda_4\text{Tr}\left(\epsilon_d\epsilon_d^\dagger Y_dY_d^\dagger\right) \\
& + \frac{4}{5}g_1^2\text{Tr}\left(\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger\right) + 64g_3^2\text{Tr}\left(\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger\right) - 24\lambda_3\text{Tr}\left(\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger\right) \\
& - 33\lambda_4\text{Tr}\left(\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger\right) - 24\lambda_6\text{Tr}\left(\epsilon_d\epsilon_u^\dagger\epsilon_uY_d^\dagger\right) + 12\lambda_7\text{Tr}\left(\epsilon_d\epsilon_u^\dagger\epsilon_uY_d^\dagger\right) \\
& + 12\lambda_6\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_u\epsilon_d^\dagger\right) - 24\lambda_7\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_u\epsilon_d^\dagger\right) - \frac{4}{5}g_1^2\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_uY_d^\dagger\right) \\
& - 64g_3^2\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_uY_d^\dagger\right) + 24\lambda_1\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_uY_d^\dagger\right) + 24\lambda_2\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_uY_d^\dagger\right) \\
& + 12\lambda_4\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_uY_d^\dagger\right) + 6\lambda_7\text{Tr}\left(\epsilon_dY_d^\dagger\epsilon_d\epsilon_d^\dagger\right) + \frac{8}{5}g_1^2\text{Tr}\left(\epsilon_dY_d^\dagger Y_d\epsilon_d^\dagger\right) \\
& - 64g_3^2\text{Tr}\left(\epsilon_dY_d^\dagger Y_d\epsilon_d^\dagger\right) + 24\lambda_3\text{Tr}\left(\epsilon_dY_d^\dagger Y_d\epsilon_d^\dagger\right) - 9\lambda_4\text{Tr}\left(\epsilon_dY_d^\dagger Y_d\epsilon_d^\dagger\right) \\
& + 12\lambda_6\text{Tr}\left(\epsilon_dY_d^\dagger Y_dY_d^\dagger\right) + 12\lambda_6\text{Tr}\left(\epsilon_dY_u^\dagger\epsilon_u\epsilon_d^\dagger\right) - 24\lambda_7\text{Tr}\left(\epsilon_dY_u^\dagger\epsilon_u\epsilon_d^\dagger\right) \\
& - 48\lambda_5\text{Tr}\left(\epsilon_dY_u^\dagger\epsilon_uY_d^\dagger\right) + 27\lambda_4\text{Tr}\left(\epsilon_dY_u^\dagger Y_u\epsilon_d^\dagger\right) + 12\lambda_6\text{Tr}\left(\epsilon_dY_u^\dagger Y_uY_d^\dagger\right) \\
& - 24\lambda_7\text{Tr}\left(\epsilon_dY_u^\dagger Y_uY_d^\dagger\right) - \frac{9}{2}\lambda_4\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\epsilon_e\epsilon_e^\dagger\right) + 2\lambda_7\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\epsilon_eY_e^\dagger\right) + 4\lambda_7\text{Tr}\left(\epsilon_e\epsilon_e^\dagger Y_e\epsilon_e^\dagger\right) \\
& + 2\lambda_4\text{Tr}\left(\epsilon_e\epsilon_e^\dagger Y_eY_e^\dagger\right) + 2\lambda_7\text{Tr}\left(\epsilon_eY_e^\dagger\epsilon_e\epsilon_e^\dagger\right) - \frac{24}{5}g_1^2\text{Tr}\left(\epsilon_eY_e^\dagger Y_e\epsilon_e^\dagger\right) \\
& + 8\lambda_3\text{Tr}\left(\epsilon_eY_e^\dagger Y_e\epsilon_e^\dagger\right) - 3\lambda_4\text{Tr}\left(\epsilon_eY_e^\dagger Y_e\epsilon_e^\dagger\right) + 4\lambda_6\text{Tr}\left(\epsilon_eY_e^\dagger Y_eY_e^\dagger\right) \\
& - 24\lambda_6\text{Tr}\left(\epsilon_u\epsilon_d^\dagger Y_d\epsilon_u^\dagger\right) + 12\lambda_7\text{Tr}\left(\epsilon_u\epsilon_d^\dagger Y_d\epsilon_u^\dagger\right) - \frac{4}{5}g_1^2\text{Tr}\left(\epsilon_u\epsilon_d^\dagger Y_dY_u^\dagger\right) \\
& - 64g_3^2\text{Tr}\left(\epsilon_u\epsilon_d^\dagger Y_dY_u^\dagger\right) + 24\lambda_1\text{Tr}\left(\epsilon_u\epsilon_d^\dagger Y_dY_u^\dagger\right) + 24\lambda_2\text{Tr}\left(\epsilon_u\epsilon_d^\dagger Y_dY_u^\dagger\right) \\
& + 12\lambda_4\text{Tr}\left(\epsilon_u\epsilon_d^\dagger Y_dY_u^\dagger\right) - \frac{27}{2}\lambda_4\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\epsilon_u\epsilon_u^\dagger\right) + 6\lambda_6\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\epsilon_uY_u^\dagger\right) + 12\lambda_6\text{Tr}\left(\epsilon_u\epsilon_u^\dagger Y_u\epsilon_u^\dagger\right) \\
& + 6\lambda_4\text{Tr}\left(\epsilon_u\epsilon_u^\dagger Y_uY_u^\dagger\right) + 27\lambda_4\text{Tr}\left(\epsilon_uY_d^\dagger Y_d\epsilon_u^\dagger\right) - 24\lambda_6\text{Tr}\left(\epsilon_uY_d^\dagger Y_dY_u^\dagger\right)
\end{aligned}$$

$$\begin{aligned}
& + 12\lambda_7 \text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger) + 6\lambda_6 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger) - \frac{16}{5} g_1^2 \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) \\
& - 64g_3^2 \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) + 24\lambda_3 \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) - 9\lambda_4 \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) \\
& + 12\lambda_7 \text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger) + 6\lambda_6 \text{Tr}(Y_d \epsilon_d^\dagger Y_d Y_d^\dagger) - 48\lambda_5 \text{Tr}(Y_d \epsilon_u^\dagger Y_u \epsilon_d^\dagger) \\
& - 24\lambda_6 \text{Tr}(Y_d \epsilon_u^\dagger Y_u Y_d^\dagger) + 12\lambda_7 \text{Tr}(Y_d \epsilon_u^\dagger Y_u Y_d^\dagger) + 6\lambda_6 \text{Tr}(Y_d Y_d^\dagger Y_d \epsilon_d^\dagger) \\
& - \frac{27}{2} \lambda_4 \text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) + 12\lambda_6 \text{Tr}(Y_d Y_u^\dagger Y_u \epsilon_d^\dagger) - 24\lambda_7 \text{Tr}(Y_d Y_u^\dagger Y_u \epsilon_d^\dagger) \\
& + \frac{4}{5} g_1^2 \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) + 64g_3^2 \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) - 24\lambda_3 \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) \\
& - 33\lambda_4 \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) + 2\lambda_6 \text{Tr}(Y_e \epsilon_e^\dagger Y_e Y_e^\dagger) + 2\lambda_6 \text{Tr}(Y_e Y_e^\dagger Y_e \epsilon_e^\dagger) - \frac{9}{2} \lambda_4 \text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) \\
& + 6\lambda_7 \text{Tr}(Y_u \epsilon_u^\dagger Y_u Y_u^\dagger) + 6\lambda_7 \text{Tr}(Y_u Y_u^\dagger Y_u \epsilon_u^\dagger) - \frac{27}{2} \lambda_4 \text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger) - 12 \text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) \\
& + 18 \text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) + 12 \text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger) + 6 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger \epsilon_d Y_d^\dagger) \\
& + 30 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) - 36 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_u^\dagger Y_u Y_d^\dagger) - 12 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger) \\
& - 36 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger Y_d Y_d^\dagger) - 24 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger \epsilon_u \epsilon_d^\dagger) + 30 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger Y_u Y_d^\dagger) \\
& - 6 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u \epsilon_d^\dagger \epsilon_d Y_d^\dagger) - 6 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u \epsilon_u^\dagger \epsilon_u Y_d^\dagger) + 12 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger \epsilon_d \epsilon_d^\dagger) \\
& + 30 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger Y_d Y_d^\dagger) - 36 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_u^\dagger \epsilon_u \epsilon_d^\dagger) + 30 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_u^\dagger Y_u Y_d^\dagger) \\
& + 30 \text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger) + 12 \text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger) + 36 \text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger Y_d Y_d^\dagger) \\
& - 6 \text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger Y_u Y_d^\dagger) + 24 \text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger Y_d \epsilon_d^\dagger) - 6 \text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger Y_d \epsilon_d^\dagger) \\
& - 6 \text{Tr}(\epsilon_d Y_u^\dagger Y_u \epsilon_u^\dagger Y_u Y_d^\dagger) + 4 \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger) + 2 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger) \\
& + 10 \text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) + 4 \text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger) + 12 \text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger Y_e Y_e^\dagger) \\
& + 8 \text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger Y_e \epsilon_e^\dagger) - 6 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d \epsilon_u^\dagger \epsilon_u Y_u^\dagger) + 30 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_d^\dagger Y_d Y_u^\dagger) \\
& + 12 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger \epsilon_u \epsilon_u^\dagger) + 30 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger Y_u Y_u^\dagger) + 18 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) \\
& + 12 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) + 6 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger) - 36 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u Y_d^\dagger Y_d Y_u^\dagger) \\
& - 6 \text{Tr}(\epsilon_u Y_d^\dagger Y_d \epsilon_d^\dagger Y_d Y_u^\dagger) + 30 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) - 6 \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_d^\dagger Y_d Y_u^\dagger) \\
& + 12 \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger) + 36 \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger Y_u Y_u^\dagger) + 24 \text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger Y_u \epsilon_u^\dagger) \\
& - 12 \text{Tr}(Y_d Y_d^\dagger Y_d Y_u^\dagger Y_u Y_d^\dagger) - 12 \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger Y_d Y_d^\dagger) - 24 \text{Tr}(Y_d Y_u^\dagger Y_u Y_u^\dagger Y_u Y_d^\dagger)
\end{aligned} \tag{19}$$

$$\beta_{\lambda_3}^{(1)} = + \frac{27}{100} g_1^4 - \frac{9}{10} g_1^2 g_2^2 + \frac{9}{4} g_2^4 - \frac{9}{5} g_1^2 \lambda_3 - 9 g_2^2 \lambda_3 + 12 \lambda_1 \lambda_3 + 12 \lambda_2 \lambda_3 + 4 \lambda_3^2 + 4 \lambda_1 \lambda_4 + 4 \lambda_2 \lambda_4 + 2 \lambda_4^2$$

$$\begin{aligned}
& + 2\lambda_5^2 + 4\lambda_6^2 + 16\lambda_6\lambda_7 + 4\lambda_7^2 + 6\lambda_3 \text{Tr}(\epsilon_d \epsilon_d^\dagger) + 6\lambda_7 \text{Tr}(\epsilon_d Y_d^\dagger) + 2\lambda_3 \text{Tr}(\epsilon_e \epsilon_e^\dagger) + 2\lambda_7 \text{Tr}(\epsilon_e Y_e^\dagger) \\
& + 6\lambda_3 \text{Tr}(\epsilon_u \epsilon_u^\dagger) + 6\lambda_6 \text{Tr}(\epsilon_u Y_u^\dagger) + 6\lambda_6 \text{Tr}(Y_d \epsilon_d^\dagger) + 6\lambda_3 \text{Tr}(Y_d Y_d^\dagger) + 2\lambda_6 \text{Tr}(Y_e \epsilon_e^\dagger) \\
& + 2\lambda_3 \text{Tr}(Y_e Y_e^\dagger) + 6\lambda_7 \text{Tr}(Y_u \epsilon_u^\dagger) + 6\lambda_3 \text{Tr}(Y_u Y_u^\dagger) - 12 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger) \\
& - 12 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) + 12 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) - 4 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger) + 12 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) \\
& - 12 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger) - 12 \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) \tag{20} \\
\beta_{\lambda_3}^{(2)} = & -\frac{3537}{1000}g_1^6 + \frac{909}{200}g_1^4g_2^2 + \frac{33}{40}g_1^2g_2^4 + \frac{291}{8}g_2^6 + \frac{27}{10}g_1^4\lambda_1 - 3g_1^2g_2^2\lambda_1 + \frac{45}{2}g_2^4\lambda_1 + \frac{27}{10}g_1^4\lambda_2 \\
& - 3g_1^2g_2^2\lambda_2 + \frac{45}{2}g_2^4\lambda_2 + \frac{1773}{200}g_1^4\lambda_3 + \frac{33}{20}g_1^2g_2^2\lambda_3 - \frac{111}{8}g_2^4\lambda_3 + \frac{72}{5}g_1^2\lambda_1\lambda_3 + 72g_2^2\lambda_1\lambda_3 \\
& - 60\lambda_1^2\lambda_3 + \frac{72}{5}g_1^2\lambda_2\lambda_3 + 72g_2^2\lambda_2\lambda_3 - 60\lambda_2^2\lambda_3 + \frac{6}{5}g_1^2\lambda_3^2 + 6g_2^2\lambda_3^2 - 72\lambda_1\lambda_3^2 - 72\lambda_2\lambda_3^2 \\
& - 12\lambda_3^3 + \frac{9}{10}g_1^4\lambda_4 - \frac{9}{5}g_1^2g_2^2\lambda_4 + \frac{15}{2}g_2^4\lambda_4 + \frac{24}{5}g_1^2\lambda_1\lambda_4 + 36g_2^2\lambda_1\lambda_4 - 16\lambda_1^2\lambda_4 + \frac{24}{5}g_1^2\lambda_2\lambda_4 \\
& + 36g_2^2\lambda_2\lambda_4 - 16\lambda_2^2\lambda_4 - 12g_2^2\lambda_3\lambda_4 - 32\lambda_1\lambda_3\lambda_4 - 32\lambda_2\lambda_3\lambda_4 - 4\lambda_3^2\lambda_4 - \frac{6}{5}g_1^2\lambda_4^2 \\
& + 6g_2^2\lambda_4^2 - 28\lambda_1\lambda_4^2 - 28\lambda_2\lambda_4^2 - 16\lambda_3\lambda_4^2 - 12\lambda_4^3 + \frac{12}{5}g_1^2\lambda_5^2 - 36\lambda_1\lambda_5^2 - 36\lambda_2\lambda_5^2 \\
& - 18\lambda_3\lambda_5^2 - 44\lambda_4\lambda_5^2 + \frac{6}{5}g_1^2\lambda_6^2 - 124\lambda_1\lambda_6^2 - 44\lambda_2\lambda_6^2 - 60\lambda_3\lambda_6^2 - 68\lambda_4\lambda_6^2 - 68\lambda_5\lambda_6^2 \\
& + \frac{96}{5}g_1^2\lambda_6\lambda_7 + 108g_2^2\lambda_6\lambda_7 - 88\lambda_1\lambda_6\lambda_7 - 88\lambda_2\lambda_6\lambda_7 - 176\lambda_3\lambda_6\lambda_7 - 88\lambda_4\lambda_6\lambda_7 - 72\lambda_5\lambda_6\lambda_7 \\
& + \frac{6}{5}g_1^2\lambda_7^2 - 44\lambda_1\lambda_7^2 - 124\lambda_2\lambda_7^2 - 60\lambda_3\lambda_7^2 - 68\lambda_4\lambda_7^2 - 68\lambda_5\lambda_7^2 \\
& + \frac{1}{20}(9g_1^4 + g_1^2(25\lambda_3 - 54g_2^2) \\
& - 5(-45g_2^2\lambda_3 + 8(-20g_3^2\lambda_3 + 3(2\lambda_3^2 + 4\lambda_2(3\lambda_3 + \lambda_4) + 4\lambda_7^2 + 8\lambda_6\lambda_7 + \lambda_4^2 + \lambda_5^2))) + 9g_2^4)) \text{Tr}(\epsilon_d \epsilon_d^\dagger) \\
& - 6(2\lambda_4\lambda_6 + 2\lambda_4\lambda_7 + 5\lambda_3(\lambda_6 + \lambda_7) + 8\lambda_1\lambda_7 + 8\lambda_2\lambda_6 + \lambda_5\lambda_6 + \lambda_5\lambda_7) \text{Tr}(\epsilon_d Y_d^\dagger) - \frac{9}{4}g_1^4 \text{Tr}(\epsilon_e \epsilon_e^\dagger) \\
& - \frac{33}{10}g_1^2g_2^2 \text{Tr}(\epsilon_e \epsilon_e^\dagger) - \frac{3}{4}g_2^4 \text{Tr}(\epsilon_e \epsilon_e^\dagger) + \frac{15}{4}g_1^2\lambda_3 \text{Tr}(\epsilon_e \epsilon_e^\dagger) + \frac{15}{4}g_2^2\lambda_3 \text{Tr}(\epsilon_e \epsilon_e^\dagger) \\
& - 24\lambda_2\lambda_3 \text{Tr}(\epsilon_e \epsilon_e^\dagger) - 4\lambda_3^2 \text{Tr}(\epsilon_e \epsilon_e^\dagger) - 8\lambda_2\lambda_4 \text{Tr}(\epsilon_e \epsilon_e^\dagger) - 2\lambda_4^2 \text{Tr}(\epsilon_e \epsilon_e^\dagger) \\
& - 2\lambda_5^2 \text{Tr}(\epsilon_e \epsilon_e^\dagger) - 16\lambda_6\lambda_7 \text{Tr}(\epsilon_e \epsilon_e^\dagger) - 8\lambda_7^2 \text{Tr}(\epsilon_e \epsilon_e^\dagger) - 16\lambda_2\lambda_6 \text{Tr}(\epsilon_e Y_e^\dagger) \\
& - 10\lambda_3\lambda_6 \text{Tr}(\epsilon_e Y_e^\dagger) - 4\lambda_4\lambda_6 \text{Tr}(\epsilon_e Y_e^\dagger) - 2\lambda_5\lambda_6 \text{Tr}(\epsilon_e Y_e^\dagger) - 16\lambda_1\lambda_7 \text{Tr}(\epsilon_e Y_e^\dagger) \\
& - 10\lambda_3\lambda_7 \text{Tr}(\epsilon_e Y_e^\dagger) - 4\lambda_4\lambda_7 \text{Tr}(\epsilon_e Y_e^\dagger) - 2\lambda_5\lambda_7 \text{Tr}(\epsilon_e Y_e^\dagger) - \frac{171}{100}g_1^4 \text{Tr}(\epsilon_u \epsilon_u^\dagger) \\
& - \frac{63}{10}g_1^2g_2^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger) - \frac{9}{4}g_2^4 \text{Tr}(\epsilon_u \epsilon_u^\dagger) + \frac{17}{4}g_1^2\lambda_3 \text{Tr}(\epsilon_u \epsilon_u^\dagger) + \frac{45}{4}g_2^2\lambda_3 \text{Tr}(\epsilon_u \epsilon_u^\dagger) \\
& + 40g_3^2\lambda_3 \text{Tr}(\epsilon_u \epsilon_u^\dagger) - 72\lambda_1\lambda_3 \text{Tr}(\epsilon_u \epsilon_u^\dagger) - 12\lambda_3^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger) - 24\lambda_1\lambda_4 \text{Tr}(\epsilon_u \epsilon_u^\dagger)
\end{aligned}$$

$$\begin{aligned}
& -6\lambda_4^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger) - 6\lambda_5^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger) - 24\lambda_6^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger) - 48\lambda_6 \lambda_7 \text{Tr}(\epsilon_u \epsilon_u^\dagger) \\
& -48\lambda_2 \lambda_6 \text{Tr}(\epsilon_u Y_u^\dagger) - 30\lambda_3 \lambda_6 \text{Tr}(\epsilon_u Y_u^\dagger) - 12\lambda_4 \lambda_6 \text{Tr}(\epsilon_u Y_u^\dagger) - 6\lambda_5 \lambda_6 \text{Tr}(\epsilon_u Y_u^\dagger) \\
& -48\lambda_1 \lambda_7 \text{Tr}(\epsilon_u Y_u^\dagger) - 30\lambda_3 \lambda_7 \text{Tr}(\epsilon_u Y_u^\dagger) - 12\lambda_4 \lambda_7 \text{Tr}(\epsilon_u Y_u^\dagger) - 6\lambda_5 \lambda_7 \text{Tr}(\epsilon_u Y_u^\dagger) \\
& -48\lambda_2 \lambda_6 \text{Tr}(Y_d \epsilon_d^\dagger) - 30\lambda_3 \lambda_6 \text{Tr}(Y_d \epsilon_d^\dagger) - 12\lambda_4 \lambda_6 \text{Tr}(Y_d \epsilon_d^\dagger) - 6\lambda_5 \lambda_6 \text{Tr}(Y_d \epsilon_d^\dagger) \\
& -48\lambda_1 \lambda_7 \text{Tr}(Y_d \epsilon_d^\dagger) - 30\lambda_3 \lambda_7 \text{Tr}(Y_d \epsilon_d^\dagger) - 12\lambda_4 \lambda_7 \text{Tr}(Y_d \epsilon_d^\dagger) - 6\lambda_5 \lambda_7 \text{Tr}(Y_d \epsilon_d^\dagger) \\
& + \frac{9}{20} g_1^4 \text{Tr}(Y_d Y_d^\dagger) - \frac{27}{10} g_1^2 g_2^2 \text{Tr}(Y_d Y_d^\dagger) - \frac{9}{4} g_2^4 \text{Tr}(Y_d Y_d^\dagger) + \frac{5}{4} g_1^2 \lambda_3 \text{Tr}(Y_d Y_d^\dagger) \\
& + \frac{45}{4} g_2^2 \lambda_3 \text{Tr}(Y_d Y_d^\dagger) + 40g_3^2 \lambda_3 \text{Tr}(Y_d Y_d^\dagger) - 72\lambda_1 \lambda_3 \text{Tr}(Y_d Y_d^\dagger) - 12\lambda_3^2 \text{Tr}(Y_d Y_d^\dagger) \\
& -24\lambda_1 \lambda_4 \text{Tr}(Y_d Y_d^\dagger) - 6\lambda_4^2 \text{Tr}(Y_d Y_d^\dagger) - 6\lambda_5^2 \text{Tr}(Y_d Y_d^\dagger) - 24\lambda_6^2 \text{Tr}(Y_d Y_d^\dagger) \\
& -48\lambda_6 \lambda_7 \text{Tr}(Y_d Y_d^\dagger) - 16\lambda_2 \lambda_6 \text{Tr}(Y_e \epsilon_e^\dagger) - 10\lambda_3 \lambda_6 \text{Tr}(Y_e \epsilon_e^\dagger) - 4\lambda_4 \lambda_6 \text{Tr}(Y_e \epsilon_e^\dagger) \\
& -2\lambda_5 \lambda_6 \text{Tr}(Y_e \epsilon_e^\dagger) - 16\lambda_1 \lambda_7 \text{Tr}(Y_e \epsilon_e^\dagger) - 10\lambda_3 \lambda_7 \text{Tr}(Y_e \epsilon_e^\dagger) - 4\lambda_4 \lambda_7 \text{Tr}(Y_e \epsilon_e^\dagger) \\
& -2\lambda_5 \lambda_7 \text{Tr}(Y_e \epsilon_e^\dagger) - \frac{9}{4} g_1^4 \text{Tr}(Y_e Y_e^\dagger) - \frac{33}{10} g_1^2 g_2^2 \text{Tr}(Y_e Y_e^\dagger) - \frac{3}{4} g_2^4 \text{Tr}(Y_e Y_e^\dagger) \\
& + \frac{15}{4} g_1^2 \lambda_3 \text{Tr}(Y_e Y_e^\dagger) + \frac{15}{4} g_2^2 \lambda_3 \text{Tr}(Y_e Y_e^\dagger) - 24\lambda_1 \lambda_3 \text{Tr}(Y_e Y_e^\dagger) - 4\lambda_3^2 \text{Tr}(Y_e Y_e^\dagger) \\
& -8\lambda_1 \lambda_4 \text{Tr}(Y_e Y_e^\dagger) - 2\lambda_4^2 \text{Tr}(Y_e Y_e^\dagger) - 2\lambda_5^2 \text{Tr}(Y_e Y_e^\dagger) - 8\lambda_6^2 \text{Tr}(Y_e Y_e^\dagger) \\
& -16\lambda_6 \lambda_7 \text{Tr}(Y_e Y_e^\dagger) - 48\lambda_2 \lambda_6 \text{Tr}(Y_u \epsilon_u^\dagger) - 30\lambda_3 \lambda_6 \text{Tr}(Y_u \epsilon_u^\dagger) - 12\lambda_4 \lambda_6 \text{Tr}(Y_u \epsilon_u^\dagger) \\
& -6\lambda_5 \lambda_6 \text{Tr}(Y_u \epsilon_u^\dagger) - 48\lambda_1 \lambda_7 \text{Tr}(Y_u \epsilon_u^\dagger) - 30\lambda_3 \lambda_7 \text{Tr}(Y_u \epsilon_u^\dagger) - 12\lambda_4 \lambda_7 \text{Tr}(Y_u \epsilon_u^\dagger) \\
& -6\lambda_5 \lambda_7 \text{Tr}(Y_u \epsilon_u^\dagger) - \frac{171}{100} g_1^4 \text{Tr}(Y_u Y_u^\dagger) - \frac{63}{10} g_1^2 g_2^2 \text{Tr}(Y_u Y_u^\dagger) - \frac{9}{4} g_2^4 \text{Tr}(Y_u Y_u^\dagger) \\
& + \frac{17}{4} g_1^2 \lambda_3 \text{Tr}(Y_u Y_u^\dagger) + \frac{45}{4} g_2^2 \lambda_3 \text{Tr}(Y_u Y_u^\dagger) + 40g_3^2 \lambda_3 \text{Tr}(Y_u Y_u^\dagger) - 72\lambda_2 \lambda_3 \text{Tr}(Y_u Y_u^\dagger) \\
& -12\lambda_3^2 \text{Tr}(Y_u Y_u^\dagger) - 24\lambda_2 \lambda_4 \text{Tr}(Y_u Y_u^\dagger) - 6\lambda_4^2 \text{Tr}(Y_u Y_u^\dagger) - 6\lambda_5^2 \text{Tr}(Y_u Y_u^\dagger) \\
& -48\lambda_6 \lambda_7 \text{Tr}(Y_u Y_u^\dagger) - 24\lambda_7^2 \text{Tr}(Y_u Y_u^\dagger) - \frac{27}{2} \lambda_3 \text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger) + 6\lambda_7 \text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d Y_d^\dagger) \\
& + 12\lambda_7 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger) + \frac{8}{5} g_1^2 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger) - 64g_3^2 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger) \\
& + 6\lambda_3 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger) - \frac{4}{5} g_1^2 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) - 64g_3^2 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) \\
& + 15\lambda_3 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) + 12\lambda_6 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_d^\dagger) - 24\lambda_7 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_d^\dagger) \\
& -24\lambda_6 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u \epsilon_d^\dagger) + 12\lambda_7 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u \epsilon_d^\dagger) + \frac{4}{5} g_1^2 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) \\
& + 64g_3^2 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) - 48\lambda_1 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) - 48\lambda_2 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger)
\end{aligned}$$

$$\begin{aligned}
& + 12\lambda_3 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) + 6\lambda_7 \text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger) - 9\lambda_3 \text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger) \\
& + 24\lambda_4 \text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger) + 12\lambda_6 \text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger) - 24\lambda_6 \text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) \\
& + 12\lambda_7 \text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) + 24\lambda_5 \text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u Y_d^\dagger) - 21\lambda_3 \text{Tr}(\epsilon_d Y_u^\dagger Y_u \epsilon_d^\dagger) \\
& - 24\lambda_4 \text{Tr}(\epsilon_d Y_u^\dagger Y_u \epsilon_d^\dagger) - 24\lambda_6 \text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger) + 12\lambda_7 \text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger) \\
& - \frac{9}{2} \lambda_3 \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger) + 2\lambda_7 \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger) + 4\lambda_7 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) - \frac{24}{5} g_1^2 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger) \\
& + 2\lambda_3 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger) + 2\lambda_7 \text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger) - 3\lambda_3 \text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger) \\
& + 8\lambda_4 \text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger) + 4\lambda_6 \text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger) + 12\lambda_6 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d \epsilon_u^\dagger) \\
& - 24\lambda_7 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d \epsilon_u^\dagger) + \frac{4}{5} g_1^2 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) + 64g_3^2 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) \\
& - 48\lambda_1 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) - 48\lambda_2 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) + 12\lambda_3 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) \\
& - \frac{27}{2} \lambda_3 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger) + 6\lambda_6 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger) + 12\lambda_6 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) - \frac{16}{5} g_1^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger) \\
& - 64g_3^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger) + 6\lambda_3 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger) - 21\lambda_3 \text{Tr}(\epsilon_u Y_d^\dagger Y_d \epsilon_u^\dagger) \\
& - 24\lambda_4 \text{Tr}(\epsilon_u Y_d^\dagger Y_d \epsilon_u^\dagger) + 12\lambda_6 \text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger) - 24\lambda_7 \text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger) \\
& + 6\lambda_6 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger) - 9\lambda_3 \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) + 24\lambda_4 \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) \\
& + 12\lambda_7 \text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger) + 6\lambda_6 \text{Tr}(Y_d \epsilon_d^\dagger Y_d Y_d^\dagger) + 24\lambda_5 \text{Tr}(Y_d \epsilon_u^\dagger Y_u \epsilon_d^\dagger) \\
& + 12\lambda_6 \text{Tr}(Y_d \epsilon_u^\dagger Y_u Y_d^\dagger) - 24\lambda_7 \text{Tr}(Y_d \epsilon_u^\dagger Y_u Y_d^\dagger) + 6\lambda_6 \text{Tr}(Y_d Y_d^\dagger Y_d \epsilon_d^\dagger) \\
& - \frac{27}{2} \lambda_3 \text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) - 24\lambda_6 \text{Tr}(Y_d Y_u^\dagger Y_u \epsilon_d^\dagger) + 12\lambda_7 \text{Tr}(Y_d Y_u^\dagger Y_u \epsilon_d^\dagger) \\
& - \frac{4}{5} g_1^2 \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) - 64g_3^2 \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) + 15\lambda_3 \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) \\
& + 2\lambda_6 \text{Tr}(Y_e \epsilon_e^\dagger Y_e Y_e^\dagger) + 2\lambda_6 \text{Tr}(Y_e Y_e^\dagger Y_e \epsilon_e^\dagger) - \frac{9}{2} \lambda_3 \text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) + 6\lambda_7 \text{Tr}(Y_u \epsilon_u^\dagger Y_u Y_u^\dagger) \\
& + 6\lambda_7 \text{Tr}(Y_u Y_u^\dagger Y_u \epsilon_u^\dagger) - \frac{27}{2} \lambda_3 \text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger) + 30 \text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger) \\
& + 18 \text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) - 18 \text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) + 6 \text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger) \\
& - 6 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d \epsilon_u^\dagger \epsilon_u Y_d^\dagger) + 12 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger) + 42 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger Y_d Y_d^\dagger) \\
& - 42 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) + 42 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_u^\dagger Y_u Y_d^\dagger) + 18 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger) \\
& + 42 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger Y_d Y_d^\dagger) + 36 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) - 42 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger Y_u Y_d^\dagger) \\
& - 6 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_d^\dagger Y_d \epsilon_d^\dagger) - 6 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_u^\dagger Y_u \epsilon_d^\dagger) + 6 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u \epsilon_u^\dagger \epsilon_u Y_d^\dagger)
\end{aligned}$$

$$\begin{aligned}
& -24\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_u Y_d^\dagger \epsilon_d \epsilon_d^\dagger\right) - 42\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_u Y_d^\dagger Y_d Y_d^\dagger\right) + 42\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_u Y_u^\dagger \epsilon_u \epsilon_d^\dagger\right) \\
& - 42\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_u Y_u^\dagger Y_u Y_d^\dagger\right) + 6\text{Tr}\left(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger\right) + 6\text{Tr}\left(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_u^\dagger Y_u \epsilon_d^\dagger\right) \\
& + 6\text{Tr}\left(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger\right) + 6\text{Tr}\left(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger Y_d Y_d^\dagger\right) - 6\text{Tr}\left(\epsilon_d Y_d^\dagger Y_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger\right) \\
& + 6\text{Tr}\left(\epsilon_d Y_d^\dagger Y_d \epsilon_u^\dagger Y_u Y_d^\dagger\right) + 12\text{Tr}\left(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger Y_d \epsilon_d^\dagger\right) - 6\text{Tr}\left(\epsilon_d Y_d^\dagger Y_d Y_u^\dagger Y_u \epsilon_d^\dagger\right) \\
& + 6\text{Tr}\left(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger Y_d \epsilon_d^\dagger\right) - 6\text{Tr}\left(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_u^\dagger Y_u \epsilon_d^\dagger\right) - 6\text{Tr}\left(\epsilon_d Y_u^\dagger Y_u \epsilon_d^\dagger Y_d Y_d^\dagger\right) \\
& - 6\text{Tr}\left(\epsilon_d Y_u^\dagger Y_u \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger\right) + 6\text{Tr}\left(\epsilon_d Y_u^\dagger Y_u \epsilon_u^\dagger Y_u Y_d^\dagger\right) - 6\text{Tr}\left(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger Y_d \epsilon_d^\dagger\right) \\
& + 10\text{Tr}\left(\epsilon_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger\right) + 2\text{Tr}\left(\epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger\right) + 4\text{Tr}\left(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger\right) \\
& + 14\text{Tr}\left(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger Y_e Y_e^\dagger\right) + 2\text{Tr}\left(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger\right) + 2\text{Tr}\left(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger\right) \\
& + 2\text{Tr}\left(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger Y_e Y_e^\dagger\right) + 4\text{Tr}\left(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger Y_e \epsilon_e^\dagger\right) - 42\text{Tr}\left(\epsilon_u \epsilon_d^\dagger Y_d Y_d^\dagger Y_d Y_u^\dagger\right) \\
& - 24\text{Tr}\left(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger \epsilon_u \epsilon_u^\dagger\right) - 42\text{Tr}\left(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger Y_u Y_u^\dagger\right) - 18\text{Tr}\left(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger\right) \\
& + 30\text{Tr}\left(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger\right) + 6\text{Tr}\left(\epsilon_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger\right) + 42\text{Tr}\left(\epsilon_u \epsilon_u^\dagger Y_u Y_d^\dagger Y_d Y_u^\dagger\right) \\
& + 12\text{Tr}\left(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger\right) + 42\text{Tr}\left(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger Y_u Y_u^\dagger\right) + 6\text{Tr}\left(\epsilon_u Y_d^\dagger Y_d \epsilon_d^\dagger Y_d Y_u^\dagger\right) \\
& - 6\text{Tr}\left(\epsilon_u Y_d^\dagger Y_d \epsilon_u^\dagger Y_u Y_u^\dagger\right) - 6\text{Tr}\left(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger Y_u \epsilon_u^\dagger\right) + 6\text{Tr}\left(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_d^\dagger Y_d \epsilon_u^\dagger\right) \\
& + 6\text{Tr}\left(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger\right) + 6\text{Tr}\left(\epsilon_u Y_u^\dagger Y_u \epsilon_d^\dagger Y_d Y_u^\dagger\right) + 6\text{Tr}\left(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger\right) \\
& + 6\text{Tr}\left(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger Y_u Y_u^\dagger\right) - 6\text{Tr}\left(\epsilon_u Y_u^\dagger Y_u Y_d^\dagger Y_d \epsilon_u^\dagger\right) + 12\text{Tr}\left(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger Y_u \epsilon_u^\dagger\right) \\
& + 18\text{Tr}\left(Y_d Y_d^\dagger Y_d Y_u^\dagger Y_u Y_d^\dagger\right) + 18\text{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger Y_d Y_d^\dagger\right) + 36\text{Tr}\left(Y_d Y_u^\dagger Y_u Y_u^\dagger Y_u Y_d^\dagger\right)
\end{aligned} \tag{21}$$

$$\begin{aligned}
\beta_{\lambda_2}^{(1)} &= +\frac{27}{200}g_1^4 + \frac{9}{20}g_1^2g_2^2 + \frac{9}{8}g_2^4 - \frac{9}{5}g_1^2\lambda_2 - 9g_2^2\lambda_2 + 24\lambda_2^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + \lambda_5^2 + 12\lambda_7^2 \\
& + 12\lambda_2\text{Tr}\left(\epsilon_d\epsilon_d^\dagger\right) + 4\lambda_2\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) + 6\lambda_7\text{Tr}\left(\epsilon_u Y_u^\dagger\right) + 6\lambda_7\text{Tr}\left(Y_d\epsilon_d^\dagger\right) + 2\lambda_7\text{Tr}\left(Y_e\epsilon_e^\dagger\right) \\
& + 12\lambda_2\text{Tr}\left(Y_u Y_u^\dagger\right) - 6\text{Tr}\left(\epsilon_d\epsilon_d^\dagger\epsilon_d\epsilon_d^\dagger\right) - 2\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\epsilon_e\epsilon_e^\dagger\right) - 6\text{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger\right)
\end{aligned} \tag{22}$$

$$\begin{aligned}
\beta_{\lambda_2}^{(2)} &= -\frac{3537}{2000}g_1^6 - \frac{1719}{400}g_1^4g_2^2 - \frac{303}{80}g_1^2g_2^4 + \frac{291}{16}g_2^6 + \frac{1953}{200}g_1^4\lambda_2 + \frac{117}{20}g_1^2g_2^2\lambda_2 - \frac{51}{8}g_2^4\lambda_2 + \frac{108}{5}g_1^2\lambda_2^2 \\
& + 108g_2^2\lambda_2^2 - 312\lambda_2^3 + \frac{9}{10}g_1^4\lambda_3 + \frac{15}{2}g_2^4\lambda_3 + \frac{12}{5}g_1^2\lambda_3^2 + 12g_2^2\lambda_3^2 - 20\lambda_2\lambda_3^2 - 8\lambda_3^3 + \frac{9}{20}g_1^4\lambda_4 \\
& + \frac{3}{2}g_1^2g_2^2\lambda_4 + \frac{15}{4}g_2^4\lambda_4 + \frac{12}{5}g_1^2\lambda_3\lambda_4 + 12g_2^2\lambda_3\lambda_4 - 20\lambda_2\lambda_3\lambda_4 - 12\lambda_3^2\lambda_4 + \frac{6}{5}g_1^2\lambda_4^2 \\
& + 3g_2^2\lambda_4^2 - 12\lambda_2\lambda_4^2 - 16\lambda_3\lambda_4^2 - 6\lambda_4^3 - \frac{3}{5}g_1^2\lambda_5^2 - 14\lambda_2\lambda_5^2 - 20\lambda_3\lambda_5^2 - 22\lambda_4\lambda_5^2 + 6\lambda_2\lambda_6^2 \\
& - 18\lambda_3\lambda_6^2 - 14\lambda_4\lambda_6^2 - 10\lambda_5\lambda_6^2 - 36\lambda_3\lambda_6\lambda_7 - 28\lambda_4\lambda_6\lambda_7 - 20\lambda_5\lambda_6\lambda_7 + \frac{54}{5}g_1^2\lambda_7^2 + 54g_2^2\lambda_7^2 \\
& - 318\lambda_2\lambda_7^2 - 66\lambda_3\lambda_7^2 - 70\lambda_4\lambda_7^2 - 74\lambda_5\lambda_7^2
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{20} \left(-5 \left(144\lambda_7^2 - 320g_3^2\lambda_2 + 576\lambda_2^2 - 90g_2^2\lambda_2 + 9g_2^4 \right) + 9g_1^4 + g_1^2 \left(50\lambda_2 + 54g_2^2 \right) \right) \text{Tr} \left(\epsilon_d \epsilon_d^\dagger \right) \\
& - 6 \left(12\lambda_2 + 2\lambda_4 + 3\lambda_3 + \lambda_5 \right) \lambda_7 \text{Tr} \left(\epsilon_d Y_d^\dagger \right) - \frac{9}{4} g_1^4 \text{Tr} \left(\epsilon_e \epsilon_e^\dagger \right) + \frac{33}{10} g_1^2 g_2^2 \text{Tr} \left(\epsilon_e \epsilon_e^\dagger \right) \\
& - \frac{3}{4} g_2^4 \text{Tr} \left(\epsilon_e \epsilon_e^\dagger \right) + \frac{15}{2} g_1^2 \lambda_2 \text{Tr} \left(\epsilon_e \epsilon_e^\dagger \right) + \frac{15}{2} g_2^2 \lambda_2 \text{Tr} \left(\epsilon_e \epsilon_e^\dagger \right) - 48\lambda_2^2 \text{Tr} \left(\epsilon_e \epsilon_e^\dagger \right) \\
& - 12\lambda_7^2 \text{Tr} \left(\epsilon_e \epsilon_e^\dagger \right) - 24\lambda_2 \lambda_7 \text{Tr} \left(\epsilon_e Y_e^\dagger \right) - 6\lambda_3 \lambda_7 \text{Tr} \left(\epsilon_e Y_e^\dagger \right) - 4\lambda_4 \lambda_7 \text{Tr} \left(\epsilon_e Y_e^\dagger \right) \\
& - 2\lambda_5 \lambda_7 \text{Tr} \left(\epsilon_e Y_e^\dagger \right) - 12\lambda_3^2 \text{Tr} \left(\epsilon_u \epsilon_u^\dagger \right) - 12\lambda_3 \lambda_4 \text{Tr} \left(\epsilon_u \epsilon_u^\dagger \right) - 6\lambda_4^2 \text{Tr} \left(\epsilon_u \epsilon_u^\dagger \right) \\
& - 6\lambda_5^2 \text{Tr} \left(\epsilon_u \epsilon_u^\dagger \right) - 36\lambda_7^2 \text{Tr} \left(\epsilon_u \epsilon_u^\dagger \right) - 72\lambda_2 \lambda_7 \text{Tr} \left(\epsilon_u Y_u^\dagger \right) - 18\lambda_3 \lambda_7 \text{Tr} \left(\epsilon_u Y_u^\dagger \right) \\
& - 12\lambda_4 \lambda_7 \text{Tr} \left(\epsilon_u Y_u^\dagger \right) - 6\lambda_5 \lambda_7 \text{Tr} \left(\epsilon_u Y_u^\dagger \right) - 72\lambda_2 \lambda_7 \text{Tr} \left(Y_d \epsilon_d^\dagger \right) - 18\lambda_3 \lambda_7 \text{Tr} \left(Y_d \epsilon_d^\dagger \right) \\
& - 12\lambda_4 \lambda_7 \text{Tr} \left(Y_d \epsilon_d^\dagger \right) - 6\lambda_5 \lambda_7 \text{Tr} \left(Y_d \epsilon_d^\dagger \right) - 12\lambda_3^2 \text{Tr} \left(Y_d Y_d^\dagger \right) - 12\lambda_3 \lambda_4 \text{Tr} \left(Y_d Y_d^\dagger \right) \\
& - 6\lambda_4^2 \text{Tr} \left(Y_d Y_d^\dagger \right) - 6\lambda_5^2 \text{Tr} \left(Y_d Y_d^\dagger \right) - 36\lambda_7^2 \text{Tr} \left(Y_d Y_d^\dagger \right) - 24\lambda_2 \lambda_7 \text{Tr} \left(Y_e \epsilon_e^\dagger \right) \\
& - 6\lambda_3 \lambda_7 \text{Tr} \left(Y_e \epsilon_e^\dagger \right) - 4\lambda_4 \lambda_7 \text{Tr} \left(Y_e \epsilon_e^\dagger \right) - 2\lambda_5 \lambda_7 \text{Tr} \left(Y_e \epsilon_e^\dagger \right) - 4\lambda_3^2 \text{Tr} \left(Y_e Y_e^\dagger \right) \\
& - 4\lambda_3 \lambda_4 \text{Tr} \left(Y_e Y_e^\dagger \right) - 2\lambda_4^2 \text{Tr} \left(Y_e Y_e^\dagger \right) - 2\lambda_5^2 \text{Tr} \left(Y_e Y_e^\dagger \right) - 12\lambda_7^2 \text{Tr} \left(Y_e Y_e^\dagger \right) \\
& - 72\lambda_2 \lambda_7 \text{Tr} \left(Y_u \epsilon_u^\dagger \right) - 18\lambda_3 \lambda_7 \text{Tr} \left(Y_u \epsilon_u^\dagger \right) - 12\lambda_4 \lambda_7 \text{Tr} \left(Y_u \epsilon_u^\dagger \right) - 6\lambda_5 \lambda_7 \text{Tr} \left(Y_u \epsilon_u^\dagger \right) \\
& - \frac{171}{100} g_1^4 \text{Tr} \left(Y_u Y_u^\dagger \right) + \frac{63}{10} g_1^2 g_2^2 \text{Tr} \left(Y_u Y_u^\dagger \right) - \frac{9}{4} g_2^4 \text{Tr} \left(Y_u Y_u^\dagger \right) + \frac{17}{2} g_1^2 \lambda_2 \text{Tr} \left(Y_u Y_u^\dagger \right) \\
& + \frac{45}{2} g_2^2 \lambda_2 \text{Tr} \left(Y_u Y_u^\dagger \right) + 80g_3^2 \lambda_2 \text{Tr} \left(Y_u Y_u^\dagger \right) - 144\lambda_2^2 \text{Tr} \left(Y_u Y_u^\dagger \right) - 36\lambda_7^2 \text{Tr} \left(Y_u Y_u^\dagger \right) \\
& + \frac{4}{5} g_1^2 \text{Tr} \left(\epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger \right) - 32g_3^2 \text{Tr} \left(\epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger \right) - 3\lambda_2 \text{Tr} \left(\epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger \right) + 6\lambda_7 \text{Tr} \left(\epsilon_d \epsilon_d^\dagger \epsilon_d Y_d^\dagger \right) \\
& + 12\lambda_7 \text{Tr} \left(\epsilon_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger \right) - 18\lambda_2 \text{Tr} \left(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger \right) - 9\lambda_2 \text{Tr} \left(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger \right) \\
& - 12\lambda_7 \text{Tr} \left(\epsilon_d \epsilon_u^\dagger Y_u \epsilon_d^\dagger \right) + 12\lambda_2 \text{Tr} \left(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger \right) - 12\lambda_3 \text{Tr} \left(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger \right) \\
& + 6\lambda_7 \text{Tr} \left(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger \right) + 6\lambda_5 \text{Tr} \left(\epsilon_d Y_d^\dagger \epsilon_d Y_d^\dagger \right) - 9\lambda_2 \text{Tr} \left(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger \right) \\
& - 12\lambda_7 \text{Tr} \left(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger \right) - 42\lambda_2 \text{Tr} \left(\epsilon_d Y_u^\dagger Y_u \epsilon_d^\dagger \right) - 12\lambda_7 \text{Tr} \left(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger \right) \\
& - \frac{12}{5} g_1^2 \text{Tr} \left(\epsilon_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger \right) - \lambda_2 \text{Tr} \left(\epsilon_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger \right) + 2\lambda_7 \text{Tr} \left(\epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger \right) + 4\lambda_7 \text{Tr} \left(\epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger \right) \\
& - 6\lambda_2 \text{Tr} \left(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger \right) + 2\lambda_7 \text{Tr} \left(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger \right) + 2\lambda_5 \text{Tr} \left(\epsilon_e Y_e^\dagger \epsilon_e Y_e^\dagger \right) \\
& - 3\lambda_2 \text{Tr} \left(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger \right) + 12\lambda_2 \text{Tr} \left(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger \right) - 12\lambda_3 \text{Tr} \left(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger \right) \\
& - 18\lambda_2 \text{Tr} \left(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger \right) + 6\lambda_5 \text{Tr} \left(\epsilon_u Y_u^\dagger \epsilon_u Y_u^\dagger \right) - 9\lambda_2 \text{Tr} \left(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger \right) + 12\lambda_7 \text{Tr} \left(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger \right) \\
& + 6\lambda_5 \text{Tr} \left(Y_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger \right) - 12\lambda_7 \text{Tr} \left(Y_d Y_u^\dagger Y_u \epsilon_d^\dagger \right) - 9\lambda_2 \text{Tr} \left(Y_d Y_u^\dagger Y_u Y_d^\dagger \right) + 2\lambda_5 \text{Tr} \left(Y_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger \right)
\end{aligned}$$

$$\begin{aligned}
& + 6\lambda_5 \text{Tr}\left(Y_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger\right) + 6\lambda_7 \text{Tr}\left(Y_u \epsilon_u^\dagger Y_u Y_u^\dagger\right) + 6\lambda_7 \text{Tr}\left(Y_u Y_u^\dagger Y_u \epsilon_u^\dagger\right) - \frac{8}{5} g_1^2 \text{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger\right) \\
& - 32g_3^2 \text{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger\right) - 3\lambda_2 \text{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger\right) + 30 \text{Tr}\left(\epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger\right) + 12 \text{Tr}\left(\epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger\right) \\
& + 6 \text{Tr}\left(\epsilon_d \epsilon_d^\dagger \epsilon_d Y_u^\dagger Y_u \epsilon_u^\dagger\right) + 6 \text{Tr}\left(\epsilon_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger \epsilon_d Y_d^\dagger\right) - 12 \text{Tr}\left(\epsilon_d \epsilon_d^\dagger Y_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger\right) \\
& + 12 \text{Tr}\left(\epsilon_d \epsilon_d^\dagger Y_d Y_u^\dagger Y_u Y_d^\dagger\right) + 6 \text{Tr}\left(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger\right) - 12 \text{Tr}\left(\epsilon_d \epsilon_u^\dagger Y_u Y_u^\dagger \epsilon_d \epsilon_d^\dagger\right) \\
& + 12 \text{Tr}\left(\epsilon_d \epsilon_u^\dagger Y_u Y_u^\dagger \epsilon_u \epsilon_d^\dagger\right) - 12 \text{Tr}\left(\epsilon_d \epsilon_u^\dagger Y_u Y_u^\dagger Y_u Y_d^\dagger\right) + 6 \text{Tr}\left(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger\right) \\
& + 6 \text{Tr}\left(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger\right) - 12 \text{Tr}\left(\epsilon_d Y_u^\dagger Y_u \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger\right) - 6 \text{Tr}\left(\epsilon_d Y_u^\dagger Y_u Y_u^\dagger Y_u \epsilon_d^\dagger\right) \\
& + 10 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger\right) + 4 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger\right) + 2 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger\right) + 2 \text{Tr}\left(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger\right) \\
& + 2 \text{Tr}\left(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger\right) - 12 \text{Tr}\left(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger Y_u Y_u^\dagger\right) + 12 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger Y_u Y_u^\dagger\right) \\
& + 12 \text{Tr}\left(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger Y_u Y_u^\dagger\right) + 6 \text{Tr}\left(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger Y_u \epsilon_u^\dagger\right) + 6 \text{Tr}\left(Y_d Y_u^\dagger Y_u Y_u^\dagger Y_u Y_d^\dagger\right) + 30 \text{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger Y_u Y_u^\dagger\right) \quad (23)
\end{aligned}$$

3.3 Yukawa Couplings

$$\begin{aligned}
\beta_{\epsilon_u}^{(1)} &= +\frac{1}{2} \left(\epsilon_u \epsilon_d^\dagger \epsilon_d + 3\epsilon_u \epsilon_u^\dagger \epsilon_u - 3\epsilon_u Y_d^\dagger Y_d + \epsilon_u Y_u^\dagger Y_u - 4Y_u Y_d^\dagger \epsilon_d + 2Y_u Y_u^\dagger \epsilon_u \right. \\
&\quad \left. + 6Y_u \text{Tr}\left(\epsilon_d Y_d^\dagger\right) + 2Y_u \text{Tr}\left(\epsilon_e Y_e^\dagger\right) + 6Y_u \text{Tr}\left(\epsilon_u Y_u^\dagger\right) \right) \\
&\quad + \epsilon_u \left(3 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) + 3 \text{Tr}\left(Y_d Y_d^\dagger\right) - 8g_3^2 - \frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 + \text{Tr}\left(Y_e Y_e^\dagger\right) \right) \quad (24) \\
\beta_{\epsilon_u}^{(2)} &= -\frac{41}{240}g_1^2 \epsilon_u \epsilon_d^\dagger \epsilon_d + \frac{33}{16}g_2^2 \epsilon_u \epsilon_d^\dagger \epsilon_d + \frac{16}{3}g_3^2 \epsilon_u \epsilon_d^\dagger \epsilon_d - 2\lambda_3 \epsilon_u \epsilon_d^\dagger \epsilon_d \\
&\quad + 2\lambda_4 \epsilon_u \epsilon_d^\dagger \epsilon_d + \frac{223}{80}g_1^2 \epsilon_u \epsilon_u^\dagger \epsilon_u + \frac{135}{16}g_2^2 \epsilon_u \epsilon_u^\dagger \epsilon_u + 16g_3^2 \epsilon_u \epsilon_u^\dagger \epsilon_u \\
&\quad - 12\lambda_1 \epsilon_u \epsilon_u^\dagger \epsilon_u - 6\lambda_6 \epsilon_u \epsilon_u^\dagger Y_u - \frac{43}{80}g_1^2 \epsilon_u Y_d^\dagger Y_d + \frac{9}{16}g_2^2 \epsilon_u Y_d^\dagger Y_d \\
&\quad - 16g_3^2 \epsilon_u Y_d^\dagger Y_d - 6\lambda_6 \epsilon_u Y_u^\dagger \epsilon_u + \frac{523}{240}g_1^2 \epsilon_u Y_u^\dagger Y_u + \frac{33}{16}g_2^2 \epsilon_u Y_u^\dagger Y_u \\
&\quad + \frac{16}{3}g_3^2 \epsilon_u Y_u^\dagger Y_u - 2\lambda_3 \epsilon_u Y_u^\dagger Y_u - 4\lambda_4 \epsilon_u Y_u^\dagger Y_u - 6\lambda_6 Y_u \epsilon_u^\dagger \epsilon_u \\
&\quad - 6\lambda_5 Y_u \epsilon_u^\dagger Y_u - \frac{11}{30}g_1^2 Y_u Y_d^\dagger \epsilon_d - \frac{3}{2}g_2^2 Y_u Y_d^\dagger \epsilon_d - \frac{64}{3}g_3^2 Y_u Y_d^\dagger \epsilon_d \\
&\quad + 2\lambda_3 Y_u Y_d^\dagger \epsilon_d - 2\lambda_4 Y_u Y_d^\dagger \epsilon_d + \frac{73}{120}g_1^2 Y_u Y_u^\dagger \epsilon_u + \frac{51}{8}g_2^2 Y_u Y_u^\dagger \epsilon_u \\
&\quad + \frac{32}{3}g_3^2 Y_u Y_u^\dagger \epsilon_u - 4\lambda_3 Y_u Y_u^\dagger \epsilon_u - 2\lambda_4 Y_u Y_u^\dagger \epsilon_u - 6\lambda_7 Y_u Y_u^\dagger Y_u \\
&\quad - \frac{1}{4} \epsilon_u \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger \epsilon_d - \frac{1}{4} \epsilon_u \epsilon_d^\dagger \epsilon_d \epsilon_u^\dagger \epsilon_u + \epsilon_u \epsilon_d^\dagger \epsilon_d Y_d^\dagger Y_d - \frac{1}{4} \epsilon_u \epsilon_d^\dagger Y_d Y_d^\dagger \epsilon_d
\end{aligned}$$

$$\begin{aligned}
& + \frac{3}{2}\epsilon_u\epsilon_d^\dagger\epsilon_u\epsilon_u^\dagger\epsilon_u - \epsilon_u\epsilon_u^\dagger\epsilon_u Y_d^\dagger Y_d - \frac{1}{4}\epsilon_u\epsilon_u^\dagger Y_u Y_u^\dagger \epsilon_u + \frac{7}{4}\epsilon_u Y_d^\dagger \epsilon_d \epsilon_d^\dagger Y_d \\
& - \frac{1}{4}\epsilon_u Y_d^\dagger Y_d \epsilon_u^\dagger \epsilon_u + \frac{11}{4}\epsilon_u Y_d^\dagger Y_d Y_d^\dagger Y_d - 2\epsilon_u Y_u^\dagger \epsilon_u \epsilon_d^\dagger Y_d + \frac{7}{4}\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger Y_u \\
& - \frac{1}{4}\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger \epsilon_u + \epsilon_u Y_u^\dagger Y_u Y_d^\dagger Y_d - \frac{1}{4}\epsilon_u Y_u^\dagger Y_u Y_u^\dagger Y_u + Y_u \epsilon_d^\dagger \epsilon_d Y_d^\dagger \epsilon_d \\
& - \frac{1}{4}Y_u \epsilon_d^\dagger \epsilon_d Y_u^\dagger \epsilon_u - Y_u \epsilon_u^\dagger \epsilon_u Y_d^\dagger \epsilon_d + \frac{7}{4}Y_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger \epsilon_u \\
& + 2Y_u Y_d^\dagger \epsilon_d \epsilon_d^\dagger \epsilon_d + 3Y_u Y_d^\dagger Y_d Y_d^\dagger \epsilon_d - \frac{1}{4}Y_u Y_d^\dagger Y_d Y_u^\dagger \epsilon_u - 2Y_u Y_u^\dagger \epsilon_u \epsilon_d^\dagger \epsilon_d \\
& + 2Y_u Y_u^\dagger \epsilon_u Y_u^\dagger Y_u + Y_u Y_u^\dagger Y_u Y_d^\dagger \epsilon_d - \frac{1}{4}Y_u Y_u^\dagger Y_u Y_u^\dagger \epsilon_u - \frac{9}{4}\epsilon_u \epsilon_d^\dagger \epsilon_d \text{Tr}(\epsilon_d \epsilon_d^\dagger) \\
& - \frac{9}{4}\epsilon_u Y_u^\dagger Y_u \text{Tr}(\epsilon_d \epsilon_d^\dagger) - \frac{9}{2}Y_u Y_u^\dagger \epsilon_u \text{Tr}(\epsilon_d \epsilon_d^\dagger) + \frac{15}{4}\epsilon_u \epsilon_d^\dagger Y_d \text{Tr}(\epsilon_d Y_d^\dagger) \\
& - \frac{9}{4}\epsilon_u \epsilon_u^\dagger Y_u \text{Tr}(\epsilon_d Y_d^\dagger) + 6Y_u \epsilon_d^\dagger \epsilon_d \text{Tr}(\epsilon_d Y_d^\dagger) - \frac{9}{2}Y_u \epsilon_u^\dagger \epsilon_u \text{Tr}(\epsilon_d Y_d^\dagger) \\
& - \frac{3}{4}\epsilon_u \epsilon_d^\dagger \epsilon_d \text{Tr}(\epsilon_e \epsilon_e^\dagger) - \frac{3}{4}\epsilon_u Y_u^\dagger Y_u \text{Tr}(\epsilon_e \epsilon_e^\dagger) - \frac{3}{2}Y_u Y_u^\dagger \epsilon_u \text{Tr}(\epsilon_e \epsilon_e^\dagger) \\
& + \frac{5}{4}\epsilon_u \epsilon_d^\dagger Y_d \text{Tr}(\epsilon_e Y_e^\dagger) - \frac{3}{4}\epsilon_u \epsilon_u^\dagger Y_u \text{Tr}(\epsilon_e Y_e^\dagger) + 2Y_u \epsilon_d^\dagger \epsilon_d \text{Tr}(\epsilon_e Y_e^\dagger) \\
& - \frac{3}{2}Y_u \epsilon_u^\dagger \epsilon_u \text{Tr}(\epsilon_e Y_e^\dagger) - \frac{27}{4}\epsilon_u \epsilon_u^\dagger \epsilon_u \text{Tr}(\epsilon_u \epsilon_u^\dagger) + \frac{15}{4}\epsilon_u Y_d^\dagger Y_d \text{Tr}(\epsilon_u \epsilon_u^\dagger) \\
& + 6Y_u Y_d^\dagger \epsilon_d \text{Tr}(\epsilon_u \epsilon_u^\dagger) + \frac{15}{4}\epsilon_u \epsilon_d^\dagger Y_d \text{Tr}(\epsilon_u Y_u^\dagger) - \frac{9}{4}\epsilon_u \epsilon_u^\dagger Y_u \text{Tr}(\epsilon_u Y_u^\dagger) \\
& + 6Y_u \epsilon_d^\dagger \epsilon_d \text{Tr}(\epsilon_u Y_u^\dagger) - \frac{9}{2}Y_u \epsilon_u^\dagger \epsilon_u \text{Tr}(\epsilon_u Y_u^\dagger) - \frac{9}{4}\epsilon_u Y_d^\dagger \epsilon_d \text{Tr}(Y_d \epsilon_d^\dagger) \\
& - \frac{27}{4}\epsilon_u Y_u^\dagger \epsilon_u \text{Tr}(Y_d \epsilon_d^\dagger) - \frac{27}{4}\epsilon_u \epsilon_u^\dagger \epsilon_u \text{Tr}(Y_d Y_d^\dagger) + \frac{15}{4}\epsilon_u Y_d^\dagger Y_d \text{Tr}(Y_d Y_d^\dagger) \\
& + 6Y_u Y_d^\dagger \epsilon_d \text{Tr}(Y_d Y_d^\dagger) - \frac{3}{4}\epsilon_u Y_d^\dagger \epsilon_d \text{Tr}(Y_e \epsilon_e^\dagger) - \frac{9}{4}\epsilon_u Y_u^\dagger \epsilon_u \text{Tr}(Y_e \epsilon_e^\dagger) \\
& - \frac{9}{4}\epsilon_u \epsilon_u^\dagger \epsilon_u \text{Tr}(Y_e Y_e^\dagger) + \frac{5}{4}\epsilon_u Y_d^\dagger Y_d \text{Tr}(Y_e Y_e^\dagger) + 2Y_u Y_d^\dagger \epsilon_d \text{Tr}(Y_e Y_e^\dagger) \\
& - \frac{9}{4}\epsilon_u Y_d^\dagger \epsilon_d \text{Tr}(Y_u \epsilon_u^\dagger) - \frac{27}{4}\epsilon_u Y_u^\dagger \epsilon_u \text{Tr}(Y_u \epsilon_u^\dagger) - \frac{9}{4}\epsilon_u \epsilon_d^\dagger \epsilon_d \text{Tr}(Y_u Y_u^\dagger) \\
& - \frac{9}{4}\epsilon_u Y_u^\dagger Y_u \text{Tr}(Y_u Y_u^\dagger) - \frac{9}{2}Y_u Y_u^\dagger \epsilon_u \text{Tr}(Y_u Y_u^\dagger) \\
& + \frac{1}{8}Y_u \left(24\lambda_1\lambda_6 + 12\lambda_3\lambda_6 + 12\lambda_4\lambda_6 + 12\lambda_5\lambda_6 + 24\lambda_2\lambda_7 + 12\lambda_3\lambda_7 + 12\lambda_4\lambda_7 + 12\lambda_5\lambda_7 \right. \\
& + 5(32g_3^2 + 9g_2^2 + g_1^2) \text{Tr}(\epsilon_d Y_d^\dagger) + 15(g_1^2 + g_2^2) \text{Tr}(\epsilon_e Y_e^\dagger) + 17g_1^2 \text{Tr}(\epsilon_u Y_u^\dagger) + 45g_2^2 \text{Tr}(\epsilon_u Y_u^\dagger) \\
& + 160g_3^2 \text{Tr}(\epsilon_u Y_u^\dagger) + 6\text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_d^\dagger) - 54\text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger) - 54\text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger) \\
& + 6\text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) + 6\text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger) - 18\text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger) - 18\text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger) \\
& \left. - 18\text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger) + 6\text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger) - 36\text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger) - 54\text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger) \right)
\end{aligned}$$

$$\begin{aligned}
& + \epsilon_u \left(\frac{1267}{600} g_1^4 - \frac{9}{20} g_1^2 g_2^2 - \frac{21}{4} g_2^4 + \frac{19}{15} g_1^2 g_3^2 + 9 g_2^2 g_3^2 - 108 g_3^4 + 6 \lambda_1^2 + \lambda_3^2 + \lambda_3 \lambda_4 + \lambda_4^2 + \frac{3}{2} \lambda_5^2 + \frac{9}{2} \lambda_6^2 \right. \\
& + \frac{3}{2} \lambda_7^2 + \frac{1}{8} (160 g_3^2 + 17 g_1^2 + 45 g_2^2) \text{Tr}(\epsilon_u \epsilon_u^\dagger) + \frac{5}{8} (32 g_3^2 + 9 g_2^2 + g_1^2) \text{Tr}(Y_d Y_d^\dagger) + \frac{15}{8} g_1^2 \text{Tr}(Y_e Y_e^\dagger) \\
& + \frac{15}{8} g_2^2 \text{Tr}(Y_e Y_e^\dagger) - \frac{9}{2} \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger) - \frac{9}{4} \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) + 3 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_u^\dagger) \\
& - \frac{9}{4} \text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger) - \frac{3}{2} \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger) - \frac{3}{4} \text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger) + 3 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_d Y_u^\dagger) \\
& - \frac{27}{4} \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger) - \frac{9}{2} \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger) + \frac{3}{2} \text{Tr}(\epsilon_u Y_d^\dagger Y_d \epsilon_u^\dagger) - \frac{9}{4} \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) \\
& \left. - \frac{27}{4} \text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) - \frac{9}{4} \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) - \frac{9}{4} \text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) \right) \quad (25)
\end{aligned}$$

$$\begin{aligned}
\beta_{Y_u}^{(1)} & = + \frac{1}{2} (2 \epsilon_u \epsilon_u^\dagger Y_u - 3 Y_u \epsilon_d^\dagger \epsilon_d + 3 Y_u Y_u^\dagger Y_u - 4 \epsilon_u \epsilon_d^\dagger Y_d + Y_u \epsilon_u^\dagger \epsilon_u + Y_u Y_d^\dagger Y_d) \\
& + \epsilon_u (3 \text{Tr}(Y_d \epsilon_d^\dagger) + 3 \text{Tr}(Y_u \epsilon_u^\dagger) + \text{Tr}(Y_e \epsilon_e^\dagger)) \\
& + Y_u (3 \text{Tr}(\epsilon_d \epsilon_d^\dagger) + 3 \text{Tr}(Y_u Y_u^\dagger) - 8 g_3^2 - \frac{17}{20} g_1^2 - \frac{9}{4} g_2^2 + \text{Tr}(\epsilon_e \epsilon_e^\dagger)) \quad (26)
\end{aligned}$$

$$\begin{aligned}
\beta_{Y_u}^{(2)} & = - \frac{11}{30} g_1^2 \epsilon_u \epsilon_d^\dagger Y_d - \frac{3}{2} g_2^2 \epsilon_u \epsilon_d^\dagger Y_d - \frac{64}{3} g_3^2 \epsilon_u \epsilon_d^\dagger Y_d + 2 \lambda_3 \epsilon_u \epsilon_d^\dagger Y_d \\
& - 2 \lambda_4 \epsilon_u \epsilon_d^\dagger Y_d - 6 \lambda_6 \epsilon_u \epsilon_u^\dagger \epsilon_u + \frac{73}{120} g_1^2 \epsilon_u \epsilon_u^\dagger Y_u + \frac{51}{8} g_2^2 \epsilon_u \epsilon_u^\dagger Y_u \\
& + \frac{32}{3} g_3^2 \epsilon_u \epsilon_u^\dagger Y_u - 4 \lambda_3 \epsilon_u \epsilon_u^\dagger Y_u - 2 \lambda_4 \epsilon_u \epsilon_u^\dagger Y_u - 6 \lambda_5 \epsilon_u Y_u^\dagger \epsilon_u \\
& - 6 \lambda_7 \epsilon_u Y_u^\dagger Y_u - \frac{43}{80} g_1^2 Y_u \epsilon_d^\dagger \epsilon_d + \frac{9}{16} g_2^2 Y_u \epsilon_d^\dagger \epsilon_d - 16 g_3^2 Y_u \epsilon_d^\dagger \epsilon_d \\
& + \frac{523}{240} g_1^2 Y_u \epsilon_u^\dagger \epsilon_u + \frac{33}{16} g_2^2 Y_u \epsilon_u^\dagger \epsilon_u + \frac{16}{3} g_3^2 Y_u \epsilon_u^\dagger \epsilon_u - 2 \lambda_3 Y_u \epsilon_u^\dagger \epsilon_u \\
& - 4 \lambda_4 Y_u \epsilon_u^\dagger \epsilon_u - 6 \lambda_7 Y_u \epsilon_u^\dagger Y_u - \frac{41}{240} g_1^2 Y_u Y_d^\dagger Y_d + \frac{33}{16} g_2^2 Y_u Y_d^\dagger Y_d \\
& + \frac{16}{3} g_3^2 Y_u Y_d^\dagger Y_d - 2 \lambda_3 Y_u Y_d^\dagger Y_d + 2 \lambda_4 Y_u Y_d^\dagger Y_d - 6 \lambda_7 Y_u Y_u^\dagger \epsilon_u \\
& + \frac{223}{80} g_1^2 Y_u Y_u^\dagger Y_u + \frac{135}{16} g_2^2 Y_u Y_u^\dagger Y_u + 16 g_3^2 Y_u Y_u^\dagger Y_u - 12 \lambda_2 Y_u Y_u^\dagger Y_u \\
& + 3 \epsilon_u \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger Y_d - \frac{1}{4} \epsilon_u \epsilon_d^\dagger \epsilon_d \epsilon_u^\dagger Y_u + 2 \epsilon_u \epsilon_d^\dagger Y_d Y_d^\dagger Y_d + \epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger Y_d \\
& - \frac{1}{4} \epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger Y_u + 2 \epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger \epsilon_u - 2 \epsilon_u \epsilon_u^\dagger Y_u Y_d^\dagger Y_d + \epsilon_u Y_d^\dagger Y_d \epsilon_d^\dagger Y_d \\
& - \frac{1}{4} \epsilon_u Y_d^\dagger Y_d \epsilon_u^\dagger Y_u - \epsilon_u Y_u^\dagger Y_u \epsilon_d^\dagger Y_d + \frac{7}{4} \epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger Y_u + \frac{11}{4} Y_u \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger \epsilon_d \\
& - \frac{1}{4} Y_u \epsilon_d^\dagger \epsilon_d Y_u^\dagger Y_u + \frac{7}{4} Y_u \epsilon_d^\dagger Y_d Y_d^\dagger \epsilon_d + Y_u \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger \epsilon_d - \frac{1}{4} Y_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger \epsilon_u \\
& - \frac{1}{4} Y_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger Y_u - 2 Y_u \epsilon_u^\dagger Y_u Y_d^\dagger \epsilon_d + \frac{7}{4} Y_u \epsilon_u^\dagger Y_u Y_u^\dagger \epsilon_u \\
& - \frac{1}{4} Y_u Y_d^\dagger \epsilon_d \epsilon_d^\dagger Y_d + Y_u Y_d^\dagger Y_d \epsilon_d^\dagger \epsilon_d - \frac{1}{4} Y_u Y_d^\dagger Y_d Y_d^\dagger Y_d - \frac{1}{4} Y_u Y_d^\dagger Y_d Y_u^\dagger Y_u
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{4}Y_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger Y_u - Y_u Y_u^\dagger Y_u \epsilon_d^\dagger \epsilon_d + \frac{3}{2}Y_u Y_u^\dagger Y_u Y_u^\dagger Y_u + 6\epsilon_u \epsilon_d^\dagger Y_d \text{Tr}(\epsilon_d \epsilon_d^\dagger) \\
& + \frac{15}{4}Y_u \epsilon_d^\dagger \epsilon_d \text{Tr}(\epsilon_d \epsilon_d^\dagger) - \frac{27}{4}Y_u Y_u^\dagger Y_u \text{Tr}(\epsilon_d \epsilon_d^\dagger) - \frac{9}{4}Y_u \epsilon_d^\dagger Y_d \text{Tr}(\epsilon_d Y_d^\dagger) \\
& - \frac{27}{4}Y_u \epsilon_u^\dagger Y_u \text{Tr}(\epsilon_d Y_d^\dagger) + 2\epsilon_u \epsilon_d^\dagger Y_d \text{Tr}(\epsilon_e \epsilon_e^\dagger) + \frac{5}{4}Y_u \epsilon_d^\dagger \epsilon_d \text{Tr}(\epsilon_e \epsilon_e^\dagger) \\
& - \frac{9}{4}Y_u Y_u^\dagger Y_u \text{Tr}(\epsilon_e \epsilon_e^\dagger) - \frac{3}{4}Y_u \epsilon_d^\dagger Y_d \text{Tr}(\epsilon_e Y_e^\dagger) - \frac{9}{4}Y_u \epsilon_u^\dagger Y_u \text{Tr}(\epsilon_e Y_e^\dagger) \\
& - \frac{9}{2}\epsilon_u \epsilon_u^\dagger Y_u \text{Tr}(\epsilon_u \epsilon_u^\dagger) - \frac{9}{4}Y_u \epsilon_u^\dagger \epsilon_u \text{Tr}(\epsilon_u \epsilon_u^\dagger) - \frac{9}{4}Y_u Y_d^\dagger Y_d \text{Tr}(\epsilon_u \epsilon_u^\dagger) \\
& - \frac{9}{4}Y_u \epsilon_d^\dagger Y_d \text{Tr}(\epsilon_u Y_u^\dagger) - \frac{27}{4}Y_u \epsilon_u^\dagger Y_u \text{Tr}(\epsilon_u Y_u^\dagger) + 6\epsilon_u Y_d^\dagger Y_d \text{Tr}(Y_d \epsilon_d^\dagger) \\
& - \frac{9}{2}\epsilon_u Y_u^\dagger Y_u \text{Tr}(Y_d \epsilon_d^\dagger) + \frac{15}{4}Y_u Y_d^\dagger \epsilon_d \text{Tr}(Y_d \epsilon_d^\dagger) - \frac{9}{4}Y_u Y_u^\dagger \epsilon_u \text{Tr}(Y_d \epsilon_d^\dagger) \\
& - \frac{9}{2}\epsilon_u \epsilon_u^\dagger Y_u \text{Tr}(Y_d Y_d^\dagger) - \frac{9}{4}Y_u \epsilon_u^\dagger \epsilon_u \text{Tr}(Y_d Y_d^\dagger) - \frac{9}{4}Y_u Y_d^\dagger Y_d \text{Tr}(Y_d Y_d^\dagger) \\
& + 2\epsilon_u Y_d^\dagger Y_d \text{Tr}(Y_e \epsilon_e^\dagger) - \frac{3}{2}\epsilon_u Y_u^\dagger Y_u \text{Tr}(Y_e \epsilon_e^\dagger) + \frac{5}{4}Y_u Y_d^\dagger \epsilon_d \text{Tr}(Y_e \epsilon_e^\dagger) \\
& - \frac{3}{4}Y_u Y_u^\dagger \epsilon_u \text{Tr}(Y_e \epsilon_e^\dagger) - \frac{3}{2}\epsilon_u \epsilon_u^\dagger Y_u \text{Tr}(Y_e Y_e^\dagger) - \frac{3}{4}Y_u \epsilon_u^\dagger \epsilon_u \text{Tr}(Y_e Y_e^\dagger) \\
& - \frac{3}{4}Y_u Y_d^\dagger Y_d \text{Tr}(Y_e Y_e^\dagger) + 6\epsilon_u Y_d^\dagger Y_d \text{Tr}(Y_u \epsilon_u^\dagger) - \frac{9}{2}\epsilon_u Y_u^\dagger Y_u \text{Tr}(Y_u \epsilon_u^\dagger) \\
& + \frac{15}{4}Y_u Y_d^\dagger \epsilon_d \text{Tr}(Y_u \epsilon_u^\dagger) - \frac{9}{4}Y_u Y_u^\dagger \epsilon_u \text{Tr}(Y_u \epsilon_u^\dagger) + 6\epsilon_u \epsilon_d^\dagger Y_d \text{Tr}(Y_u Y_u^\dagger) \\
& + \frac{15}{4}Y_u \epsilon_d^\dagger \epsilon_d \text{Tr}(Y_u Y_u^\dagger) - \frac{27}{4}Y_u Y_u^\dagger Y_u \text{Tr}(Y_u Y_u^\dagger) \\
& + \frac{1}{8}\epsilon_u (24\lambda_1\lambda_6 + 12\lambda_3\lambda_6 + 12\lambda_4\lambda_6 + 12\lambda_5\lambda_6 + 24\lambda_2\lambda_7 + 12\lambda_3\lambda_7 + 12\lambda_4\lambda_7 + 12\lambda_5\lambda_7 \\
& + 5(32g_3^2 + 9g_2^2 + g_1^2)\text{Tr}(Y_d \epsilon_d^\dagger) + 15(g_1^2 + g_2^2)\text{Tr}(Y_e \epsilon_e^\dagger) + 17g_1^2\text{Tr}(Y_u \epsilon_u^\dagger) + 45g_2^2\text{Tr}(Y_u \epsilon_u^\dagger) \\
& + 160g_3^2\text{Tr}(Y_u \epsilon_u^\dagger) - 54\text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger) + 6\text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u \epsilon_d^\dagger) - 18\text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) \\
& + 6\text{Tr}(\epsilon_u \epsilon_u^\dagger Y_d \epsilon_u^\dagger) - 54\text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) - 54\text{Tr}(Y_d \epsilon_d^\dagger Y_d Y_d^\dagger) + 6\text{Tr}(Y_d \epsilon_u^\dagger Y_u Y_d^\dagger) \\
& + 6\text{Tr}(Y_d Y_u^\dagger Y_u \epsilon_d^\dagger) - 18\text{Tr}(Y_e \epsilon_e^\dagger Y_e Y_e^\dagger) - 36\text{Tr}(Y_u \epsilon_u^\dagger Y_u Y_u^\dagger) - 18\text{Tr}(Y_u Y_u^\dagger Y_u \epsilon_u^\dagger)) \\
& + Y_u \left(\frac{1267}{600}g_1^4 - \frac{9}{20}g_1^2 g_2^2 - \frac{21}{4}g_2^4 + \frac{19}{15}g_1^2 g_3^2 + 9g_2^2 g_3^2 - 108g_3^4 + 6\lambda_2^2 + \lambda_3^2 + \lambda_3\lambda_4 + \lambda_4^2 + \frac{3}{2}\lambda_5^2 + \frac{3}{2}\lambda_6^2 \right. \\
& + \frac{9}{2}\lambda_7^2 + \frac{5}{8}(32g_3^2 + 9g_2^2 + g_1^2)\text{Tr}(\epsilon_d \epsilon_d^\dagger) + \frac{15}{8}(g_1^2 + g_2^2)\text{Tr}(\epsilon_e \epsilon_e^\dagger) + \frac{17}{8}g_1^2\text{Tr}(Y_u Y_u^\dagger) \\
& + \frac{45}{8}g_2^2\text{Tr}(Y_u Y_u^\dagger) + 20g_3^2\text{Tr}(Y_u Y_u^\dagger) - \frac{27}{4}\text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger) - \frac{9}{2}\text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger) \\
& - \frac{9}{4}\text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) + 3\text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) - \frac{9}{4}\text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger) + \frac{3}{2}\text{Tr}(\epsilon_d Y_u^\dagger Y_u \epsilon_d^\dagger) \\
& \left. - \frac{9}{4}\text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger) - \frac{3}{2}\text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger) - \frac{3}{4}\text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger) + 3\text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) \right)
\end{aligned}$$

$$-\frac{9}{2}\text{Tr}\left(\epsilon_u\epsilon_u^\dagger Y_u Y_u^\dagger\right) - \frac{9}{4}\text{Tr}\left(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger\right) - \frac{9}{4}\text{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger\right) - \frac{27}{4}\text{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger\right) \quad (27)$$

$$\begin{aligned} \beta_{Y_d}^{(1)} = & +\frac{1}{4}\left(2\left(2\epsilon_d\epsilon_d^\dagger Y_d - 3Y_d\epsilon_u^\dagger\epsilon_u + 3Y_d Y_d^\dagger Y_d - 4\epsilon_d\epsilon_u^\dagger Y_u + Y_d\epsilon_d^\dagger\epsilon_d + Y_d Y_u^\dagger Y_u\right)\right. \\ & \left.- Y_d\left(-12\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) - 12\text{Tr}\left(Y_d Y_d^\dagger\right) + 32g_3^2 - 4\text{Tr}\left(Y_e Y_e^\dagger\right) + 9g_2^2 + g_1^2\right)\right) \\ & + \epsilon_d\left(3\text{Tr}\left(Y_d\epsilon_d^\dagger\right) + 3\text{Tr}\left(Y_u\epsilon_u^\dagger\right) + \text{Tr}\left(Y_e\epsilon_e^\dagger\right)\right) \end{aligned} \quad (28)$$

$$\begin{aligned} \beta_{Y_d}^{(2)} = & -6\lambda_7\epsilon_d\epsilon_d^\dagger\epsilon_d + \frac{157}{120}g_1^2\epsilon_d\epsilon_d^\dagger Y_d + \frac{51}{8}g_2^2\epsilon_d\epsilon_d^\dagger Y_d + \frac{32}{3}g_3^2\epsilon_d\epsilon_d^\dagger Y_d \\ & - 4\lambda_3\epsilon_d\epsilon_d^\dagger Y_d - 2\lambda_4\epsilon_d\epsilon_d^\dagger Y_d - \frac{23}{30}g_1^2\epsilon_d\epsilon_u^\dagger Y_u - \frac{3}{2}g_2^2\epsilon_d\epsilon_u^\dagger Y_u \\ & - \frac{64}{3}g_3^2\epsilon_d\epsilon_u^\dagger Y_u + 2\lambda_3\epsilon_d\epsilon_u^\dagger Y_u - 2\lambda_4\epsilon_d\epsilon_u^\dagger Y_u - 6\lambda_5\epsilon_d Y_d^\dagger \epsilon_d \\ & - 6\lambda_6\epsilon_d Y_d^\dagger Y_d + \frac{247}{240}g_1^2 Y_d\epsilon_d^\dagger\epsilon_d + \frac{33}{16}g_2^2 Y_d\epsilon_d^\dagger\epsilon_d + \frac{16}{3}g_3^2 Y_d\epsilon_d^\dagger\epsilon_d \\ & - 2\lambda_3 Y_d\epsilon_d^\dagger\epsilon_d - 4\lambda_4 Y_d\epsilon_d^\dagger\epsilon_d - 6\lambda_6 Y_d\epsilon_d^\dagger Y_d - \frac{79}{80}g_1^2 Y_d\epsilon_u^\dagger\epsilon_u \\ & + \frac{9}{16}g_2^2 Y_d\epsilon_u^\dagger\epsilon_u - 16g_3^2 Y_d\epsilon_u^\dagger\epsilon_u - 6\lambda_6 Y_d Y_d^\dagger \epsilon_d + \frac{187}{80}g_1^2 Y_d Y_d^\dagger Y_d \\ & + \frac{135}{16}g_2^2 Y_d Y_d^\dagger Y_d + 16g_3^2 Y_d Y_d^\dagger Y_d - 12\lambda_1 Y_d Y_d^\dagger Y_d - \frac{53}{240}g_1^2 Y_d Y_u^\dagger Y_u \\ & + \frac{33}{16}g_2^2 Y_d Y_u^\dagger Y_u + \frac{16}{3}g_3^2 Y_d Y_u^\dagger Y_u - 2\lambda_3 Y_d Y_u^\dagger Y_u + 2\lambda_4 Y_d Y_u^\dagger Y_u \\ & - \frac{1}{4}\epsilon_d\epsilon_d^\dagger\epsilon_d\epsilon_d^\dagger Y_d + \epsilon_d\epsilon_d^\dagger\epsilon_d\epsilon_u^\dagger Y_u + 2\epsilon_d\epsilon_d^\dagger Y_d\epsilon_d^\dagger\epsilon_d - 2\epsilon_d\epsilon_d^\dagger Y_d Y_u^\dagger Y_u \\ & - \frac{1}{4}\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger Y_d + 3\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_u^\dagger Y_u + 2\epsilon_d\epsilon_u^\dagger Y_u Y_u^\dagger Y_u + \frac{7}{4}\epsilon_d Y_d^\dagger Y_d\epsilon_d^\dagger Y_d \\ & - \epsilon_d Y_d^\dagger Y_d\epsilon_u^\dagger Y_u - \frac{1}{4}\epsilon_d Y_u^\dagger Y_u\epsilon_d^\dagger Y_d + \epsilon_d Y_u^\dagger Y_u\epsilon_u^\dagger Y_u - \frac{1}{4}Y_d\epsilon_d^\dagger\epsilon_d\epsilon_d^\dagger\epsilon_d \\ & + Y_d\epsilon_d^\dagger\epsilon_d\epsilon_u^\dagger\epsilon_u - \frac{1}{4}Y_d\epsilon_d^\dagger\epsilon_d Y_d^\dagger Y_d + \frac{7}{4}Y_d\epsilon_d^\dagger Y_d Y_d^\dagger \epsilon_d - 2Y_d\epsilon_d^\dagger Y_d Y_u^\dagger \epsilon_u \\ & + \frac{11}{4}Y_d\epsilon_u^\dagger\epsilon_u\epsilon_u^\dagger\epsilon_u - \frac{1}{4}Y_d\epsilon_u^\dagger\epsilon_u Y_d^\dagger Y_d + \frac{7}{4}Y_d\epsilon_u^\dagger Y_u Y_u^\dagger \epsilon_u - \frac{1}{4}Y_d Y_d^\dagger \epsilon_d\epsilon_d^\dagger Y_d \\ & - Y_d Y_d^\dagger Y_d\epsilon_u^\dagger\epsilon_u + \frac{3}{2}Y_d Y_d^\dagger Y_d Y_d^\dagger Y_d - \frac{1}{4}Y_d Y_u^\dagger \epsilon_u\epsilon_u^\dagger Y_u + Y_d Y_u^\dagger Y_u\epsilon_u^\dagger\epsilon_u \\ & - \frac{1}{4}Y_d Y_u^\dagger Y_u Y_d^\dagger Y_d - \frac{1}{4}Y_d Y_u^\dagger Y_u Y_u^\dagger Y_u - \frac{9}{2}\epsilon_d\epsilon_d^\dagger Y_d \text{Tr}\left(\epsilon_d\epsilon_d^\dagger\right) \\ & - \frac{9}{4}Y_d\epsilon_d^\dagger\epsilon_d \text{Tr}\left(\epsilon_d\epsilon_d^\dagger\right) - \frac{9}{4}Y_d Y_u^\dagger Y_u \text{Tr}\left(\epsilon_d\epsilon_d^\dagger\right) - \frac{27}{4}Y_d\epsilon_d^\dagger Y_d \text{Tr}\left(\epsilon_d Y_d^\dagger\right) \\ & - \frac{9}{4}Y_d\epsilon_u^\dagger Y_u \text{Tr}\left(\epsilon_d Y_d^\dagger\right) - \frac{3}{2}\epsilon_d\epsilon_d^\dagger Y_d \text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - \frac{3}{4}Y_d\epsilon_d^\dagger\epsilon_d \text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) \\ & - \frac{3}{4}Y_d Y_u^\dagger Y_u \text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - \frac{9}{4}Y_d\epsilon_d^\dagger Y_d \text{Tr}\left(\epsilon_e Y_e^\dagger\right) - \frac{3}{4}Y_d\epsilon_u^\dagger Y_u \text{Tr}\left(\epsilon_e Y_e^\dagger\right) \\ & + 6\epsilon_d\epsilon_u^\dagger Y_u \text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) + \frac{15}{4}Y_d\epsilon_u^\dagger\epsilon_u \text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) - \frac{27}{4}Y_d Y_d^\dagger Y_d \text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) \end{aligned}$$

$$\begin{aligned}
& -\frac{27}{4}Y_d\epsilon_d^\dagger Y_d\text{Tr}(\epsilon_u Y_u^\dagger) - \frac{9}{4}Y_d\epsilon_u^\dagger Y_u\text{Tr}(\epsilon_u Y_u^\dagger) - \frac{9}{2}\epsilon_d Y_d^\dagger Y_d\text{Tr}(Y_d\epsilon_d^\dagger) \\
& + 6\epsilon_d Y_u^\dagger Y_u\text{Tr}(Y_d\epsilon_d^\dagger) - \frac{9}{4}Y_d Y_d^\dagger \epsilon_d\text{Tr}(Y_d\epsilon_d^\dagger) + \frac{15}{4}Y_d Y_u^\dagger \epsilon_u\text{Tr}(Y_d\epsilon_d^\dagger) \\
& + 6\epsilon_d\epsilon_u^\dagger Y_u\text{Tr}(Y_d Y_d^\dagger) + \frac{15}{4}Y_d\epsilon_u^\dagger \epsilon_u\text{Tr}(Y_d Y_d^\dagger) - \frac{27}{4}Y_d Y_d^\dagger Y_d\text{Tr}(Y_d Y_d^\dagger) \\
& - \frac{3}{2}\epsilon_d Y_d^\dagger Y_d\text{Tr}(Y_e\epsilon_e^\dagger) + 2\epsilon_d Y_u^\dagger Y_u\text{Tr}(Y_e\epsilon_e^\dagger) - \frac{3}{4}Y_d Y_d^\dagger \epsilon_d\text{Tr}(Y_e\epsilon_e^\dagger) \\
& + \frac{5}{4}Y_d Y_u^\dagger \epsilon_u\text{Tr}(Y_e\epsilon_e^\dagger) + 2\epsilon_d\epsilon_u^\dagger Y_u\text{Tr}(Y_e Y_e^\dagger) + \frac{5}{4}Y_d\epsilon_u^\dagger \epsilon_u\text{Tr}(Y_e Y_e^\dagger) \\
& - \frac{9}{4}Y_d Y_d^\dagger Y_d\text{Tr}(Y_e Y_e^\dagger) - \frac{9}{2}\epsilon_d Y_d^\dagger Y_d\text{Tr}(Y_u\epsilon_u^\dagger) + 6\epsilon_d Y_u^\dagger Y_u\text{Tr}(Y_u\epsilon_u^\dagger) \\
& - \frac{9}{4}Y_d Y_d^\dagger \epsilon_d\text{Tr}(Y_u\epsilon_u^\dagger) + \frac{15}{4}Y_d Y_u^\dagger \epsilon_u\text{Tr}(Y_u\epsilon_u^\dagger) - \frac{9}{2}\epsilon_d\epsilon_u^\dagger Y_d\text{Tr}(Y_u Y_u^\dagger) \\
& - \frac{9}{4}Y_d\epsilon_d^\dagger \epsilon_d\text{Tr}(Y_u Y_u^\dagger) - \frac{9}{4}Y_d Y_u^\dagger Y_u\text{Tr}(Y_u Y_u^\dagger) \\
& + Y_d\left(-\frac{113}{600}g_1^4 - \frac{27}{20}g_1^2 g_2^2 - \frac{21}{4}g_2^4 + \frac{31}{15}g_1^2 g_3^2 + 9g_2^2 g_3^2 - 108g_3^4 + 6\lambda_1^2 + \lambda_3^2 + \lambda_3\lambda_4 + \lambda_4^2 + \frac{3}{2}\lambda_5^2 + \frac{9}{2}\lambda_6^2\right. \\
& \left. + \frac{3}{2}\lambda_7^2 + \frac{1}{8}(160g_3^2 + 17g_1^2 + 45g_2^2)\text{Tr}(\epsilon_u\epsilon_u^\dagger) + \frac{5}{8}(32g_3^2 + 9g_2^2 + g_1^2)\text{Tr}(Y_d Y_d^\dagger) + \frac{15}{8}g_1^2\text{Tr}(Y_e Y_e^\dagger)\right. \\
& \left. + \frac{15}{8}g_2^2\text{Tr}(Y_e Y_e^\dagger) - \frac{9}{2}\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d Y_d^\dagger) - \frac{9}{4}\text{Tr}(\epsilon_d\epsilon_u^\dagger \epsilon_u\epsilon_d^\dagger) + 3\text{Tr}(\epsilon_d\epsilon_u^\dagger Y_u Y_d^\dagger)\right. \\
& \left. - \frac{9}{4}\text{Tr}(\epsilon_d Y_d^\dagger Y_d\epsilon_d^\dagger) - \frac{3}{2}\text{Tr}(\epsilon_e\epsilon_e^\dagger Y_e Y_e^\dagger) - \frac{3}{4}\text{Tr}(\epsilon_e Y_e^\dagger Y_e\epsilon_e^\dagger) + 3\text{Tr}(\epsilon_u\epsilon_d^\dagger Y_d Y_u^\dagger)\right. \\
& \left. - \frac{27}{4}\text{Tr}(\epsilon_u\epsilon_u^\dagger \epsilon_u\epsilon_u^\dagger) - \frac{9}{2}\text{Tr}(\epsilon_u\epsilon_u^\dagger Y_u Y_u^\dagger) + \frac{3}{2}\text{Tr}(\epsilon_u Y_d^\dagger Y_d\epsilon_u^\dagger) - \frac{9}{4}\text{Tr}(\epsilon_u Y_u^\dagger Y_u\epsilon_u^\dagger)\right. \\
& \left. - \frac{27}{4}\text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) - \frac{9}{4}\text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) - \frac{9}{4}\text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger)\right) \\
& + \frac{1}{8}\epsilon_d(24\lambda_1\lambda_6 + 12\lambda_3\lambda_6 + 12\lambda_4\lambda_6 + 12\lambda_5\lambda_6 + 24\lambda_2\lambda_7 + 12\lambda_3\lambda_7 + 12\lambda_4\lambda_7 + 12\lambda_5\lambda_7 \\
& + 5(32g_3^2 + 9g_2^2 + g_1^2)\text{Tr}(Y_d\epsilon_d^\dagger) + 15(g_1^2 + g_2^2)\text{Tr}(Y_e\epsilon_e^\dagger) + 17g_1^2\text{Tr}(Y_u\epsilon_u^\dagger) + 45g_2^2\text{Tr}(Y_u\epsilon_u^\dagger) \\
& + 160g_3^2\text{Tr}(Y_u\epsilon_u^\dagger) - 54\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d\epsilon_d^\dagger) + 6\text{Tr}(\epsilon_d\epsilon_u^\dagger Y_u\epsilon_d^\dagger) - 18\text{Tr}(\epsilon_e\epsilon_e^\dagger Y_e\epsilon_e^\dagger) \\
& + 6\text{Tr}(\epsilon_u\epsilon_d^\dagger Y_d\epsilon_u^\dagger) - 54\text{Tr}(\epsilon_u\epsilon_u^\dagger Y_u\epsilon_u^\dagger) - 36\text{Tr}(Y_d\epsilon_d^\dagger Y_d Y_d^\dagger) + 6\text{Tr}(Y_d\epsilon_u^\dagger Y_u Y_d^\dagger) \\
& - 18\text{Tr}(Y_d Y_d^\dagger Y_d\epsilon_d^\dagger) + 6\text{Tr}(Y_d Y_u^\dagger Y_u\epsilon_d^\dagger) - 12\text{Tr}(Y_e\epsilon_e^\dagger Y_e Y_e^\dagger) - 6\text{Tr}(Y_e Y_e^\dagger Y_e\epsilon_e^\dagger) \\
& \left. - 54\text{Tr}(Y_u\epsilon_u^\dagger Y_u Y_u^\dagger)\right) \tag{29}
\end{aligned}$$

$$\begin{aligned}
\beta_{Y_e}^{(1)} & = +\frac{1}{2}(2\epsilon_e\epsilon_e^\dagger Y_e + 3Y_e Y_e^\dagger Y_e + Y_e\epsilon_e^\dagger \epsilon_e) \\
& + Y_e\left(3\text{Tr}(\epsilon_u\epsilon_u^\dagger) + 3\text{Tr}(Y_d Y_d^\dagger) - \frac{9}{4}g_1^2 - \frac{9}{4}g_2^2 + \text{Tr}(Y_e Y_e^\dagger)\right) \\
& + \epsilon_e\left(3\text{Tr}(Y_d\epsilon_d^\dagger) + 3\text{Tr}(Y_u\epsilon_u^\dagger) + \text{Tr}(Y_e\epsilon_e^\dagger)\right) \tag{30}
\end{aligned}$$

$$\begin{aligned}
\beta_{Y_e}^{(2)} = & -6\lambda_7\epsilon_e\epsilon_e^\dagger\epsilon_e + \frac{39}{40}g_1^2\epsilon_e\epsilon_e^\dagger Y_e + \frac{51}{8}g_2^2\epsilon_e\epsilon_e^\dagger Y_e - 4\lambda_3\epsilon_e\epsilon_e^\dagger Y_e \\
& - 2\lambda_4\epsilon_e\epsilon_e^\dagger Y_e - 6\lambda_5\epsilon_e Y_e^\dagger\epsilon_e - 6\lambda_6\epsilon_e Y_e^\dagger Y_e + \frac{309}{80}g_1^2 Y_e\epsilon_e^\dagger\epsilon_e \\
& + \frac{33}{16}g_2^2 Y_e\epsilon_e^\dagger\epsilon_e - 2\lambda_3 Y_e\epsilon_e^\dagger\epsilon_e - 4\lambda_4 Y_e\epsilon_e^\dagger\epsilon_e - 6\lambda_6 Y_e\epsilon_e^\dagger Y_e \\
& - 6\lambda_6 Y_e Y_e^\dagger\epsilon_e + \frac{387}{80}g_1^2 Y_e Y_e^\dagger Y_e + \frac{135}{16}g_2^2 Y_e Y_e^\dagger Y_e - 12\lambda_1 Y_e Y_e^\dagger Y_e \\
& - \frac{1}{4}\epsilon_e\epsilon_e^\dagger\epsilon_e\epsilon_e^\dagger Y_e + 2\epsilon_e\epsilon_e^\dagger Y_e\epsilon_e^\dagger\epsilon_e + \frac{7}{4}\epsilon_e Y_e^\dagger Y_e\epsilon_e^\dagger Y_e - \frac{1}{4}Y_e\epsilon_e^\dagger\epsilon_e\epsilon_e^\dagger\epsilon_e \\
& - \frac{1}{4}Y_e\epsilon_e^\dagger\epsilon_e Y_e^\dagger Y_e + \frac{7}{4}Y_e\epsilon_e^\dagger Y_e Y_e^\dagger\epsilon_e - \frac{1}{4}Y_e Y_e^\dagger\epsilon_e\epsilon_e^\dagger Y_e + \frac{3}{2}Y_e Y_e^\dagger Y_e Y_e^\dagger Y_e \\
& - \frac{9}{2}\epsilon_e\epsilon_e^\dagger Y_e \text{Tr}(\epsilon_d\epsilon_d^\dagger) - \frac{9}{4}Y_e\epsilon_e^\dagger\epsilon_e \text{Tr}(\epsilon_d\epsilon_d^\dagger) - \frac{27}{4}Y_e\epsilon_e^\dagger Y_e \text{Tr}(\epsilon_d Y_d^\dagger) \\
& - \frac{3}{2}\epsilon_e\epsilon_e^\dagger Y_e \text{Tr}(\epsilon_e\epsilon_e^\dagger) - \frac{3}{4}Y_e\epsilon_e^\dagger\epsilon_e \text{Tr}(\epsilon_e\epsilon_e^\dagger) - \frac{9}{4}Y_e\epsilon_e^\dagger Y_e \text{Tr}(\epsilon_e Y_e^\dagger) \\
& - \frac{27}{4}Y_e Y_e^\dagger Y_e \text{Tr}(\epsilon_u\epsilon_u^\dagger) - \frac{27}{4}Y_e\epsilon_e^\dagger Y_e \text{Tr}(\epsilon_u Y_u^\dagger) - \frac{9}{2}\epsilon_e Y_e^\dagger Y_e \text{Tr}(Y_d\epsilon_d^\dagger) \\
& - \frac{9}{4}Y_e Y_e^\dagger\epsilon_e \text{Tr}(Y_d\epsilon_d^\dagger) - \frac{27}{4}Y_e Y_e^\dagger Y_e \text{Tr}(Y_d Y_d^\dagger) - \frac{3}{2}\epsilon_e Y_e^\dagger Y_e \text{Tr}(Y_e\epsilon_e^\dagger) \\
& - \frac{3}{4}Y_e Y_e^\dagger\epsilon_e \text{Tr}(Y_e\epsilon_e^\dagger) - \frac{9}{4}Y_e Y_e^\dagger Y_e \text{Tr}(Y_e Y_e^\dagger) - \frac{9}{2}\epsilon_e Y_e^\dagger Y_e \text{Tr}(Y_u\epsilon_u^\dagger) \\
& - \frac{9}{4}Y_e Y_e^\dagger\epsilon_e \text{Tr}(Y_u\epsilon_u^\dagger) - \frac{9}{2}\epsilon_e\epsilon_e^\dagger Y_e \text{Tr}(Y_u Y_u^\dagger) - \frac{9}{4}Y_e\epsilon_e^\dagger\epsilon_e \text{Tr}(Y_u Y_u^\dagger) \\
& + Y_e \left(\frac{1449}{200}g_1^4 + \frac{27}{20}g_1^2 g_2^2 - \frac{21}{4}g_2^4 + 6\lambda_1^2 + \lambda_3^2 + \lambda_3\lambda_4 + \lambda_4^2 + \frac{3}{2}\lambda_5^2 + \frac{9}{2}\lambda_6^2 + \frac{3}{2}\lambda_7^2 \right) \\
& + \frac{1}{8} \left(160g_3^2 + 17g_1^2 + 45g_2^2 \right) \text{Tr}(\epsilon_u\epsilon_u^\dagger) + \frac{5}{8} \left(32g_3^2 + 9g_2^2 + g_1^2 \right) \text{Tr}(Y_d Y_d^\dagger) + \frac{15}{8}g_1^2 \text{Tr}(Y_e Y_e^\dagger) \\
& + \frac{15}{8}g_2^2 \text{Tr}(Y_e Y_e^\dagger) - \frac{9}{2}\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d Y_d^\dagger) - \frac{9}{4}\text{Tr}(\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger) + 3\text{Tr}(\epsilon_d\epsilon_u^\dagger Y_u Y_d^\dagger) \\
& - \frac{9}{4}\text{Tr}(\epsilon_d Y_d^\dagger Y_d\epsilon_d^\dagger) - \frac{3}{2}\text{Tr}(\epsilon_e\epsilon_e^\dagger Y_e Y_e^\dagger) - \frac{3}{4}\text{Tr}(\epsilon_e Y_e^\dagger Y_e\epsilon_e^\dagger) + 3\text{Tr}(\epsilon_u\epsilon_d^\dagger Y_d Y_u^\dagger) \\
& - \frac{27}{4}\text{Tr}(\epsilon_u\epsilon_u^\dagger\epsilon_u\epsilon_u^\dagger) - \frac{9}{2}\text{Tr}(\epsilon_u\epsilon_u^\dagger Y_u Y_u^\dagger) + \frac{3}{2}\text{Tr}(\epsilon_u Y_d^\dagger Y_d\epsilon_u^\dagger) - \frac{9}{4}\text{Tr}(\epsilon_u Y_u^\dagger Y_u\epsilon_u^\dagger) \\
& - \frac{27}{4}\text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) - \frac{9}{4}\text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) - \frac{9}{4}\text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) \\
& + \frac{1}{8}\epsilon_e \left(24\lambda_1\lambda_6 + 12\lambda_3\lambda_6 + 12\lambda_4\lambda_6 + 12\lambda_5\lambda_6 + 24\lambda_2\lambda_7 + 12\lambda_3\lambda_7 + 12\lambda_4\lambda_7 + 12\lambda_5\lambda_7 \right) \\
& + 5 \left(32g_3^2 + 9g_2^2 + g_1^2 \right) \text{Tr}(Y_d\epsilon_d^\dagger) + 15 \left(g_1^2 + g_2^2 \right) \text{Tr}(Y_e\epsilon_e^\dagger) + 17g_1^2 \text{Tr}(Y_u\epsilon_u^\dagger) + 45g_2^2 \text{Tr}(Y_u\epsilon_u^\dagger) \\
& + 160g_3^2 \text{Tr}(Y_u\epsilon_u^\dagger) - 54\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d\epsilon_d^\dagger) + 6\text{Tr}(\epsilon_d\epsilon_u^\dagger Y_u\epsilon_d^\dagger) - 18\text{Tr}(\epsilon_e\epsilon_e^\dagger Y_e\epsilon_e^\dagger) \\
& + 6\text{Tr}(\epsilon_u\epsilon_d^\dagger Y_d\epsilon_u^\dagger) - 54\text{Tr}(\epsilon_u\epsilon_u^\dagger Y_u\epsilon_u^\dagger) - 36\text{Tr}(Y_d\epsilon_d^\dagger Y_d Y_d^\dagger) + 6\text{Tr}(Y_d\epsilon_u^\dagger Y_u Y_d^\dagger) \\
& - 18\text{Tr}(Y_d Y_d^\dagger Y_d\epsilon_d^\dagger) + 6\text{Tr}(Y_d Y_u^\dagger Y_u\epsilon_d^\dagger) - 12\text{Tr}(Y_e\epsilon_e^\dagger Y_e Y_e^\dagger) - 6\text{Tr}(Y_e Y_e^\dagger Y_e\epsilon_e^\dagger)
\end{aligned}$$

$$- 54\text{Tr}\left(Y_u\epsilon_u^\dagger Y_u Y_u^\dagger\right) \quad (31)$$

$$\begin{aligned} \beta_{\epsilon_d}^{(1)} = & \frac{1}{2}\left(3\epsilon_d\epsilon_d^\dagger\epsilon_d + \epsilon_d\epsilon_u^\dagger\epsilon_u + \epsilon_d Y_d^\dagger Y_d - 3\epsilon_d Y_u^\dagger Y_u + 2Y_d Y_d^\dagger \epsilon_d - 4Y_d Y_u^\dagger \epsilon_u\right. \\ & + 6Y_d \text{Tr}\left(\epsilon_d Y_d^\dagger\right) + 2Y_d \text{Tr}\left(\epsilon_e Y_e^\dagger\right) + 6Y_d \text{Tr}\left(\epsilon_u Y_u^\dagger\right) \\ & \left. + \epsilon_d\left(3\text{Tr}\left(\epsilon_d\epsilon_d^\dagger\right) + 3\text{Tr}\left(Y_u Y_u^\dagger\right) - 8g_3^2 - \frac{1}{4}g_1^2 - \frac{9}{4}g_2^2 + \text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right)\right) \end{aligned} \quad (32)$$

$$\begin{aligned} \beta_{\epsilon_d}^{(2)} = & \frac{187}{80}g_1^2\epsilon_d\epsilon_d^\dagger\epsilon_d + \frac{135}{16}g_2^2\epsilon_d\epsilon_d^\dagger\epsilon_d + 16g_3^2\epsilon_d\epsilon_d^\dagger\epsilon_d - 12\lambda_2\epsilon_d\epsilon_d^\dagger\epsilon_d \\ & - 6\lambda_7\epsilon_d\epsilon_d^\dagger Y_d - \frac{53}{240}g_1^2\epsilon_d\epsilon_u^\dagger\epsilon_u + \frac{33}{16}g_2^2\epsilon_d\epsilon_u^\dagger\epsilon_u + \frac{16}{3}g_3^2\epsilon_d\epsilon_u^\dagger\epsilon_u \\ & - 2\lambda_3\epsilon_d\epsilon_u^\dagger\epsilon_u + 2\lambda_4\epsilon_d\epsilon_u^\dagger\epsilon_u - 6\lambda_7\epsilon_d Y_d^\dagger \epsilon_d + \frac{247}{240}g_1^2\epsilon_d Y_d^\dagger Y_d \\ & + \frac{33}{16}g_2^2\epsilon_d Y_d^\dagger Y_d + \frac{16}{3}g_3^2\epsilon_d Y_d^\dagger Y_d - 2\lambda_3\epsilon_d Y_d^\dagger Y_d - 4\lambda_4\epsilon_d Y_d^\dagger Y_d \\ & - \frac{79}{80}g_1^2\epsilon_d Y_u^\dagger Y_u + \frac{9}{16}g_2^2\epsilon_d Y_u^\dagger Y_u - 16g_3^2\epsilon_d Y_u^\dagger Y_u - 6\lambda_7 Y_d \epsilon_d^\dagger \epsilon_d \\ & - 6\lambda_5 Y_d \epsilon_d^\dagger Y_d + \frac{157}{120}g_1^2 Y_d Y_d^\dagger \epsilon_d + \frac{51}{8}g_2^2 Y_d Y_d^\dagger \epsilon_d + \frac{32}{3}g_3^2 Y_d Y_d^\dagger \epsilon_d \\ & - 4\lambda_3 Y_d Y_d^\dagger \epsilon_d - 2\lambda_4 Y_d Y_d^\dagger \epsilon_d - 6\lambda_6 Y_d Y_d^\dagger Y_d - \frac{23}{30}g_1^2 Y_d Y_u^\dagger \epsilon_u \\ & - \frac{3}{2}g_2^2 Y_d Y_u^\dagger \epsilon_u - \frac{64}{3}g_3^2 Y_d Y_u^\dagger \epsilon_u + 2\lambda_3 Y_d Y_u^\dagger \epsilon_u - 2\lambda_4 Y_d Y_u^\dagger \epsilon_u \\ & + \frac{3}{2}\epsilon_d\epsilon_d^\dagger\epsilon_d\epsilon_d^\dagger\epsilon_d - \epsilon_d\epsilon_d^\dagger\epsilon_d Y_u^\dagger Y_u - \frac{1}{4}\epsilon_d\epsilon_d^\dagger Y_d Y_d^\dagger \epsilon_d - \frac{1}{4}\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger\epsilon_d \\ & - \frac{1}{4}\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_u^\dagger\epsilon_u + \epsilon_d\epsilon_u^\dagger\epsilon_u Y_u^\dagger Y_u - \frac{1}{4}\epsilon_d\epsilon_u^\dagger Y_u Y_u^\dagger \epsilon_u + \frac{7}{4}\epsilon_d Y_d^\dagger \epsilon_d\epsilon_d^\dagger Y_d \\ & - 2\epsilon_d Y_d^\dagger \epsilon_d\epsilon_d^\dagger Y_u - \frac{1}{4}\epsilon_d Y_d^\dagger Y_d\epsilon_d^\dagger\epsilon_d - \frac{1}{4}\epsilon_d Y_d^\dagger Y_d Y_d^\dagger Y_d + \epsilon_d Y_d^\dagger Y_d Y_u^\dagger Y_u \\ & + \frac{7}{4}\epsilon_d Y_u^\dagger \epsilon_u\epsilon_u^\dagger Y_u - \frac{1}{4}\epsilon_d Y_u^\dagger Y_u\epsilon_d^\dagger\epsilon_d + \frac{11}{4}\epsilon_d Y_u^\dagger Y_u Y_u^\dagger Y_u + \frac{7}{4}Y_d\epsilon_d^\dagger\epsilon_d Y_d^\dagger \epsilon_d \\ & - Y_d\epsilon_d^\dagger\epsilon_d Y_u^\dagger \epsilon_u - \frac{1}{4}Y_d\epsilon_u^\dagger\epsilon_u Y_d^\dagger \epsilon_d + Y_d\epsilon_u^\dagger\epsilon_u Y_u^\dagger \epsilon_u \\ & - 2Y_d Y_d^\dagger \epsilon_d\epsilon_u^\dagger\epsilon_u + 2Y_d Y_d^\dagger \epsilon_d Y_d^\dagger Y_d - \frac{1}{4}Y_d Y_d^\dagger Y_d Y_d^\dagger \epsilon_d + Y_d Y_d^\dagger Y_d Y_u^\dagger \epsilon_u \\ & + 2Y_d Y_u^\dagger \epsilon_u\epsilon_u^\dagger\epsilon_u - \frac{1}{4}Y_d Y_u^\dagger Y_u Y_d^\dagger \epsilon_d + 3Y_d Y_u^\dagger Y_u Y_u^\dagger \epsilon_u \\ & - \frac{27}{4}\epsilon_d\epsilon_d^\dagger\epsilon_d \text{Tr}\left(\epsilon_d\epsilon_d^\dagger\right) + \frac{15}{4}\epsilon_d Y_u^\dagger Y_u \text{Tr}\left(\epsilon_d\epsilon_d^\dagger\right) + 6Y_d Y_u^\dagger \epsilon_u \text{Tr}\left(\epsilon_d\epsilon_d^\dagger\right) \\ & - \frac{9}{4}\epsilon_d\epsilon_d^\dagger Y_d \text{Tr}\left(\epsilon_d Y_d^\dagger\right) + \frac{15}{4}\epsilon_d\epsilon_u^\dagger Y_u \text{Tr}\left(\epsilon_d Y_d^\dagger\right) - \frac{9}{2}Y_d\epsilon_d^\dagger\epsilon_d \text{Tr}\left(\epsilon_d Y_d^\dagger\right) \\ & + 6Y_d\epsilon_u^\dagger\epsilon_u \text{Tr}\left(\epsilon_d Y_d^\dagger\right) - \frac{9}{4}\epsilon_d\epsilon_d^\dagger\epsilon_d \text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) + \frac{5}{4}\epsilon_d Y_u^\dagger Y_u \text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) \\ & + 2Y_d Y_u^\dagger \epsilon_u \text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - \frac{3}{4}\epsilon_d\epsilon_d^\dagger Y_d \text{Tr}\left(\epsilon_e Y_e^\dagger\right) + \frac{5}{4}\epsilon_d\epsilon_u^\dagger Y_u \text{Tr}\left(\epsilon_e Y_e^\dagger\right) \end{aligned}$$

$$\begin{aligned}
& -\frac{3}{2}Y_d\epsilon_d^\dagger\epsilon_d\text{Tr}(\epsilon_e Y_e^\dagger) + 2Y_d\epsilon_u^\dagger\epsilon_u\text{Tr}(\epsilon_e Y_e^\dagger) - \frac{9}{4}\epsilon_d\epsilon_d^\dagger\epsilon_u\text{Tr}(\epsilon_u\epsilon_u^\dagger) \\
& -\frac{9}{4}\epsilon_d Y_d^\dagger Y_d\text{Tr}(\epsilon_u\epsilon_u^\dagger) - \frac{9}{2}Y_d Y_d^\dagger \epsilon_d\text{Tr}(\epsilon_u\epsilon_u^\dagger) - \frac{9}{4}\epsilon_d\epsilon_d^\dagger Y_d\text{Tr}(\epsilon_u Y_u^\dagger) \\
& +\frac{15}{4}\epsilon_d\epsilon_u^\dagger Y_u\text{Tr}(\epsilon_u Y_u^\dagger) - \frac{9}{2}Y_d\epsilon_d^\dagger\epsilon_d\text{Tr}(\epsilon_u Y_u^\dagger) + 6Y_d\epsilon_u^\dagger\epsilon_u\text{Tr}(\epsilon_u Y_u^\dagger) \\
& -\frac{27}{4}\epsilon_d Y_d^\dagger \epsilon_d\text{Tr}(Y_d\epsilon_d^\dagger) - \frac{9}{4}\epsilon_d Y_u^\dagger \epsilon_u\text{Tr}(Y_d\epsilon_d^\dagger) - \frac{9}{4}\epsilon_d\epsilon_u^\dagger\epsilon_u\text{Tr}(Y_d Y_d^\dagger) \\
& -\frac{9}{4}\epsilon_d Y_d^\dagger Y_d\text{Tr}(Y_d Y_d^\dagger) - \frac{9}{2}Y_d Y_d^\dagger \epsilon_d\text{Tr}(Y_d Y_d^\dagger) - \frac{9}{4}\epsilon_d Y_d^\dagger \epsilon_d\text{Tr}(Y_e\epsilon_e^\dagger) \\
& -\frac{3}{4}\epsilon_d Y_u^\dagger \epsilon_u\text{Tr}(Y_e\epsilon_e^\dagger) - \frac{3}{4}\epsilon_d\epsilon_u^\dagger\epsilon_u\text{Tr}(Y_e Y_e^\dagger) - \frac{3}{4}\epsilon_d Y_d^\dagger Y_d\text{Tr}(Y_e Y_e^\dagger) \\
& -\frac{3}{2}Y_d Y_d^\dagger \epsilon_d\text{Tr}(Y_e Y_e^\dagger) - \frac{27}{4}\epsilon_d Y_d^\dagger \epsilon_d\text{Tr}(Y_u\epsilon_u^\dagger) - \frac{9}{4}\epsilon_d Y_u^\dagger \epsilon_u\text{Tr}(Y_u\epsilon_u^\dagger) \\
& -\frac{27}{4}\epsilon_d\epsilon_d^\dagger\epsilon_d\text{Tr}(Y_u Y_u^\dagger) + \frac{15}{4}\epsilon_d Y_u^\dagger Y_u\text{Tr}(Y_u Y_u^\dagger) + 6Y_d Y_u^\dagger \epsilon_u\text{Tr}(Y_u Y_u^\dagger) \\
& +\frac{1}{8}Y_d(24\lambda_1\lambda_6 + 12\lambda_3\lambda_6 + 12\lambda_4\lambda_6 + 12\lambda_5\lambda_6 + 24\lambda_2\lambda_7 + 12\lambda_3\lambda_7 + 12\lambda_4\lambda_7 + 12\lambda_5\lambda_7 \\
& + 5(32g_3^2 + 9g_2^2 + g_1^2)\text{Tr}(\epsilon_d Y_d^\dagger) + 15(g_1^2 + g_2^2)\text{Tr}(\epsilon_e Y_e^\dagger) + 17g_1^2\text{Tr}(\epsilon_u Y_u^\dagger) + 45g_2^2\text{Tr}(\epsilon_u Y_u^\dagger) \\
& + 160g_3^2\text{Tr}(\epsilon_u Y_u^\dagger) - 18\text{Tr}(\epsilon_d\epsilon_d^\dagger\epsilon_d Y_d^\dagger) + 6\text{Tr}(\epsilon_d\epsilon_u^\dagger\epsilon_u Y_d^\dagger) - 36\text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d\epsilon_d^\dagger) \\
& - 54\text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger) + 6\text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u\epsilon_d^\dagger) + 6\text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger) - 6\text{Tr}(\epsilon_e\epsilon_e^\dagger\epsilon_e Y_e^\dagger) \\
& - 12\text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e\epsilon_e^\dagger) - 18\text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger) + 6\text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger) - 54\text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u\epsilon_u^\dagger) \\
& - 54\text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger)) \\
& +\epsilon_d\left(-\frac{113}{600}g_1^4 - \frac{27}{20}g_1^2 g_2^2 - \frac{21}{4}g_2^4 + \frac{31}{15}g_1^2 g_3^2 + 9g_2^2 g_3^2 - 108g_3^4 + 6\lambda_2^2 + \lambda_3^2 + \lambda_3\lambda_4 + \lambda_4^2 + \frac{3}{2}\lambda_5^2 + \frac{3}{2}\lambda_6^2\right. \\
& +\frac{9}{2}\lambda_7^2 + \frac{5}{8}(32g_3^2 + 9g_2^2 + g_1^2)\text{Tr}(\epsilon_d\epsilon_d^\dagger) + \frac{15}{8}(g_1^2 + g_2^2)\text{Tr}(\epsilon_e\epsilon_e^\dagger) + \frac{17}{8}g_1^2\text{Tr}(Y_u Y_u^\dagger) \\
& +\frac{45}{8}g_2^2\text{Tr}(Y_u Y_u^\dagger) + 20g_3^2\text{Tr}(Y_u Y_u^\dagger) - \frac{27}{4}\text{Tr}(\epsilon_d\epsilon_d^\dagger\epsilon_d\epsilon_d^\dagger) - \frac{9}{2}\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d Y_d^\dagger) \\
& -\frac{9}{4}\text{Tr}(\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger) + 3\text{Tr}(\epsilon_d\epsilon_u^\dagger Y_u Y_d^\dagger) - \frac{9}{4}\text{Tr}(\epsilon_d Y_d^\dagger Y_d\epsilon_d^\dagger) + \frac{3}{2}\text{Tr}(\epsilon_d Y_u^\dagger Y_u\epsilon_d^\dagger) \\
& -\frac{9}{4}\text{Tr}(\epsilon_e\epsilon_e^\dagger\epsilon_e\epsilon_e^\dagger) - \frac{3}{2}\text{Tr}(\epsilon_e\epsilon_e^\dagger Y_e Y_e^\dagger) - \frac{3}{4}\text{Tr}(\epsilon_e Y_e^\dagger Y_e\epsilon_e^\dagger) + 3\text{Tr}(\epsilon_u\epsilon_d^\dagger Y_d Y_u^\dagger) \\
& \left.-\frac{9}{2}\text{Tr}(\epsilon_u\epsilon_u^\dagger Y_u Y_u^\dagger) - \frac{9}{4}\text{Tr}(\epsilon_u Y_u^\dagger Y_u\epsilon_u^\dagger) - \frac{9}{4}\text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) - \frac{27}{4}\text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger)\right) \tag{33}
\end{aligned}$$

$$\beta_{\epsilon_e}^{(1)} = +\frac{3}{2}\epsilon_e\epsilon_e^\dagger\epsilon_e + \frac{1}{2}\epsilon_e Y_e^\dagger Y_e + Y_e Y_e^\dagger \epsilon_e + 3Y_e\text{Tr}(\epsilon_d Y_d^\dagger) + Y_e\text{Tr}(\epsilon_e Y_e^\dagger) + 3Y_e\text{Tr}(\epsilon_u Y_u^\dagger)$$

$$+ \epsilon_e\left(3\text{Tr}(\epsilon_d\epsilon_d^\dagger) + 3\text{Tr}(Y_u Y_u^\dagger) - \frac{9}{4}g_1^2 - \frac{9}{4}g_2^2 + \text{Tr}(\epsilon_e\epsilon_e^\dagger)\right) \tag{34}$$

$$\beta_{\epsilon_e}^{(2)} = +\frac{1}{80}\left(-480\lambda_7\epsilon_e\epsilon_e^\dagger Y_e - 480\lambda_7\epsilon_e Y_e^\dagger \epsilon_e + 309g_1^2\epsilon_e Y_e^\dagger Y_e + 165g_2^2\epsilon_e Y_e^\dagger Y_e\right)$$

$$\begin{aligned}
& -160\lambda_3\epsilon_e Y_e^\dagger Y_e - 320\lambda_4\epsilon_e Y_e^\dagger Y_e - 480\lambda_7 Y_e \epsilon_e^\dagger \epsilon_e - 480\lambda_5 Y_e \epsilon_e^\dagger Y_e \\
& + 78g_1^2 Y_e Y_e^\dagger \epsilon_e + 510g_2^2 Y_e Y_e^\dagger \epsilon_e - 320\lambda_3 Y_e Y_e^\dagger \epsilon_e - 160\lambda_4 Y_e Y_e^\dagger \epsilon_e \\
& - 480\lambda_6 Y_e Y_e^\dagger Y_e + 120\epsilon_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger \epsilon_e - 20\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger \epsilon_e + 140\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger Y_e \\
& - 20\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger \epsilon_e - 20\epsilon_e Y_e^\dagger Y_e Y_e^\dagger Y_e + 140Y_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger \epsilon_e + 160Y_e Y_e^\dagger \epsilon_e Y_e^\dagger Y_e \\
& - 20Y_e Y_e^\dagger Y_e Y_e^\dagger \epsilon_e - 180\epsilon_e \epsilon_e^\dagger Y_e \text{Tr}(\epsilon_d Y_d^\dagger) - 360Y_e \epsilon_e^\dagger \epsilon_e \text{Tr}(\epsilon_d Y_d^\dagger) \\
& - 60\epsilon_e \epsilon_e^\dagger Y_e \text{Tr}(\epsilon_e Y_e^\dagger) - 120Y_e \epsilon_e^\dagger \epsilon_e \text{Tr}(\epsilon_e Y_e^\dagger) - 180\epsilon_e Y_e^\dagger Y_e \text{Tr}(\epsilon_u \epsilon_u^\dagger) \\
& - 360Y_e Y_e^\dagger \epsilon_e \text{Tr}(\epsilon_u \epsilon_u^\dagger) - 180\epsilon_e \epsilon_e^\dagger Y_e \text{Tr}(\epsilon_u Y_u^\dagger) - 360Y_e \epsilon_e^\dagger \epsilon_e \text{Tr}(\epsilon_u Y_u^\dagger) \\
& - 540\epsilon_e Y_e^\dagger \epsilon_e \text{Tr}(Y_d \epsilon_d^\dagger) - 180\epsilon_e Y_e^\dagger Y_e \text{Tr}(Y_d Y_d^\dagger) - 360Y_e Y_e^\dagger \epsilon_e \text{Tr}(Y_d Y_d^\dagger) \\
& - 180\epsilon_e Y_e^\dagger \epsilon_e \text{Tr}(Y_e \epsilon_e^\dagger) - 60\epsilon_e Y_e^\dagger Y_e \text{Tr}(Y_e Y_e^\dagger) - 120Y_e Y_e^\dagger \epsilon_e \text{Tr}(Y_e Y_e^\dagger) \\
& - 540\epsilon_e Y_e^\dagger \epsilon_e \text{Tr}(Y_u \epsilon_u^\dagger) \\
& + 3\epsilon_e \epsilon_e^\dagger \epsilon_e (129g_1^2 - 180\text{Tr}(\epsilon_d \epsilon_d^\dagger) - 180\text{Tr}(Y_u Y_u^\dagger) + 225g_2^2 - 320\lambda_2 - 60\text{Tr}(\epsilon_e \epsilon_e^\dagger)) \\
& + 10Y_e (24\lambda_1 \lambda_6 + 12\lambda_3 \lambda_6 + 12\lambda_4 \lambda_6 + 12\lambda_5 \lambda_6 + 24\lambda_2 \lambda_7 + 12\lambda_3 \lambda_7 + 12\lambda_4 \lambda_7 + 12\lambda_5 \lambda_7 \\
& + 5(32g_3^2 + 9g_2^2 + g_1^2)\text{Tr}(\epsilon_d Y_d^\dagger) + 15(g_1^2 + g_2^2)\text{Tr}(\epsilon_e Y_e^\dagger) + 17g_1^2 \text{Tr}(\epsilon_u Y_u^\dagger) + 45g_2^2 \text{Tr}(\epsilon_u Y_u^\dagger) \\
& + 160g_3^2 \text{Tr}(\epsilon_u Y_u^\dagger) - 18\text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d Y_d^\dagger) + 6\text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_d^\dagger) - 36\text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger) \\
& - 54\text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger) + 6\text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) + 6\text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger) - 6\text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger) \\
& - 12\text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger) - 18\text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger) + 6\text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger) - 54\text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger) \\
& - 54\text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger)) \\
& + \epsilon_e \left(\frac{1449}{200} g_1^4 + \frac{27}{20} g_1^2 g_2^2 - \frac{21}{4} g_2^4 + 6\lambda_2^2 + \lambda_3^2 + \lambda_3 \lambda_4 + \lambda_4^2 + \frac{3}{2} \lambda_5^2 + \frac{3}{2} \lambda_6^2 + \frac{9}{2} \lambda_7^2 \right. \\
& + \frac{5}{8} (32g_3^2 + 9g_2^2 + g_1^2) \text{Tr}(\epsilon_d \epsilon_d^\dagger) + \frac{15}{8} (g_1^2 + g_2^2) \text{Tr}(\epsilon_e \epsilon_e^\dagger) + \frac{17}{8} g_1^2 \text{Tr}(Y_u Y_u^\dagger) + \frac{45}{8} g_2^2 \text{Tr}(Y_u Y_u^\dagger) \\
& + 20g_3^2 \text{Tr}(Y_u Y_u^\dagger) - \frac{27}{4} \text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger) - \frac{9}{2} \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger) - \frac{9}{4} \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) \\
& + 3\text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) - \frac{9}{4} \text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger) + \frac{3}{2} \text{Tr}(\epsilon_d Y_u^\dagger Y_u \epsilon_d^\dagger) - \frac{9}{4} \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger) \\
& - \frac{3}{2} \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger) - \frac{3}{4} \text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger) + 3\text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) - \frac{9}{2} \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger) \\
& \left. - \frac{9}{4} \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) - \frac{9}{4} \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) - \frac{27}{4} \text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger) \right)
\end{aligned} \tag{35}$$

3.4 Scalar Mass Terms

$$\begin{aligned}
\beta_{m_{12}}^{(1)} = & -12\lambda_6 m_1^2 - \frac{9}{10}g_1^2 m_{12} - \frac{9}{2}g_2^2 m_{12} + 2\lambda_3 m_{12} + 4\lambda_4 m_{12} + 6\lambda_5 m_{12} - 12\lambda_7 m_2^2 - 6m_1^2 \text{Tr}(\epsilon_d Y_d^\dagger) \\
& - 2m_1^2 \text{Tr}(\epsilon_e Y_e^\dagger) - 6m_2^2 \text{Tr}(\epsilon_u Y_u^\dagger) - 6m_2^2 \text{Tr}(Y_d \epsilon_d^\dagger) + 6m_{12} \text{Tr}(Y_d Y_d^\dagger) - 2m_2^2 \text{Tr}(Y_e \epsilon_e^\dagger) \\
& + 2m_{12} \text{Tr}(Y_e Y_e^\dagger) - 6m_1^2 \text{Tr}(Y_u \epsilon_u^\dagger) + 6m_{12} \text{Tr}(Y_u Y_u^\dagger)
\end{aligned} \tag{36}$$

$$\begin{aligned}
\beta_{m_{12}}^{(2)} = & -\frac{72}{5}g_1^2 \lambda_6 m_1^2 - 72g_2^2 \lambda_6 m_1^2 + 72\lambda_1 \lambda_6 m_1^2 + 24\lambda_3 \lambda_6 m_1^2 + 24\lambda_4 \lambda_6 m_1^2 + 24\lambda_5 \lambda_6 m_1^2 + 12\lambda_3 \lambda_7 m_1^2 \\
& + 12\lambda_4 \lambda_7 m_1^2 + 12\lambda_5 \lambda_7 m_1^2 + \frac{1377}{400}g_1^4 m_{12} + \frac{9}{8}g_1^2 g_2^2 m_{12} - \frac{243}{16}g_2^4 m_{12} + 6\lambda_1^2 m_{12} + 6\lambda_2^2 m_{12} + \frac{12}{5}g_1^2 \lambda_3 m_{12} \\
& + 12g_2^2 \lambda_3 m_{12} - 12\lambda_1 \lambda_3 m_{12} - 12\lambda_2 \lambda_3 m_{12} + \frac{24}{5}g_1^2 \lambda_4 m_{12} + 24g_2^2 \lambda_4 m_{12} - 12\lambda_1 \lambda_4 m_{12} - 12\lambda_2 \lambda_4 m_{12} \\
& - 6\lambda_3 \lambda_4 m_{12} + \frac{36}{5}g_1^2 \lambda_5 m_{12} + 36g_2^2 \lambda_5 m_{12} - 12\lambda_1 \lambda_5 m_{12} - 12\lambda_2 \lambda_5 m_{12} - 12\lambda_3 \lambda_5 m_{12} - 12\lambda_4 \lambda_5 m_{12} \\
& + 3\lambda_5^2 m_{12} - 12\lambda_6^2 m_{12} - 36\lambda_6 \lambda_7 m_{12} - 12\lambda_7^2 m_{12} + 12\lambda_3 \lambda_6 m_2^2 + 12\lambda_4 \lambda_6 m_2^2 + 12\lambda_5 \lambda_6 m_2^2 \\
& - \frac{72}{5}g_1^2 \lambda_7 m_2^2 - 72g_2^2 \lambda_7 m_2^2 + 72\lambda_2 \lambda_7 m_2^2 + 24\lambda_3 \lambda_7 m_2^2 + 24\lambda_4 \lambda_7 m_2^2 + 24\lambda_5 \lambda_7 m_2^2 \\
& + \left(-12\lambda_4 m_{12} - 18\lambda_5 m_{12} + 20g_3^2 m_{12} - 6\lambda_3 m_{12} + 72\lambda_7 m_2^2 + \frac{45}{8}g_2^2 m_{12} + \frac{5}{8}g_1^2 m_{12} \right) \text{Tr}(\epsilon_d \epsilon_d^\dagger) \\
& + 6 \left(2\lambda_4 (m_1^2 + m_2^2) + 3 \left(-2(\lambda_6 + \lambda_7) m_{12} + \lambda_5 (m_1^2 + m_2^2) \right) + \lambda_3 (m_1^2 + m_2^2) \right) \text{Tr}(\epsilon_d Y_d^\dagger) + \frac{15}{8}g_1^2 m_{12} \text{Tr}(\epsilon_e \epsilon_e^\dagger) \\
& + \frac{15}{8}g_2^2 m_{12} \text{Tr}(\epsilon_e \epsilon_e^\dagger) - 2\lambda_3 m_{12} \text{Tr}(\epsilon_e \epsilon_e^\dagger) - 4\lambda_4 m_{12} \text{Tr}(\epsilon_e \epsilon_e^\dagger) - 6\lambda_5 m_{12} \text{Tr}(\epsilon_e \epsilon_e^\dagger) \\
& + 24\lambda_7 m_2^2 \text{Tr}(\epsilon_e \epsilon_e^\dagger) + 2\lambda_3 m_1^2 \text{Tr}(\epsilon_e Y_e^\dagger) + 4\lambda_4 m_1^2 \text{Tr}(\epsilon_e Y_e^\dagger) + 6\lambda_5 m_1^2 \text{Tr}(\epsilon_e Y_e^\dagger) \\
& - 12\lambda_6 m_{12} \text{Tr}(\epsilon_e Y_e^\dagger) - 12\lambda_7 m_{12} \text{Tr}(\epsilon_e Y_e^\dagger) + 2\lambda_3 m_2^2 \text{Tr}(\epsilon_e Y_e^\dagger) + 4\lambda_4 m_2^2 \text{Tr}(\epsilon_e Y_e^\dagger) \\
& + 6\lambda_5 m_2^2 \text{Tr}(\epsilon_e Y_e^\dagger) + 72\lambda_6 m_1^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger) + \frac{17}{8}g_1^2 m_{12} \text{Tr}(\epsilon_u \epsilon_u^\dagger) + \frac{45}{8}g_2^2 m_{12} \text{Tr}(\epsilon_u \epsilon_u^\dagger) \\
& + 20g_3^2 m_{12} \text{Tr}(\epsilon_u \epsilon_u^\dagger) - 6\lambda_3 m_{12} \text{Tr}(\epsilon_u \epsilon_u^\dagger) - 12\lambda_4 m_{12} \text{Tr}(\epsilon_u \epsilon_u^\dagger) - 18\lambda_5 m_{12} \text{Tr}(\epsilon_u \epsilon_u^\dagger) \\
& + 6\lambda_3 m_1^2 \text{Tr}(\epsilon_u Y_u^\dagger) + 12\lambda_4 m_1^2 \text{Tr}(\epsilon_u Y_u^\dagger) + 18\lambda_5 m_1^2 \text{Tr}(\epsilon_u Y_u^\dagger) - 36\lambda_6 m_{12} \text{Tr}(\epsilon_u Y_u^\dagger) \\
& - 36\lambda_7 m_{12} \text{Tr}(\epsilon_u Y_u^\dagger) + 6\lambda_3 m_2^2 \text{Tr}(\epsilon_u Y_u^\dagger) + 12\lambda_4 m_2^2 \text{Tr}(\epsilon_u Y_u^\dagger) + 18\lambda_5 m_2^2 \text{Tr}(\epsilon_u Y_u^\dagger) \\
& + 6\lambda_3 m_1^2 \text{Tr}(Y_d \epsilon_d^\dagger) + 12\lambda_4 m_1^2 \text{Tr}(Y_d \epsilon_d^\dagger) + 18\lambda_5 m_1^2 \text{Tr}(Y_d \epsilon_d^\dagger) + 6\lambda_3 m_2^2 \text{Tr}(Y_d \epsilon_d^\dagger) \\
& + 12\lambda_4 m_2^2 \text{Tr}(Y_d \epsilon_d^\dagger) + 18\lambda_5 m_2^2 \text{Tr}(Y_d \epsilon_d^\dagger) + 72\lambda_6 m_1^2 \text{Tr}(Y_d Y_d^\dagger) + \frac{5}{8}g_1^2 m_{12} \text{Tr}(Y_d Y_d^\dagger) \\
& + \frac{45}{8}g_2^2 m_{12} \text{Tr}(Y_d Y_d^\dagger) + 20g_3^2 m_{12} \text{Tr}(Y_d Y_d^\dagger) - 6\lambda_3 m_{12} \text{Tr}(Y_d Y_d^\dagger) - 12\lambda_4 m_{12} \text{Tr}(Y_d Y_d^\dagger) \\
& - 18\lambda_5 m_{12} \text{Tr}(Y_d Y_d^\dagger) + 2\lambda_3 m_1^2 \text{Tr}(Y_e \epsilon_e^\dagger) + 4\lambda_4 m_1^2 \text{Tr}(Y_e \epsilon_e^\dagger) + 6\lambda_5 m_1^2 \text{Tr}(Y_e \epsilon_e^\dagger) \\
& + 2\lambda_3 m_2^2 \text{Tr}(Y_e \epsilon_e^\dagger) + 4\lambda_4 m_2^2 \text{Tr}(Y_e \epsilon_e^\dagger) + 6\lambda_5 m_2^2 \text{Tr}(Y_e \epsilon_e^\dagger) + 24\lambda_6 m_1^2 \text{Tr}(Y_e Y_e^\dagger)
\end{aligned}$$

$$\begin{aligned}
& + \frac{15}{8}g_1^2m_{12}\text{Tr}(Y_eY_e^\dagger) + \frac{15}{8}g_2^2m_{12}\text{Tr}(Y_eY_e^\dagger) - 2\lambda_3m_{12}\text{Tr}(Y_eY_e^\dagger) - 4\lambda_4m_{12}\text{Tr}(Y_eY_e^\dagger) \\
& - 6\lambda_5m_{12}\text{Tr}(Y_eY_e^\dagger) + 6\lambda_3m_1^2\text{Tr}(Y_u\epsilon_u^\dagger) + 12\lambda_4m_1^2\text{Tr}(Y_u\epsilon_u^\dagger) + 18\lambda_5m_1^2\text{Tr}(Y_u\epsilon_u^\dagger) \\
& + 6\lambda_3m_2^2\text{Tr}(Y_u\epsilon_u^\dagger) + 12\lambda_4m_2^2\text{Tr}(Y_u\epsilon_u^\dagger) + 18\lambda_5m_2^2\text{Tr}(Y_u\epsilon_u^\dagger) + \frac{17}{8}g_1^2m_{12}\text{Tr}(Y_uY_u^\dagger) \\
& + \frac{45}{8}g_2^2m_{12}\text{Tr}(Y_uY_u^\dagger) + 20g_3^2m_{12}\text{Tr}(Y_uY_u^\dagger) - 6\lambda_3m_{12}\text{Tr}(Y_uY_u^\dagger) - 12\lambda_4m_{12}\text{Tr}(Y_uY_u^\dagger) \\
& - 18\lambda_5m_{12}\text{Tr}(Y_uY_u^\dagger) + 72\lambda_7m_2^2\text{Tr}(Y_uY_u^\dagger) - \frac{27}{4}m_{12}\text{Tr}(\epsilon_d\epsilon_d^\dagger\epsilon_d\epsilon_d^\dagger) - 9m_{12}\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d Y_d^\dagger) \\
& - \frac{33}{2}m_{12}\text{Tr}(\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger) + 12m_1^2\text{Tr}(\epsilon_d\epsilon_u^\dagger\epsilon_u Y_d^\dagger) + 12m_2^2\text{Tr}(\epsilon_d\epsilon_u^\dagger Y_u\epsilon_d^\dagger) \\
& + 6m_{12}\text{Tr}(\epsilon_d\epsilon_u^\dagger Y_u Y_d^\dagger) - \frac{9}{2}m_{12}\text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger) + 12m_2^2\text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) \\
& - 24m_{12}\text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u Y_d^\dagger) + \frac{3}{2}m_{12}\text{Tr}(\epsilon_d Y_u^\dagger Y_u \epsilon_d^\dagger) + 12m_2^2\text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger) \\
& - \frac{9}{4}m_{12}\text{Tr}(\epsilon_e\epsilon_e^\dagger\epsilon_e\epsilon_e^\dagger) - 3m_{12}\text{Tr}(\epsilon_e\epsilon_e^\dagger Y_e Y_e^\dagger) - \frac{3}{2}m_{12}\text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger) \\
& + 12m_1^2\text{Tr}(\epsilon_u\epsilon_u^\dagger Y_d \epsilon_u^\dagger) + 6m_{12}\text{Tr}(\epsilon_u\epsilon_u^\dagger Y_d Y_u^\dagger) - \frac{27}{4}m_{12}\text{Tr}(\epsilon_u\epsilon_u^\dagger\epsilon_u\epsilon_u^\dagger) \\
& - 9m_{12}\text{Tr}(\epsilon_u\epsilon_u^\dagger Y_u Y_u^\dagger) + \frac{3}{2}m_{12}\text{Tr}(\epsilon_u Y_d^\dagger Y_d \epsilon_u^\dagger) + 12m_1^2\text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger) \\
& - \frac{9}{2}m_{12}\text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) + 12m_1^2\text{Tr}(Y_d\epsilon_u^\dagger Y_u Y_d^\dagger) - \frac{27}{4}m_{12}\text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) \\
& + 12m_2^2\text{Tr}(Y_d Y_u^\dagger Y_u \epsilon_d^\dagger) - \frac{33}{2}m_{12}\text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) - \frac{9}{4}m_{12}\text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) - \frac{27}{4}m_{12}\text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger)
\end{aligned} \tag{37}$$

$$\begin{aligned}
\beta_{m_1^2}^{(1)} &= -\frac{9}{10}g_1^2m_1^2 - \frac{9}{2}g_2^2m_1^2 + 12\lambda_1m_1^2 - 6\lambda_6m_{12} + 4\lambda_3m_2^2 + 2\lambda_4m_2^2 + 6m_1^2\text{Tr}(\epsilon_u\epsilon_u^\dagger) - 6m_{12}\text{Tr}(\epsilon_u Y_u^\dagger) \\
& + 6m_1^2\text{Tr}(Y_d Y_d^\dagger) + 2m_1^2\text{Tr}(Y_e Y_e^\dagger)
\end{aligned} \tag{38}$$

$$\begin{aligned}
\beta_{m_1^2}^{(2)} &= +\frac{1737}{400}g_1^4m_1^2 + \frac{9}{8}g_1^2g_2^2m_1^2 - \frac{123}{16}g_2^4m_1^2 + \frac{72}{5}g_1^2\lambda_1m_1^2 + 72g_2^2\lambda_1m_1^2 - 60\lambda_1^2m_1^2 - 2\lambda_3^2m_1^2 \\
& - 2\lambda_3\lambda_4m_1^2 - 2\lambda_4^2m_1^2 - 3\lambda_5^2m_1^2 - 27\lambda_6^2m_1^2 + 3\lambda_7^2m_1^2 - \frac{36}{5}g_1^2\lambda_6m_{12} - 36g_2^2\lambda_6m_{12} \\
& + 36\lambda_1\lambda_6m_{12} + 12\lambda_3\lambda_6m_{12} + 12\lambda_4\lambda_6m_{12} + 12\lambda_5\lambda_6m_{12} + 6\lambda_3\lambda_7m_{12} + 6\lambda_4\lambda_7m_{12} + 6\lambda_5\lambda_7m_{12} + \frac{9}{10}g_1^4m_2^2 \\
& + \frac{15}{2}g_2^4m_2^2 + \frac{24}{5}g_1^2\lambda_3m_2^2 + 24g_2^2\lambda_3m_2^2 - 8\lambda_3^2m_2^2 + \frac{12}{5}g_1^2\lambda_4m_2^2 + 12g_2^2\lambda_4m_2^2 - 8\lambda_3\lambda_4m_2^2 \\
& - 8\lambda_4^2m_2^2 - 12\lambda_5^2m_2^2 - 18\lambda_6^2m_2^2 - 18\lambda_7^2m_2^2 + 6\left(-2(2\lambda_3 + \lambda_4)m_2^2 + 3\lambda_6m_{12}\right)\text{Tr}(\epsilon_d\epsilon_d^\dagger) \\
& - 6\left(-\left(2\lambda_3 + 6\lambda_1 + \lambda_4\right)m_{12} + 3\lambda_6\left(m_1^2 + m_2^2\right)\right)\text{Tr}(\epsilon_d Y_d^\dagger) + 6\lambda_6m_{12}\text{Tr}(\epsilon_e\epsilon_e^\dagger) - 8\lambda_3m_2^2\text{Tr}(\epsilon_e\epsilon_e^\dagger) \\
& - 4\lambda_4m_2^2\text{Tr}(\epsilon_e\epsilon_e^\dagger) - 6\lambda_6m_1^2\text{Tr}(\epsilon_e Y_e^\dagger) + 12\lambda_1m_{12}\text{Tr}(\epsilon_e Y_e^\dagger) + 4\lambda_3m_{12}\text{Tr}(\epsilon_e Y_e^\dagger) \\
& + 2\lambda_4m_{12}\text{Tr}(\epsilon_e Y_e^\dagger) - 6\lambda_6m_2^2\text{Tr}(\epsilon_e Y_e^\dagger) + \frac{17}{4}g_1^2m_1^2\text{Tr}(\epsilon_u\epsilon_u^\dagger) + \frac{45}{4}g_2^2m_1^2\text{Tr}(\epsilon_u\epsilon_u^\dagger)
\end{aligned}$$

$$\begin{aligned}
& + 40g_3^2 m_1^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger) - 72\lambda_1 m_1^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger) + 18\lambda_6 m_{12} \text{Tr}(\epsilon_u \epsilon_u^\dagger) - 18\lambda_6 m_1^2 \text{Tr}(\epsilon_u Y_u^\dagger) \\
& + 36\lambda_1 m_{12} \text{Tr}(\epsilon_u Y_u^\dagger) + 12\lambda_3 m_{12} \text{Tr}(\epsilon_u Y_u^\dagger) + 6\lambda_4 m_{12} \text{Tr}(\epsilon_u Y_u^\dagger) - 18\lambda_6 m_2^2 \text{Tr}(\epsilon_u Y_u^\dagger) \\
& - 18\lambda_6 m_1^2 \text{Tr}(Y_d \epsilon_d^\dagger) - 18\lambda_6 m_2^2 \text{Tr}(Y_d \epsilon_d^\dagger) + \frac{5}{4} g_1^2 m_1^2 \text{Tr}(Y_d Y_d^\dagger) + \frac{45}{4} g_2^2 m_1^2 \text{Tr}(Y_d Y_d^\dagger) \\
& + 40g_3^2 m_1^2 \text{Tr}(Y_d Y_d^\dagger) - 72\lambda_1 m_1^2 \text{Tr}(Y_d Y_d^\dagger) + 18\lambda_6 m_{12} \text{Tr}(Y_d Y_d^\dagger) - 6\lambda_6 m_1^2 \text{Tr}(Y_e \epsilon_e^\dagger) \\
& - 6\lambda_6 m_2^2 \text{Tr}(Y_e \epsilon_e^\dagger) + \frac{15}{4} g_1^2 m_1^2 \text{Tr}(Y_e Y_e^\dagger) + \frac{15}{4} g_2^2 m_1^2 \text{Tr}(Y_e Y_e^\dagger) - 24\lambda_1 m_1^2 \text{Tr}(Y_e Y_e^\dagger) \\
& + 6\lambda_6 m_{12} \text{Tr}(Y_e Y_e^\dagger) - 18\lambda_6 m_1^2 \text{Tr}(Y_u \epsilon_u^\dagger) - 18\lambda_6 m_2^2 \text{Tr}(Y_u \epsilon_u^\dagger) + 18\lambda_6 m_{12} \text{Tr}(Y_u Y_u^\dagger) \\
& - 24\lambda_3 m_2^2 \text{Tr}(Y_u Y_u^\dagger) - 12\lambda_4 m_2^2 \text{Tr}(Y_u Y_u^\dagger) - 9m_1^2 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger) \\
& - \frac{9}{2} m_1^2 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) + 12m_{12} \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_d^\dagger) + 6m_1^2 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) \\
& - 12m_2^2 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) - \frac{9}{2} m_1^2 \text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger) - 3m_1^2 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger) \\
& - \frac{3}{2} m_1^2 \text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger) + 6m_1^2 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) - 12m_2^2 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) \\
& - \frac{27}{2} m_1^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger) - 9m_2^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger) - 21m_1^2 \text{Tr}(\epsilon_u Y_d^\dagger Y_d \epsilon_u^\dagger) \\
& + 12m_{12} \text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger) - \frac{9}{2} m_1^2 \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) - \frac{27}{2} m_1^2 \text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) \\
& - \frac{9}{2} m_1^2 \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) - \frac{9}{2} m_1^2 \text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger)
\end{aligned} \tag{39}$$

$$\begin{aligned}
\beta_{m_2^2}^{(1)} & = +4\lambda_3 m_1^2 + 2\lambda_4 m_1^2 - 6\lambda_7 m_{12} - \frac{9}{10} g_1^2 m_2^2 - \frac{9}{2} g_2^2 m_2^2 + 12\lambda_2 m_2^2 + 6m_2^2 \text{Tr}(\epsilon_d \epsilon_d^\dagger) - 6m_{12} \text{Tr}(\epsilon_d Y_d^\dagger) \\
& + 2m_2^2 \text{Tr}(\epsilon_e \epsilon_e^\dagger) - 2m_{12} \text{Tr}(\epsilon_e Y_e^\dagger) + 6m_2^2 \text{Tr}(Y_u Y_u^\dagger)
\end{aligned} \tag{40}$$

$$\begin{aligned}
\beta_{m_2^2}^{(2)} & = +\frac{9}{10} g_1^4 m_1^2 + \frac{15}{2} g_2^4 m_1^2 + \frac{24}{5} g_1^2 \lambda_3 m_1^2 + 24g_2^2 \lambda_3 m_1^2 - 8\lambda_3^2 m_1^2 + \frac{12}{5} g_1^2 \lambda_4 m_1^2 + 12g_2^2 \lambda_4 m_1^2 \\
& - 8\lambda_3 \lambda_4 m_1^2 - 8\lambda_4^2 m_1^2 - 12\lambda_5^2 m_1^2 - 18\lambda_6^2 m_1^2 - 18\lambda_7^2 m_1^2 + 6\lambda_3 \lambda_6 m_{12} + 6\lambda_4 \lambda_6 m_{12} + 6\lambda_5 \lambda_6 m_{12} \\
& - \frac{36}{5} g_1^2 \lambda_7 m_{12} - 36g_2^2 \lambda_7 m_{12} + 36\lambda_2 \lambda_7 m_{12} + 12\lambda_3 \lambda_7 m_{12} + 12\lambda_4 \lambda_7 m_{12} + 12\lambda_5 \lambda_7 m_{12} + \frac{1737}{400} g_1^4 m_2^2 \\
& + \frac{9}{8} g_1^2 g_2^2 m_2^2 - \frac{123}{16} g_2^4 m_2^2 + \frac{72}{5} g_1^2 \lambda_2 m_2^2 + 72g_2^2 \lambda_2 m_2^2 - 60\lambda_2^2 m_2^2 - 2\lambda_3^2 m_2^2 - 2\lambda_3 \lambda_4 m_2^2 \\
& - 2\lambda_4^2 m_2^2 - 3\lambda_5^2 m_2^2 + 3\lambda_6^2 m_2^2 - 27\lambda_7^2 m_2^2 \\
& + \left(18\lambda_7 m_{12} + \frac{1}{4} (160g_3^2 - 288\lambda_2 + 45g_2^2 + 5g_1^2) m_2^2\right) \text{Tr}(\epsilon_d \epsilon_d^\dagger) \\
& - 6 \left(- (2\lambda_3 + 6\lambda_2 + \lambda_4) m_{12} + 3\lambda_7 (m_1^2 + m_2^2) \right) \text{Tr}(\epsilon_d Y_d^\dagger) + 6\lambda_7 m_{12} \text{Tr}(\epsilon_e \epsilon_e^\dagger) + \frac{15}{4} g_1^2 m_2^2 \text{Tr}(\epsilon_e \epsilon_e^\dagger) \\
& + \frac{15}{4} g_2^2 m_2^2 \text{Tr}(\epsilon_e \epsilon_e^\dagger) - 24\lambda_2 m_2^2 \text{Tr}(\epsilon_e \epsilon_e^\dagger) - 6\lambda_7 m_1^2 \text{Tr}(\epsilon_e Y_e^\dagger) + 12\lambda_2 m_{12} \text{Tr}(\epsilon_e Y_e^\dagger) \\
& + 4\lambda_3 m_{12} \text{Tr}(\epsilon_e Y_e^\dagger) + 2\lambda_4 m_{12} \text{Tr}(\epsilon_e Y_e^\dagger) - 6\lambda_7 m_2^2 \text{Tr}(\epsilon_e Y_e^\dagger) - 24\lambda_3 m_1^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger)
\end{aligned}$$

$$\begin{aligned}
& -12\lambda_4 m_1^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger) + 18\lambda_7 m_{12} \text{Tr}(\epsilon_u \epsilon_u^\dagger) - 18\lambda_7 m_1^2 \text{Tr}(\epsilon_u Y_u^\dagger) + 36\lambda_2 m_{12} \text{Tr}(\epsilon_u Y_u^\dagger) \\
& + 12\lambda_3 m_{12} \text{Tr}(\epsilon_u Y_u^\dagger) + 6\lambda_4 m_{12} \text{Tr}(\epsilon_u Y_u^\dagger) - 18\lambda_7 m_2^2 \text{Tr}(\epsilon_u Y_u^\dagger) - 18\lambda_7 m_1^2 \text{Tr}(Y_d \epsilon_d^\dagger) \\
& - 18\lambda_7 m_2^2 \text{Tr}(Y_d \epsilon_d^\dagger) - 24\lambda_3 m_1^2 \text{Tr}(Y_d Y_d^\dagger) - 12\lambda_4 m_1^2 \text{Tr}(Y_d Y_d^\dagger) + 18\lambda_7 m_{12} \text{Tr}(Y_d Y_d^\dagger) \\
& - 6\lambda_7 m_1^2 \text{Tr}(Y_e \epsilon_e^\dagger) - 6\lambda_7 m_2^2 \text{Tr}(Y_e \epsilon_e^\dagger) - 8\lambda_3 m_1^2 \text{Tr}(Y_e Y_e^\dagger) - 4\lambda_4 m_1^2 \text{Tr}(Y_e Y_e^\dagger) \\
& + 6\lambda_7 m_{12} \text{Tr}(Y_e Y_e^\dagger) - 18\lambda_7 m_1^2 \text{Tr}(Y_u \epsilon_u^\dagger) - 18\lambda_7 m_2^2 \text{Tr}(Y_u \epsilon_u^\dagger) + 18\lambda_7 m_{12} \text{Tr}(Y_u Y_u^\dagger) \\
& + \frac{17}{4} g_1^2 m_2^2 \text{Tr}(Y_u Y_u^\dagger) + \frac{45}{4} g_2^2 m_2^2 \text{Tr}(Y_u Y_u^\dagger) + 40 g_3^2 m_2^2 \text{Tr}(Y_u Y_u^\dagger) - 72\lambda_2 m_2^2 \text{Tr}(Y_u Y_u^\dagger) \\
& - \frac{27}{2} m_2^2 \text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger) - 9m_2^2 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger) - \frac{9}{2} m_2^2 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) \\
& - 12m_1^2 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) + 6m_2^2 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) - \frac{9}{2} m_2^2 \text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger) \\
& + 12m_{12} \text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) - 21m_2^2 \text{Tr}(\epsilon_d Y_u^\dagger Y_u \epsilon_d^\dagger) + 12m_{12} \text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger) \\
& - \frac{9}{2} m_2^2 \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger) - 3m_2^2 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger) - \frac{3}{2} m_2^2 \text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger) \\
& - 12m_1^2 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) + 6m_2^2 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) - 9m_2^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger) \\
& - \frac{9}{2} m_2^2 \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) - \frac{9}{2} m_2^2 \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) - \frac{27}{2} m_2^2 \text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger)
\end{aligned} \tag{41}$$

3.5 Vacuum expectation values

$$\begin{aligned}
\beta_{v_1}^{(1)} &= \frac{1}{20} \left(9g_1^2 v_1 + 45g_2^2 v_1 + 3g_1^2 v_1 \text{Xi} + 15g_2^2 v_1 \text{Xi} - 30v_2 \text{Tr}(\epsilon_d Y_d^\dagger) - 10v_2 \text{Tr}(\epsilon_e Y_e^\dagger) \right. \\
& - 60v_1 \text{Tr}(\epsilon_u \epsilon_u^\dagger) - 30v_2 \text{Tr}(\epsilon_u Y_u^\dagger) - 30v_2 \text{Tr}(Y_d \epsilon_d^\dagger) - 60v_1 \text{Tr}(Y_d Y_d^\dagger) - 10v_2 \text{Tr}(Y_e \epsilon_e^\dagger) \\
& \left. - 20v_1 \text{Tr}(Y_e Y_e^\dagger) - 30v_2 \text{Tr}(Y_u \epsilon_u^\dagger) \right) \\
\beta_{v_1}^{(2)} &= -\frac{1359}{800} g_1^4 v_1 - \frac{27}{80} g_1^2 g_2^2 v_1 + \frac{249}{32} g_2^4 v_1 - 6\lambda_1^2 v_1 - \lambda_3^2 v_1 - \lambda_3 \lambda_4 v_1 - \lambda_4^2 v_1 - \frac{3}{2} \lambda_5^2 v_1 \\
& - \frac{9}{2} \lambda_6^2 v_1 - \frac{3}{2} \lambda_7^2 v_1 - 3\lambda_1 \lambda_6 v_2 - \frac{3}{2} \lambda_3 \lambda_6 v_2 - \frac{3}{2} \lambda_4 \lambda_6 v_2 - \frac{3}{2} \lambda_5 \lambda_6 v_2 - 3\lambda_2 \lambda_7 v_2 - \frac{3}{2} \lambda_3 \lambda_7 v_2 \\
& - \frac{3}{2} \lambda_4 \lambda_7 v_2 - \frac{3}{2} \lambda_5 \lambda_7 v_2 + \frac{9}{400} g_1^4 v_1 \text{Xi} + \frac{9}{40} g_1^2 g_2^2 v_1 \text{Xi} + \frac{45}{16} g_2^4 v_1 \text{Xi} + \frac{9}{400} g_1^4 v_1 \text{Xi}^2 + \frac{9}{40} g_1^2 g_2^2 v_1 \text{Xi}^2 \\
& - \frac{9}{16} g_2^4 v_1 \text{Xi}^2 - \frac{1}{80} \left(45g_2^2 (2 - 2iv_1 \text{Xi} + 5v_2) + 800g_3^2 v_2 + g_1^2 (18 - 18iv_1 \text{Xi} + 25v_2) \right) \text{Tr}(\epsilon_d Y_d^\dagger) \\
& - \frac{3}{80} \left(5g_2^2 (2 - 2iv_1 \text{Xi} + 5v_2) + g_1^2 (2 - 2iv_1 \text{Xi} + 25v_2) \right) \text{Tr}(\epsilon_e Y_e^\dagger) - \frac{17}{8} g_1^2 v_1 \text{Tr}(\epsilon_u \epsilon_u^\dagger) - \frac{45}{8} g_2^2 v_1 \text{Tr}(\epsilon_u \epsilon_u^\dagger) \\
& - 20g_3^2 v_1 \text{Tr}(\epsilon_u \epsilon_u^\dagger) - \frac{9}{20} g_1^2 v_1 \text{Xi} \text{Tr}(\epsilon_u \epsilon_u^\dagger) - \frac{9}{4} g_2^2 v_1 \text{Xi} \text{Tr}(\epsilon_u \epsilon_u^\dagger) - \frac{17}{16} g_1^2 v_2 \text{Tr}(\epsilon_u Y_u^\dagger)
\end{aligned} \tag{42}$$

$$\begin{aligned}
& -\frac{45}{16}g_2^2v_2\text{Tr}(\epsilon_u Y_u^\dagger) - 10g_3^2v_2\text{Tr}(\epsilon_u Y_u^\dagger) - \frac{9}{40} + \frac{9i}{40}g_1^2v_1\text{XiTr}(\epsilon_u Y_u^\dagger) - \frac{9}{8} + \frac{9i}{8}g_2^2v_1\text{XiTr}(\epsilon_u Y_u^\dagger) \\
& -\frac{5}{16}g_1^2v_2\text{Tr}(Y_d\epsilon_d^\dagger) - \frac{45}{16}g_2^2v_2\text{Tr}(Y_d\epsilon_d^\dagger) - 10g_3^2v_2\text{Tr}(Y_d\epsilon_d^\dagger) - \frac{9}{40} - \frac{9i}{40}g_1^2v_1\text{XiTr}(Y_d\epsilon_d^\dagger) \\
& -\frac{9}{8} - \frac{9i}{8}g_2^2v_1\text{XiTr}(Y_d\epsilon_d^\dagger) - \frac{5}{8}g_1^2v_1\text{Tr}(Y_d Y_d^\dagger) - \frac{45}{8}g_2^2v_1\text{Tr}(Y_d Y_d^\dagger) - 20g_3^2v_1\text{Tr}(Y_d Y_d^\dagger) \\
& -\frac{9}{20}g_1^2v_1\text{XiTr}(Y_d Y_d^\dagger) - \frac{9}{4}g_2^2v_1\text{XiTr}(Y_d Y_d^\dagger) - \frac{15}{16}g_1^2v_2\text{Tr}(Y_e\epsilon_e^\dagger) - \frac{15}{16}g_2^2v_2\text{Tr}(Y_e\epsilon_e^\dagger) \\
& -\frac{3}{40} - \frac{3i}{40}g_1^2v_1\text{XiTr}(Y_e\epsilon_e^\dagger) - \frac{3}{8} - \frac{3i}{8}g_2^2v_1\text{XiTr}(Y_e\epsilon_e^\dagger) - \frac{15}{8}g_1^2v_1\text{Tr}(Y_e Y_e^\dagger) - \frac{15}{8}g_2^2v_1\text{Tr}(Y_e Y_e^\dagger) \\
& -\frac{3}{20}g_1^2v_1\text{XiTr}(Y_e Y_e^\dagger) - \frac{3}{4}g_2^2v_1\text{XiTr}(Y_e Y_e^\dagger) - \frac{17}{16}g_1^2v_2\text{Tr}(Y_u\epsilon_u^\dagger) - \frac{45}{16}g_2^2v_2\text{Tr}(Y_u\epsilon_u^\dagger) \\
& -10g_3^2v_2\text{Tr}(Y_u\epsilon_u^\dagger) - \frac{9}{40} - \frac{9i}{40}g_1^2v_1\text{XiTr}(Y_u\epsilon_u^\dagger) - \frac{9}{8} - \frac{9i}{8}g_2^2v_1\text{XiTr}(Y_u\epsilon_u^\dagger) + \frac{9}{8}v_2\text{Tr}(\epsilon_d\epsilon_d^\dagger\epsilon_d Y_d^\dagger) \\
& +\frac{27}{8}v_2\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d\epsilon_d^\dagger) + \frac{9}{2}v_1\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d Y_d^\dagger) + \frac{9}{4}v_1\text{Tr}(\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger) \\
& -\frac{3}{8}v_2\text{Tr}(\epsilon_d\epsilon_u^\dagger\epsilon_u Y_d^\dagger) - \frac{3}{8}v_2\text{Tr}(\epsilon_d\epsilon_u^\dagger Y_u\epsilon_d^\dagger) - 3v_1\text{Tr}(\epsilon_d\epsilon_u^\dagger Y_u Y_d^\dagger) \\
& +\frac{9}{4}v_2\text{Tr}(\epsilon_d Y_d^\dagger\epsilon_d\epsilon_d^\dagger) + \frac{9}{4}v_1\text{Tr}(\epsilon_d Y_d^\dagger Y_d\epsilon_d^\dagger) + \frac{27}{8}v_2\text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger) \\
& -\frac{3}{8}v_2\text{Tr}(\epsilon_d Y_u^\dagger\epsilon_u\epsilon_d^\dagger) - \frac{3}{8}v_2\text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger) + \frac{3}{8}v_2\text{Tr}(\epsilon_e\epsilon_e^\dagger\epsilon_e Y_e^\dagger) \\
& +\frac{9}{8}v_2\text{Tr}(\epsilon_e\epsilon_e^\dagger Y_e\epsilon_e^\dagger) + \frac{3}{2}v_1\text{Tr}(\epsilon_e\epsilon_e^\dagger Y_e Y_e^\dagger) + \frac{3}{4}v_2\text{Tr}(\epsilon_e Y_e^\dagger\epsilon_e\epsilon_e^\dagger) \\
& +\frac{3}{4}v_1\text{Tr}(\epsilon_e Y_e^\dagger Y_e\epsilon_e^\dagger) + \frac{9}{8}v_2\text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger) - \frac{3}{8}v_2\text{Tr}(\epsilon_u\epsilon_d^\dagger Y_d\epsilon_u^\dagger) \\
& -3v_1\text{Tr}(\epsilon_u\epsilon_d^\dagger Y_d Y_u^\dagger) + \frac{27}{4}v_1\text{Tr}(\epsilon_u\epsilon_u^\dagger\epsilon_u\epsilon_u^\dagger) + \frac{27}{8}v_2\text{Tr}(\epsilon_u\epsilon_u^\dagger Y_u\epsilon_u^\dagger) + \frac{9}{2}v_1\text{Tr}(\epsilon_u\epsilon_u^\dagger Y_u Y_u^\dagger) \\
& -\frac{3}{2}v_1\text{Tr}(\epsilon_u Y_d^\dagger Y_d\epsilon_u^\dagger) - \frac{3}{8}v_2\text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger) + \frac{27}{8}v_2\text{Tr}(\epsilon_u Y_u^\dagger\epsilon_u\epsilon_u^\dagger) \\
& +\frac{9}{4}v_1\text{Tr}(\epsilon_u Y_u^\dagger Y_u\epsilon_u^\dagger) + \frac{27}{8}v_2\text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger) + \frac{27}{8}v_2\text{Tr}(Y_d\epsilon_d^\dagger Y_d Y_d^\dagger) \\
& -\frac{3}{8}v_2\text{Tr}(Y_d\epsilon_u^\dagger Y_u Y_d^\dagger) + \frac{27}{4}v_1\text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) - \frac{3}{8}v_2\text{Tr}(Y_d Y_u^\dagger Y_u\epsilon_d^\dagger) \\
& +\frac{9}{4}v_1\text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) + \frac{9}{8}v_2\text{Tr}(Y_e\epsilon_e^\dagger Y_e Y_e^\dagger) + \frac{9}{4}v_1\text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) \\
& +\frac{9}{4}v_2\text{Tr}(Y_u\epsilon_u^\dagger Y_u Y_u^\dagger) + \frac{9}{8}v_2\text{Tr}(Y_u Y_u^\dagger Y_u\epsilon_u^\dagger)
\end{aligned} \tag{43}$$

$$\begin{aligned}
\beta_{v_2}^{(1)} &= \frac{1}{20}\left(9g_1^2v_2 + 45g_2^2v_2 + 3g_1^2v_2\text{Xi} + 15g_2^2v_2\text{Xi} - 60v_2\text{Tr}(\epsilon_d\epsilon_d^\dagger) - 30v_1\text{Tr}(\epsilon_d Y_d^\dagger)\right) \\
& -20v_2\text{Tr}(\epsilon_e\epsilon_e^\dagger) - 10v_1\text{Tr}(\epsilon_e Y_e^\dagger) - 30v_1\text{Tr}(\epsilon_u Y_u^\dagger) - 30v_1\text{Tr}(Y_d\epsilon_d^\dagger) - 10v_1\text{Tr}(Y_e\epsilon_e^\dagger) \\
& -30v_1\text{Tr}(Y_u\epsilon_u^\dagger) - 60v_2\text{Tr}(Y_u Y_u^\dagger)
\end{aligned} \tag{44}$$

$$\beta_{v_2}^{(2)} = -3\lambda_1\lambda_6v_1 - \frac{3}{2}\lambda_3\lambda_6v_1 - \frac{3}{2}\lambda_4\lambda_6v_1 - \frac{3}{2}\lambda_5\lambda_6v_1 - 3\lambda_2\lambda_7v_1 - \frac{3}{2}\lambda_3\lambda_7v_1 - \frac{3}{2}\lambda_4\lambda_7v_1 - \frac{3}{2}\lambda_5\lambda_7v_1$$

$$\begin{aligned}
& -\frac{1359}{800}g_1^4v_2 - \frac{27}{80}g_1^2g_2^2v_2 + \frac{249}{32}g_2^4v_2 - 6\lambda_2^2v_2 - \lambda_3^2v_2 - \lambda_3\lambda_4v_2 - \lambda_4^2v_2 - \frac{3}{2}\lambda_5^2v_2 \\
& -\frac{3}{2}\lambda_6^2v_2 - \frac{9}{2}\lambda_7^2v_2 + \frac{9}{400}g_1^4v_2\text{Xi} + \frac{9}{40}g_1^2g_2^2v_2\text{Xi} + \frac{45}{16}g_2^4v_2\text{Xi} + \frac{9}{400}g_1^4v_2\text{Xi}^2 + \frac{9}{40}g_1^2g_2^2v_2\text{Xi}^2 \\
& -\frac{9}{16}g_2^4v_2\text{Xi}^2 - \frac{1}{40}v_2\left(45g_2^2(2\text{Xi} + 5) + 800g_3^2 + g_1^2(18\text{Xi} + 25)\right)\text{Tr}\left(\epsilon_d\epsilon_d^\dagger\right) \\
& -\frac{1}{80}\left(45g_2^2(2 + 2iv_2\text{Xi} + 5v_1) + 800g_3^2v_1 + g_1^2(18 + 18iv_2\text{Xi} + 25v_1)\right)\text{Tr}\left(\epsilon_dY_d^\dagger\right) - \frac{15}{8}g_1^2v_2\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) \\
& -\frac{15}{8}g_2^2v_2\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - \frac{3}{20}g_1^2v_2\text{Xi}\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - \frac{3}{4}g_2^2v_2\text{Xi}\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - \frac{15}{16}g_1^2v_1\text{Tr}\left(\epsilon_eY_e^\dagger\right) \\
& -\frac{15}{16}g_2^2v_1\text{Tr}\left(\epsilon_eY_e^\dagger\right) - \frac{3}{40} - \frac{3i}{40}g_1^2v_2\text{Xi}\text{Tr}\left(\epsilon_eY_e^\dagger\right) - \frac{3}{8} - \frac{3i}{8}g_2^2v_2\text{Xi}\text{Tr}\left(\epsilon_eY_e^\dagger\right) - \frac{17}{16}g_1^2v_1\text{Tr}\left(\epsilon_uY_u^\dagger\right) \\
& -\frac{45}{16}g_2^2v_1\text{Tr}\left(\epsilon_uY_u^\dagger\right) - 10g_3^2v_1\text{Tr}\left(\epsilon_uY_u^\dagger\right) - \frac{9}{40} - \frac{9i}{40}g_1^2v_2\text{Xi}\text{Tr}\left(\epsilon_uY_u^\dagger\right) - \frac{9}{8} - \frac{9i}{8}g_2^2v_2\text{Xi}\text{Tr}\left(\epsilon_uY_u^\dagger\right) \\
& -\frac{5}{16}g_1^2v_1\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - \frac{45}{16}g_2^2v_1\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - 10g_3^2v_1\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - \frac{9}{40} + \frac{9i}{40}g_1^2v_2\text{Xi}\text{Tr}\left(Y_d\epsilon_d^\dagger\right) \\
& -\frac{9}{8} + \frac{9i}{8}g_2^2v_2\text{Xi}\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - \frac{15}{16}g_1^2v_1\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - \frac{15}{16}g_2^2v_1\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - \frac{3}{40} + \frac{3i}{40}g_1^2v_2\text{Xi}\text{Tr}\left(Y_e\epsilon_e^\dagger\right) \\
& -\frac{3}{8} + \frac{3i}{8}g_2^2v_2\text{Xi}\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - \frac{17}{16}g_1^2v_1\text{Tr}\left(Y_u\epsilon_u^\dagger\right) - \frac{45}{16}g_2^2v_1\text{Tr}\left(Y_u\epsilon_u^\dagger\right) - 10g_3^2v_1\text{Tr}\left(Y_u\epsilon_u^\dagger\right) \\
& -\frac{9}{40} + \frac{9i}{40}g_1^2v_2\text{Xi}\text{Tr}\left(Y_u\epsilon_u^\dagger\right) - \frac{9}{8} + \frac{9i}{8}g_2^2v_2\text{Xi}\text{Tr}\left(Y_u\epsilon_u^\dagger\right) - \frac{17}{8}g_1^2v_2\text{Tr}\left(Y_uY_u^\dagger\right) - \frac{45}{8}g_2^2v_2\text{Tr}\left(Y_uY_u^\dagger\right) \\
& -20g_3^2v_2\text{Tr}\left(Y_uY_u^\dagger\right) - \frac{9}{20}g_1^2v_2\text{Xi}\text{Tr}\left(Y_uY_u^\dagger\right) - \frac{9}{4}g_2^2v_2\text{Xi}\text{Tr}\left(Y_uY_u^\dagger\right) + \frac{27}{4}v_2\text{Tr}\left(\epsilon_d\epsilon_d^\dagger\epsilon_d\epsilon_d^\dagger\right) \\
& +\frac{27}{8}v_1\text{Tr}\left(\epsilon_d\epsilon_d^\dagger Y_d\epsilon_d^\dagger\right) + \frac{9}{2}v_2\text{Tr}\left(\epsilon_d\epsilon_d^\dagger Y_d Y_d^\dagger\right) + \frac{9}{4}v_2\text{Tr}\left(\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger\right) \\
& -\frac{3}{8}v_1\text{Tr}\left(\epsilon_d\epsilon_u^\dagger\epsilon_u Y_d^\dagger\right) - \frac{3}{8}v_1\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_u\epsilon_d^\dagger\right) - 3v_2\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_u Y_d^\dagger\right) \\
& +\frac{27}{8}v_1\text{Tr}\left(\epsilon_d Y_d^\dagger\epsilon_d\epsilon_d^\dagger\right) + \frac{9}{4}v_2\text{Tr}\left(\epsilon_d Y_d^\dagger Y_d\epsilon_d^\dagger\right) + \frac{27}{8}v_1\text{Tr}\left(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger\right) \\
& -\frac{3}{8}v_1\text{Tr}\left(\epsilon_d Y_u^\dagger\epsilon_u\epsilon_d^\dagger\right) - \frac{3}{2}v_2\text{Tr}\left(\epsilon_d Y_u^\dagger Y_u\epsilon_d^\dagger\right) - \frac{3}{8}v_1\text{Tr}\left(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger\right) \\
& +\frac{9}{4}v_2\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\epsilon_e\epsilon_e^\dagger\right) + \frac{9}{8}v_1\text{Tr}\left(\epsilon_e\epsilon_e^\dagger Y_e\epsilon_e^\dagger\right) + \frac{3}{2}v_2\text{Tr}\left(\epsilon_e\epsilon_e^\dagger Y_e Y_e^\dagger\right) + \frac{9}{8}v_1\text{Tr}\left(\epsilon_e Y_e^\dagger\epsilon_e\epsilon_e^\dagger\right) \\
& +\frac{3}{4}v_2\text{Tr}\left(\epsilon_e Y_e^\dagger Y_e\epsilon_e^\dagger\right) + \frac{9}{8}v_1\text{Tr}\left(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger\right) - \frac{3}{8}v_1\text{Tr}\left(\epsilon_u\epsilon_d^\dagger Y_d\epsilon_u^\dagger\right) \\
& -3v_2\text{Tr}\left(\epsilon_u\epsilon_d^\dagger Y_d Y_u^\dagger\right) + \frac{9}{8}v_1\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\epsilon_u Y_u^\dagger\right) + \frac{27}{8}v_1\text{Tr}\left(\epsilon_u\epsilon_u^\dagger Y_u\epsilon_u^\dagger\right) \\
& +\frac{9}{2}v_2\text{Tr}\left(\epsilon_u\epsilon_u^\dagger Y_u Y_u^\dagger\right) - \frac{3}{8}v_1\text{Tr}\left(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger\right) + \frac{9}{4}v_1\text{Tr}\left(\epsilon_u Y_u^\dagger\epsilon_u\epsilon_u^\dagger\right) \\
& +\frac{9}{4}v_2\text{Tr}\left(\epsilon_u Y_u^\dagger Y_u\epsilon_u^\dagger\right) + \frac{27}{8}v_1\text{Tr}\left(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger\right) + \frac{9}{4}v_1\text{Tr}\left(Y_d\epsilon_d^\dagger Y_d Y_d^\dagger\right) \\
& -\frac{3}{8}v_1\text{Tr}\left(Y_d\epsilon_u^\dagger Y_u Y_d^\dagger\right) + \frac{9}{8}v_1\text{Tr}\left(Y_d Y_d^\dagger Y_d\epsilon_d^\dagger\right) - \frac{3}{8}v_1\text{Tr}\left(Y_d Y_u^\dagger Y_u\epsilon_d^\dagger\right) \\
& +\frac{9}{4}v_2\text{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger\right) + \frac{3}{4}v_1\text{Tr}\left(Y_e\epsilon_e^\dagger Y_e Y_e^\dagger\right) + \frac{3}{8}v_1\text{Tr}\left(Y_e Y_e^\dagger Y_e\epsilon_e^\dagger\right)
\end{aligned}$$

$$+ \frac{27}{8} v_1 \text{Tr} \left(Y_u \epsilon_u^\dagger Y_u Y_u^\dagger \right) + \frac{27}{4} v_2 \text{Tr} \left(Y_u Y_u^\dagger Y_u Y_u^\dagger \right) \quad (45)$$

4 Field Rotations

4.1 Rotations in gauge sector for eigenstates 'EWSB'

$$\begin{pmatrix} B_\rho \\ W_{3\rho} \end{pmatrix} = Z^{\gamma Z} \begin{pmatrix} \gamma_\rho \\ Z_\rho \end{pmatrix} \quad (46)$$

$$\begin{pmatrix} W_{1\rho} \\ W_{2\rho} \end{pmatrix} = Z^W \begin{pmatrix} W_\rho^- \\ W_\rho^- \end{pmatrix} \quad (47)$$

$$(48)$$

The mixing matrices are parametrized by

$$Z^{\gamma Z} = \begin{pmatrix} \cos \Theta_W & -\sin \Theta_W \\ \sin \Theta_W & \cos \Theta_W \end{pmatrix} \quad (49)$$

$$Z^W = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -i\frac{1}{\sqrt{2}} & i\frac{1}{\sqrt{2}} \end{pmatrix} \quad (50)$$

$$(51)$$

4.2 Rotations in Mass sector for eigenstates 'EWSB'

4.2.1 Mass Matrices for Scalars

- Mass matrix for Higgs, Basis: $(\phi_1, \phi_2, \sigma_1, \sigma_2), (\phi_1, \phi_2, \sigma_1, \sigma_2)$

$$m_h^2 = \begin{pmatrix} m_{\phi_1\phi_1} & m_{\phi_2\phi_1} & m_{\sigma_1\phi_1} & m_{\sigma_2\phi_1} \\ m_{\phi_1\phi_2} & m_{\phi_2\phi_2} & m_{\sigma_1\phi_2} & m_{\sigma_2\phi_2} \\ m_{\phi_1\sigma_1} & m_{\phi_2\sigma_1} & m_{\sigma_1\sigma_1} & m_{\sigma_2\sigma_1} \\ m_{\phi_1\sigma_2} & m_{\phi_2\sigma_2} & m_{\sigma_1\sigma_2} & m_{\sigma_2\sigma_2} \end{pmatrix} + \xi_Z m^2(Z) \quad (52)$$

$$m_{\phi_1\phi_1} = \frac{1}{4} \left(12\lambda_1 v_1^2 + e^{-2i\eta} v_2^2 \lambda_5^* + v_2 \left(\left(2(\lambda_3 + \lambda_4) + e^{2i\eta} \lambda_5 \right) v_2 + 6v_1 \left(e^{-i\eta} \lambda_6 + e^{i\eta} \lambda_6^* \right) \right) \right) + m_1^2 \quad (53)$$

$$m_{\phi_1\phi_2} = \frac{1}{4} e^{-2i\eta} \left(2v_1 v_2 \lambda_5^* + e^{i\eta} \left(-2m_{12} + 3\lambda_6 v_1^2 + e^{2i\eta} \left(-2m_{12}^* + 3 \left(v_1^2 \lambda_6^* + v_2^2 \lambda_7^* \right) \right) + v_2 \left(\left(2e^{3i\eta} \lambda_5 + 4e^{i\eta} (\lambda_3 + \lambda_4) \right) v_1 + 3\lambda_7 v_2 \right) \right) \right) \quad (54)$$

$$m_{\phi_2\phi_2} = \frac{1}{4} \left(12\lambda_2 v_2^2 + 6v_1 v_2 \left(e^{-i\eta} \lambda_7 + e^{i\eta} \lambda_7^* \right) + v_1^2 \left(2 \left(\lambda_3 + \lambda_4 \right) + e^{-2i\eta} \lambda_5^* + e^{2i\eta} \lambda_5 \right) \right) + m_2^2 \quad (55)$$

$$m_{\phi_1\sigma_1} = \frac{i}{4} e^{-2i\eta} v_2 \left(v_1 \left(2e^{3i\eta} \lambda_6^* - 2e^{i\eta} \lambda_6 \right) + v_2 \left(e^{4i\eta} \lambda_5 - \lambda_5^* \right) \right) \quad (56)$$

$$m_{\phi_2\sigma_1} = \frac{i}{4} e^{-2i\eta} \left(-2v_1 v_2 \lambda_5^* + e^{i\eta} \left(2 \left(e^{3i\eta} \lambda_5 v_1 v_2 + m_{12} \right) - 3\lambda_7 v_2^2 + e^{2i\eta} \left(-2m_{12}^* + 3v_2^2 \lambda_7^* + v_1^2 \lambda_6^* \right) - \lambda_6 v_1^2 \right) \right) \quad (57)$$

$$m_{\sigma_1\sigma_1} = \frac{1}{4} \left(4\lambda_1 v_1^2 - e^{-2i\eta} v_2^2 \lambda_5^* + v_2 \left(2 \left(\left(\lambda_3 + \lambda_4 \right) v_2 + v_1 \left(e^{-i\eta} \lambda_6 + e^{i\eta} \lambda_6^* \right) \right) - e^{2i\eta} \lambda_5 v_2 \right) \right) + m_1^2 \quad (58)$$

$$m_{\phi_1\sigma_2} = -\frac{i}{4} e^{-2i\eta} \left(-2v_1 v_2 \lambda_5^* + e^{i\eta} \left(2 \left(e^{3i\eta} \lambda_5 v_1 v_2 + m_{12} \right) - 3\lambda_6 v_1^2 + e^{2i\eta} \left(-2m_{12}^* + 3v_1^2 \lambda_6^* + v_2^2 \lambda_7^* \right) - \lambda_7 v_2^2 \right) \right) \quad (59)$$

$$m_{\phi_2\sigma_2} = -\frac{i}{4} e^{-2i\eta} v_1 \left(v_1 \left(e^{4i\eta} \lambda_5 - \lambda_5^* \right) + v_2 \left(2e^{3i\eta} \lambda_7^* - 2e^{i\eta} \lambda_7 \right) \right) \quad (60)$$

$$m_{\sigma_1\sigma_2} = \frac{1}{4} e^{-2i\eta} \left(2v_1 v_2 \lambda_5^* + e^{i\eta} \left(2e^{3i\eta} \lambda_5 v_1 v_2 - 2m_{12} + e^{2i\eta} \left(-2m_{12}^* + v_1^2 \lambda_6^* + v_2^2 \lambda_7^* \right) + \lambda_6 v_1^2 + \lambda_7 v_2^2 \right) \right) \quad (61)$$

$$m_{\sigma_2\sigma_2} = +m_2^2 + \frac{1}{4} \left(2e^{-i\eta} \lambda_7 v_1 v_2 + 2e^{i\eta} v_1 v_2 \lambda_7^* + 2 \left(\lambda_3 v_1^2 + \lambda_4 v_1^2 \right) + 4\lambda_2 v_2^2 + v_1^2 \left(-e^{-2i\eta} \lambda_5^* - e^{2i\eta} \lambda_5 \right) \right) \quad (62)$$

Gauge fixing contributions:

$$m^2(\xi_Z) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & m_{\sigma_1\sigma_1} & m_{\sigma_2\sigma_1} \\ 0 & 0 & m_{\sigma_1\sigma_2} & m_{\sigma_2\sigma_2} \end{pmatrix} \quad (63)$$

$$m_{\sigma_1\sigma_1} = \frac{1}{4} v_1^2 \left(g_1 \sin \Theta_W + g_2 \cos \Theta_W \right)^2 \quad (64)$$

$$m_{\sigma_1\sigma_2} = \frac{1}{4} v_1 v_2 \left(g_1 \sin \Theta_W + g_2 \cos \Theta_W \right)^2 \quad (65)$$

$$m_{\sigma_2\sigma_2} = \frac{1}{4} v_2^2 \left(g_1 \sin \Theta_W + g_2 \cos \Theta_W \right)^2 \quad (66)$$

This matrix is diagonalized by Z^H :

$$Z^H m_h^2 Z^{H,\dagger} = m_{2,h}^{dia} \quad (67)$$

with

$$\phi_1 = \sum_j Z_{j1}^H h_j, \quad \phi_2 = \sum_j Z_{j2}^H h_j, \quad \sigma_1 = \sum_j Z_{j3}^H h_j \quad (68)$$

$$\sigma_2 = \sum_j Z_{j4}^H h_j \quad (69)$$

- **Mass matrix for Charged Higgs**, Basis: $(H_1^{+,*}, H_2^{+,*}), (H_1^+, H_2^+)$

$$m_{H^-}^2 = \begin{pmatrix} m_{H_1^+,*H_1^+} & m_{H_2^+,*H_1^+}^* \\ m_{H_1^+,*H_2^+} & m_{H_2^+,*H_2^+} \end{pmatrix} + \xi_{W^-} m^2(W^-) \quad (70)$$

$$m_{H_1^+,*H_1^+} = \frac{1}{2} \left(2\lambda_1 v_1^2 + v_2 \left(\lambda_3 v_2 + v_1 \left(e^{-i\eta} \lambda_6 + e^{i\eta} \lambda_6^* \right) \right) \right) + m_1^2 \quad (71)$$

$$m_{H_1^+,*H_2^+} = \frac{1}{2} e^{-2i\eta} \left(e^{i\eta} \left(-2m_{12} + e^{i\eta} \lambda_4 v_1 v_2 + \lambda_6 v_1^2 + \lambda_7 v_2^2 \right) + v_1 v_2 \lambda_5^* \right) \quad (72)$$

$$m_{H_2^+,*H_2^+} = \frac{1}{2} \left(\lambda_3 v_1^2 + v_2 \left(2\lambda_2 v_2 + v_1 \left(e^{-i\eta} \lambda_7 + e^{i\eta} \lambda_7^* \right) \right) \right) + m_2^2 \quad (73)$$

Gauge fixing contributions:

$$m^2(\xi_{W^-}) = \begin{pmatrix} \frac{1}{4} g_2^2 v_1^2 & \frac{1}{4} g_2^2 v_1 v_2 \\ \frac{1}{4} g_2^2 v_1 v_2 & \frac{1}{4} g_2^2 v_2^2 \end{pmatrix} \quad (74)$$

This matrix is diagonalized by Z^+ :

$$Z^+ m_{H^-}^2 Z^{+,\dagger} = m_{2,H^-}^{dia} \quad (75)$$

with

$$H_1^+ = \sum_j Z_{j1}^+ H_j^+, \quad H_2^+ = \sum_j Z_{j2}^+ H_j^+ \quad (76)$$

4.2.2 Mass Matrices for Fermions

- **Mass matrix for Down-Quarks**, Basis: $(d_{L,\alpha_1}), (d_{R,\beta_1}^*)$

$$m_d = \left(\frac{1}{\sqrt{2}} e^{-i\eta} \delta_{\alpha_1 \beta_1} \left(e^{i\eta} v_2 \epsilon_d^T + v_1 Y_d^T \right) \right) \quad (77)$$

This matrix is diagonalized by U_L^d and U_R^d

$$U_L^{d,*} m_d U_R^{d,\dagger} = m_d^{dia} \quad (78)$$

with

$$d_{L,i\alpha} = \sum_{t_2} U_{L,ji}^{d,*} D_{L,j\alpha} \quad (79)$$

$$d_{R,i\alpha} = \sum_{t_2} U_{R,ij}^d D_{R,j\alpha}^* \quad (80)$$

- **Mass matrix for Up-Quarks**, Basis: $(u_{L,\alpha_1}), (u_{R,\beta_1}^*)$

$$m_u = \left(-\frac{1}{\sqrt{2}} \delta_{\alpha_1 \beta_1} \left(e^{i\eta} v_1 \epsilon_u^T + v_2 Y_u^T \right) \right) \quad (81)$$

This matrix is diagonalized by U_L^u and U_R^u

$$U_L^{u,*} m_u U_R^{u,\dagger} = m_u^{dia} \quad (82)$$

with

$$u_{L,i\alpha} = \sum_{t_2} U_{L,ji}^{u,*} U_{L,j\alpha} \quad (83)$$

$$u_{R,i\alpha} = \sum_{t_2} U_{R,ij}^u U_{R,j\alpha}^* \quad (84)$$

- **Mass matrix for Leptons**, Basis: $(e_L), (e_R^*)$

$$m_e = \left(\frac{1}{\sqrt{2}} \left(e^{-i\eta} v_1 Y_e^T + v_2 \epsilon_e^T \right) \right) \quad (85)$$

This matrix is diagonalized by U_L^e and U_R^e

$$U_L^{e,*} m_e U_R^{e,\dagger} = m_e^{dia} \quad (86)$$

with

$$e_{L,i} = \sum_{t_2} U_{L,ji}^{e,*} E_{L,j} \quad (87)$$

$$e_{R,i} = \sum_{t_2} U_{R,ij}^e E_{R,j}^* \quad (88)$$

5 Vacuum Expectation Values

$$H_1^0 = \frac{1}{\sqrt{2}} \phi_1 + \frac{1}{\sqrt{2}} v_1 + i \frac{1}{\sqrt{2}} \sigma_1 \quad (89)$$

$$H_2^0 = \frac{1}{\sqrt{2}} \phi_2 + \frac{1}{\sqrt{2}} v_2 + i \frac{1}{\sqrt{2}} \sigma_2 \quad (90)$$

6 Tadpole Equations

$$\begin{aligned} \frac{\partial V}{\partial \phi_1} = & \frac{1}{4} e^{-2i\eta} \left(v_1 \left(2e^{2i\eta} \left(2(\lambda_1 v_1^2 + m_1^2) + (\lambda_3 + \lambda_4) v_2^2 \right) + v_2^2 (e^{4i\eta} \lambda_5 + \lambda_5^*) \right) \right. \\ & \left. + v_2 \left(e^{3i\eta} \left(-2m_{12}^* + 3v_1^2 \lambda_6^* + v_2^2 \lambda_7^* \right) + e^{i\eta} \left(-2m_{12} + 3\lambda_6 v_1^2 + \lambda_7 v_2^2 \right) \right) \right) \end{aligned} \quad (91)$$

$$\begin{aligned} \frac{\partial V}{\partial \phi_2} = & \frac{1}{4} e^{-2i\eta} \left(v_2 \left(2e^{2i\eta} \left(2(\lambda_2 v_2^2 + m_2^2) + (\lambda_3 + \lambda_4) v_1^2 \right) + v_1^2 (e^{4i\eta} \lambda_5 + \lambda_5^*) \right) \right. \\ & \left. + v_1 \left(e^{3i\eta} \left(-2m_{12}^* + 3v_2^2 \lambda_7^* + v_1^2 \lambda_6^* \right) + e^{i\eta} \left(-2m_{12} + 3\lambda_7 v_2^2 + \lambda_6 v_1^2 \right) \right) \right) \end{aligned} \quad (92)$$

$$(93)$$

7 Particle content for eigenstates 'EWSB'

Name	Type	complex/real	Generations	Indices
h	Scalar	real	4	generation, 4
H^-	Scalar	complex	2	generation, 2
ν	Fermion	Dirac	3	generation, 3
d	Fermion	Dirac	3	generation, 3, color, 3
u	Fermion	Dirac	3	generation, 3, color, 3
e	Fermion	Dirac	3	generation, 3
g	Vector	real	1	color, 8, lorentz, 4
γ	Vector	real	1	lorentz, 4
Z	Vector	real	1	lorentz, 4
W^-	Vector	complex	1	lorentz, 4
η^G	Ghost	real	1	color, 8
η^γ	Ghost	real	1	
η^Z	Ghost	real	1	
η^-	Ghost	complex	1	
η^+	Ghost	complex	1	

8 One Loop Self-Energy and One Loop Tadpoles for eigenstates 'EWSB'

8.1 One Loop Self-Energy

- Self-Energy for Higgs (h)

$$\begin{aligned}
\Pi_{i,j}(p^2) = & +2\left(-\frac{1}{2}\text{rMS} + B_0(p^2, m_Z^2, m_Z^2)\right)\Gamma_{\check{h}_j, Z, Z}^* \Gamma_{\check{h}_i, Z, Z} + 4\left(-\frac{1}{2}\text{rMS} + B_0(p^2, m_{W^-}^2, m_{W^-}^2)\right)\Gamma_{\check{h}_j, W^+, W^-}^* \Gamma_{\check{h}_i, W^+, W^-} \\
& - B_0(p^2, m_{\eta^-}^2, m_{\eta^-}^2)\Gamma_{\check{h}_i, \eta^-, \eta^-} \Gamma_{\check{h}_j, \eta^-, \eta^-} - B_0(p^2, m_{\eta^+}^2, m_{\eta^+}^2)\Gamma_{\check{h}_i, \eta^+, \eta^+} \Gamma_{\check{h}_j, \eta^+, \eta^+} \\
& - B_0(p^2, m_{\eta^Z}^2, m_{\eta^Z}^2)\Gamma_{\check{h}_i, \eta^Z, \eta^Z} \Gamma_{\check{h}_j, \eta^Z, \eta^Z} + 4\Gamma_{\check{h}_i, \check{h}_j, W^+, W^-} \left(-\frac{1}{2}\text{rMS}m_{W^-}^2 + A_0(m_{W^-}^2)\right) \\
& + 2\Gamma_{\check{h}_i, \check{h}_j, Z, Z} \left(-\frac{1}{2}\text{rMS}m_Z^2 + A_0(m_Z^2)\right) - \sum_{a=1}^2 A_0(m_{H_a^-}^2) \Gamma_{\check{h}_i, \check{h}_j, H_a^+, H_a^-} \\
& + \sum_{a=1}^2 \sum_{b=1}^2 B_0(p^2, m_{H_a^-}^2, m_{H_b^-}^2) \Gamma_{\check{h}_j, H_a^+, H_b^-}^* \Gamma_{\check{h}_i, H_a^+, H_b^-} \\
& - 6 \sum_{a=1}^3 m_{d_a} \sum_{b=1}^3 B_0(p^2, m_{d_a}^2, m_{d_b}^2) m_{d_b} \left(\Gamma_{\check{h}_j, \bar{d}_a, d_b}^{L*} \Gamma_{\check{h}_i, \bar{d}_a, d_b}^R + \Gamma_{\check{h}_j, \bar{d}_a, d_b}^{R*} \Gamma_{\check{h}_i, \bar{d}_a, d_b}^L\right)
\end{aligned}$$

$$\begin{aligned}
& + 3 \sum_{a=1}^3 \sum_{b=1}^3 G_0(p^2, m_{d_a}^2, m_{d_b}^2) \left(\Gamma_{\check{h}_j, \bar{d}_a, d_b}^{L*} \Gamma_{\check{h}_i, \bar{d}_a, d_b}^L + \Gamma_{\check{h}_j, \bar{d}_a, d_b}^{R*} \Gamma_{\check{h}_i, \bar{d}_a, d_b}^R \right) \\
& - 2 \sum_{a=1}^3 m_{e_a} \sum_{b=1}^3 B_0(p^2, m_{e_a}^2, m_{e_b}^2) m_{e_b} \left(\Gamma_{\check{h}_j, \bar{e}_a, e_b}^{L*} \Gamma_{\check{h}_i, \bar{e}_a, e_b}^R + \Gamma_{\check{h}_j, \bar{e}_a, e_b}^{R*} \Gamma_{\check{h}_i, \bar{e}_a, e_b}^L \right) \\
& + \sum_{a=1}^3 \sum_{b=1}^3 G_0(p^2, m_{e_a}^2, m_{e_b}^2) \left(\Gamma_{\check{h}_j, \bar{e}_a, e_b}^{L*} \Gamma_{\check{h}_i, \bar{e}_a, e_b}^L + \Gamma_{\check{h}_j, \bar{e}_a, e_b}^{R*} \Gamma_{\check{h}_i, \bar{e}_a, e_b}^R \right) \\
& - 6 \sum_{a=1}^3 m_{u_a} \sum_{b=1}^3 B_0(p^2, m_{u_a}^2, m_{u_b}^2) m_{u_b} \left(\Gamma_{\check{h}_j, \bar{u}_a, u_b}^{L*} \Gamma_{\check{h}_i, \bar{u}_a, u_b}^R + \Gamma_{\check{h}_j, \bar{u}_a, u_b}^{R*} \Gamma_{\check{h}_i, \bar{u}_a, u_b}^L \right) \\
& + 3 \sum_{a=1}^3 \sum_{b=1}^3 G_0(p^2, m_{u_a}^2, m_{u_b}^2) \left(\Gamma_{\check{h}_j, \bar{u}_a, u_b}^{L*} \Gamma_{\check{h}_i, \bar{u}_a, u_b}^L + \Gamma_{\check{h}_j, \bar{u}_a, u_b}^{R*} \Gamma_{\check{h}_i, \bar{u}_a, u_b}^R \right) \\
& - \frac{1}{2} \sum_{a=1}^4 A_0(m_{h_a}^2) \Gamma_{\check{h}_i, \check{h}_j, h_a, h_a} + \frac{1}{2} \sum_{a=1}^4 \sum_{b=1}^4 B_0(p^2, m_{h_a}^2, m_{h_b}^2) \Gamma_{\check{h}_j, h_a, h_b}^* \Gamma_{\check{h}_i, h_a, h_b} \\
& + 2 \sum_{b=1}^2 \Gamma_{\check{h}_j, W^+, H_b^-}^* \Gamma_{\check{h}_i, W^+, H_b^-} F_0(p^2, m_{H_b^-}^2, m_{W^-}^2) + \sum_{b=1}^4 \Gamma_{\check{h}_j, Z, h_b}^* \Gamma_{\check{h}_i, Z, h_b} F_0(p^2, m_{h_b}^2, m_Z^2) \tag{94}
\end{aligned}$$

• Self-Energy for Charged Higgs (H^-)

$$\begin{aligned}
\Pi_{i,j}(p^2) = & +4 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, 0, m_{W^-}^2) \right) \Gamma_{\check{H}_j^+, W^-, \gamma}^* \Gamma_{\check{H}_i^+, W^-, \gamma} + 4 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{W^-}^2, m_Z^2) \right) \Gamma_{\check{H}_j^+, Z, W^-}^* \Gamma_{\check{H}_i^+, Z, W^-} \\
& - B_0(p^2, m_{\eta^+}^2, m_{\eta^+}^2) \Gamma_{\check{H}_i^+, \eta^+, \eta^+} \Gamma_{\check{H}_j^-, \eta^+, \eta^+} - B_0(p^2, m_{\eta^-}^2, m_{\eta^-}^2) \Gamma_{\check{H}_i^+, \eta^+, \eta^-} \Gamma_{\check{H}_j^-, \eta^+, \eta^-} \\
& + 4 \Gamma_{\check{H}_i^-, \check{H}_j^+, W^+, W^-} \left(-\frac{1}{2} \text{rMS} m_{W^-}^2 + A_0(m_{W^-}^2) \right) + 2 \Gamma_{\check{H}_i^-, \check{H}_j^+, Z, Z} \left(-\frac{1}{2} \text{rMS} m_Z^2 + A_0(m_Z^2) \right) \\
& - \sum_{a=1}^2 A_0(m_{H_a^-}^2) \Gamma_{\check{H}_i^-, \check{H}_j^+, H_a^+, H_a^-} + \sum_{a=1}^2 \sum_{b=1}^4 B_0(p^2, m_{H_a^-}^2, m_{h_b}^2) \Gamma_{\check{H}_j^+, H_a^-, h_b}^* \Gamma_{\check{H}_i^+, H_a^-, h_b} \\
& - 6 \sum_{a=1}^3 m_{u_a} \sum_{b=1}^3 B_0(p^2, m_{u_a}^2, m_{d_b}^2) m_{d_b} \left(\Gamma_{\check{H}_j^+, \bar{u}_a, d_b}^{L*} \Gamma_{\check{H}_i^+, \bar{u}_a, d_b}^R + \Gamma_{\check{H}_j^+, \bar{u}_a, d_b}^{R*} \Gamma_{\check{H}_i^+, \bar{u}_a, d_b}^L \right) \\
& + 3 \sum_{a=1}^3 \sum_{b=1}^3 G_0(p^2, m_{u_a}^2, m_{d_b}^2) \left(\Gamma_{\check{H}_j^+, \bar{u}_a, d_b}^{L*} \Gamma_{\check{H}_i^+, \bar{u}_a, d_b}^L + \Gamma_{\check{H}_j^+, \bar{u}_a, d_b}^{R*} \Gamma_{\check{H}_i^+, \bar{u}_a, d_b}^R \right) \\
& - 2 \sum_{a=1}^3 m_{\nu_a} \sum_{b=1}^3 B_0(p^2, m_{\nu_a}^2, m_{e_b}^2) m_{e_b} \left(\Gamma_{\check{H}_j^+, \bar{\nu}_a, e_b}^{L*} \Gamma_{\check{H}_i^+, \bar{\nu}_a, e_b}^R + \Gamma_{\check{H}_j^+, \bar{\nu}_a, e_b}^{R*} \Gamma_{\check{H}_i^+, \bar{\nu}_a, e_b}^L \right) \\
& + \sum_{a=1}^3 \sum_{b=1}^3 G_0(p^2, m_{\nu_a}^2, m_{e_b}^2) \left(\Gamma_{\check{H}_j^+, \bar{\nu}_a, e_b}^{L*} \Gamma_{\check{H}_i^+, \bar{\nu}_a, e_b}^L + \Gamma_{\check{H}_j^+, \bar{\nu}_a, e_b}^{R*} \Gamma_{\check{H}_i^+, \bar{\nu}_a, e_b}^R \right)
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{2} \sum_{a=1}^4 A_0(m_{h_a}^2) \Gamma_{\tilde{H}_i^-, \tilde{H}_j^+, h_a, h_a} + \sum_{b=1}^2 \Gamma_{\tilde{H}_j^+, \gamma, H_b^-}^* \Gamma_{\tilde{H}_i^+, \gamma, H_b^-} F_0(p^2, m_{H_b^-}^2, 0) \\
& + \sum_{b=1}^2 \Gamma_{\tilde{H}_j^+, Z, H_b^-}^* \Gamma_{\tilde{H}_i^+, Z, H_b^-} F_0(p^2, m_{H_b^-}^2, m_Z^2) + \sum_{b=1}^4 \Gamma_{\tilde{H}_j^+, W^-, h_b}^* \Gamma_{\tilde{H}_i^+, W^-, h_b} F_0(p^2, m_{h_b}^2, m_{W^-}^2)
\end{aligned} \tag{95}$$

• Self-Energy for Down-Quarks (d)

$$\begin{aligned}
\Sigma_{i,j}^S(p^2) &= + \sum_{a=1}^2 \sum_{b=1}^3 B_0(p^2, m_{u_b}^2, m_{H_a^-}^2) \Gamma_{\tilde{d}_j, H_a^-, u_b}^{L*} m_{u_b} \Gamma_{\tilde{d}_i, H_a^-, u_b}^R \\
& + \sum_{a=1}^4 \sum_{b=1}^3 B_0(p^2, m_{d_b}^2, m_{h_a}^2) \Gamma_{\tilde{d}_j, h_a, d_b}^{L*} m_{d_b} \Gamma_{\tilde{d}_i, h_a, d_b}^R \\
& - \frac{16}{3} \sum_{b=1}^3 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{d_b}^2, 0) \right) \Gamma_{\tilde{d}_j, g, d_b}^{R*} m_{d_b} \Gamma_{\tilde{d}_i, g, d_b}^L - 4 \sum_{b=1}^3 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{d_b}^2, 0) \right) \Gamma_{\tilde{d}_j, \gamma, d_b}^{R*} m_{d_b} \Gamma_{\tilde{d}_i, \gamma, d_b}^L \\
& - 4 \sum_{b=1}^3 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{u_b}^2, m_{W^-}^2) \right) \Gamma_{\tilde{d}_j, W^-, u_b}^{R*} m_{u_b} \Gamma_{\tilde{d}_i, W^-, u_b}^L \\
& - 4 \sum_{b=1}^3 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{d_b}^2, m_Z^2) \right) \Gamma_{\tilde{d}_j, Z, d_b}^{R*} m_{d_b} \Gamma_{\tilde{d}_i, Z, d_b}^L
\end{aligned} \tag{96}$$

$$\begin{aligned}
\Sigma_{i,j}^R(p^2) &= -\frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, m_{H_a^-}^2) \Gamma_{\tilde{d}_j, H_a^-, u_b}^{R*} \Gamma_{\tilde{d}_i, H_a^-, u_b}^R \\
& - \frac{1}{2} \sum_{a=1}^4 \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, m_{h_a}^2) \Gamma_{\tilde{d}_j, h_a, d_b}^{R*} \Gamma_{\tilde{d}_i, h_a, d_b}^R - \frac{4}{3} \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, 0) \Gamma_{\tilde{d}_j, g, d_b}^{L*} \Gamma_{\tilde{d}_i, g, d_b}^L \\
& - \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, 0) \Gamma_{\tilde{d}_j, \gamma, d_b}^{L*} \Gamma_{\tilde{d}_i, \gamma, d_b}^L - \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, m_{W^-}^2) \Gamma_{\tilde{d}_j, W^-, u_b}^{L*} \Gamma_{\tilde{d}_i, W^-, u_b}^L \\
& - \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, m_Z^2) \Gamma_{\tilde{d}_j, Z, d_b}^{L*} \Gamma_{\tilde{d}_i, Z, d_b}^L
\end{aligned} \tag{97}$$

$$\begin{aligned}
\Sigma_{i,j}^L(p^2) &= -\frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, m_{H_a^-}^2) \Gamma_{\tilde{d}_j, H_a^-, u_b}^{L*} \Gamma_{\tilde{d}_i, H_a^-, u_b}^L \\
& - \frac{1}{2} \sum_{a=1}^4 \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, m_{h_a}^2) \Gamma_{\tilde{d}_j, h_a, d_b}^{L*} \Gamma_{\tilde{d}_i, h_a, d_b}^L - \frac{4}{3} \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, 0) \Gamma_{\tilde{d}_j, g, d_b}^{R*} \Gamma_{\tilde{d}_i, g, d_b}^R \\
& - \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, 0) \Gamma_{\tilde{d}_j, \gamma, d_b}^{R*} \Gamma_{\tilde{d}_i, \gamma, d_b}^R - \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, m_{W^-}^2) \Gamma_{\tilde{d}_j, W^-, u_b}^{R*} \Gamma_{\tilde{d}_i, W^-, u_b}^R
\end{aligned}$$

$$- \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, m_Z^2) \Gamma_{\check{d}_j, Z, d_b}^{R*} \Gamma_{\check{d}_i, Z, d_b}^R \quad (98)$$

• Self-Energy for Up-Quarks (u)

$$\begin{aligned} \Sigma_{i,j}^S(p^2) = & + \sum_{a=1}^2 \sum_{b=1}^3 B_0(p^2, m_{d_b}^2, m_{H_a^-}^2) \Gamma_{\check{u}_j, H_a^+, d_b}^{L*} m_{d_b} \Gamma_{\check{u}_i, H_a^+, d_b}^R \\ & + \sum_{a=1}^4 \sum_{b=1}^3 B_0(p^2, m_{u_b}^2, m_{h_a}^2) \Gamma_{\check{u}_j, h_a, u_b}^{L*} m_{u_b} \Gamma_{\check{u}_i, h_a, u_b}^R \\ & - \frac{16}{3} \sum_{b=1}^3 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{u_b}^2, 0) \right) \Gamma_{\check{u}_j, g, u_b}^{R*} m_{u_b} \Gamma_{\check{u}_i, g, u_b}^L - 4 \sum_{b=1}^3 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{u_b}^2, 0) \right) \Gamma_{\check{u}_j, \gamma, u_b}^{R*} m_{u_b} \Gamma_{\check{u}_i, \gamma, u_b}^L \\ & - 4 \sum_{b=1}^3 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{u_b}^2, m_Z^2) \right) \Gamma_{\check{u}_j, Z, u_b}^{R*} m_{u_b} \Gamma_{\check{u}_i, Z, u_b}^L \\ & - 4 \sum_{b=1}^3 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{d_b}^2, m_{W^-}^2) \right) \Gamma_{\check{u}_j, W^+, d_b}^{R*} m_{d_b} \Gamma_{\check{u}_i, W^+, d_b}^L \quad (99) \end{aligned}$$

$$\begin{aligned} \Sigma_{i,j}^R(p^2) = & -\frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, m_{H_a^-}^2) \Gamma_{\check{u}_j, H_a^+, d_b}^{R*} \Gamma_{\check{u}_i, H_a^+, d_b}^R \\ & - \frac{1}{2} \sum_{a=1}^4 \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, m_{h_a}^2) \Gamma_{\check{u}_j, h_a, u_b}^{R*} \Gamma_{\check{u}_i, h_a, u_b}^R - \frac{4}{3} \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, 0) \Gamma_{\check{u}_j, g, u_b}^{L*} \Gamma_{\check{u}_i, g, u_b}^L \\ & - \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, 0) \Gamma_{\check{u}_j, \gamma, u_b}^{L*} \Gamma_{\check{u}_i, \gamma, u_b}^L - \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, m_Z^2) \Gamma_{\check{u}_j, Z, u_b}^{L*} \Gamma_{\check{u}_i, Z, u_b}^L \\ & - \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, m_{W^-}^2) \Gamma_{\check{u}_j, W^+, d_b}^{L*} \Gamma_{\check{u}_i, W^+, d_b}^L \quad (100) \end{aligned}$$

$$\begin{aligned} \Sigma_{i,j}^L(p^2) = & -\frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, m_{H_a^-}^2) \Gamma_{\check{u}_j, H_a^+, d_b}^{L*} \Gamma_{\check{u}_i, H_a^+, d_b}^L \\ & - \frac{1}{2} \sum_{a=1}^4 \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, m_{h_a}^2) \Gamma_{\check{u}_j, h_a, u_b}^{L*} \Gamma_{\check{u}_i, h_a, u_b}^L - \frac{4}{3} \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, 0) \Gamma_{\check{u}_j, g, u_b}^{R*} \Gamma_{\check{u}_i, g, u_b}^R \\ & - \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, 0) \Gamma_{\check{u}_j, \gamma, u_b}^{R*} \Gamma_{\check{u}_i, \gamma, u_b}^R - \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, m_Z^2) \Gamma_{\check{u}_j, Z, u_b}^{R*} \Gamma_{\check{u}_i, Z, u_b}^R \\ & - \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, m_{W^-}^2) \Gamma_{\check{u}_j, W^+, d_b}^{R*} \Gamma_{\check{u}_i, W^+, d_b}^R \quad (101) \end{aligned}$$

• Self-Energy for Leptons (e)

$$\begin{aligned}
\Sigma_{i,j}^S(p^2) = & + \sum_{a=1}^2 \sum_{b=1}^3 B_0(p^2, m_{\nu_b}^2, m_{H_a^-}^2) \Gamma_{\tilde{e}_j, H_a^-, \nu_b}^{L*} m_{\nu_b} \Gamma_{\tilde{e}_i, H_a^-, \nu_b}^R \\
& + \sum_{a=1}^4 \sum_{b=1}^3 B_0(p^2, m_{e_b}^2, m_{h_a}^2) \Gamma_{\tilde{e}_j, h_a, e_b}^{L*} m_{e_b} \Gamma_{\tilde{e}_i, h_a, e_b}^R \\
& - 4 \sum_{b=1}^3 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{e_b}^2, 0) \right) \Gamma_{\tilde{e}_j, \gamma, e_b}^{R*} m_{e_b} \Gamma_{\tilde{e}_i, \gamma, e_b}^L \\
& - 4 \sum_{b=1}^3 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{\nu_b}^2, m_{W^-}^2) \right) \Gamma_{\tilde{e}_j, W^-, \nu_b}^{R*} m_{\nu_b} \Gamma_{\tilde{e}_i, W^-, \nu_b}^L \\
& - 4 \sum_{b=1}^3 \left(-\frac{1}{2} \text{rMS} + B_0(p^2, m_{e_b}^2, m_Z^2) \right) \Gamma_{\tilde{e}_j, Z, e_b}^{R*} m_{e_b} \Gamma_{\tilde{e}_i, Z, e_b}^L \tag{102}
\end{aligned}$$

$$\begin{aligned}
\Sigma_{i,j}^R(p^2) = & -\frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^3 B_1(p^2, m_{\nu_b}^2, m_{H_a^-}^2) \Gamma_{\tilde{e}_j, H_a^-, \nu_b}^{R*} \Gamma_{\tilde{e}_i, H_a^-, \nu_b}^R \\
& - \frac{1}{2} \sum_{a=1}^4 \sum_{b=1}^3 B_1(p^2, m_{e_b}^2, m_{h_a}^2) \Gamma_{\tilde{e}_j, h_a, e_b}^{R*} \Gamma_{\tilde{e}_i, h_a, e_b}^R - \sum_{b=1}^3 B_1(p^2, m_{e_b}^2, 0) \Gamma_{\tilde{e}_j, \gamma, e_b}^{L*} \Gamma_{\tilde{e}_i, \gamma, e_b}^L \\
& - \sum_{b=1}^3 B_1(p^2, m_{\nu_b}^2, m_{W^-}^2) \Gamma_{\tilde{e}_j, W^-, \nu_b}^{L*} \Gamma_{\tilde{e}_i, W^-, \nu_b}^L - \sum_{b=1}^3 B_1(p^2, m_{e_b}^2, m_Z^2) \Gamma_{\tilde{e}_j, Z, e_b}^{L*} \Gamma_{\tilde{e}_i, Z, e_b}^L \tag{103}
\end{aligned}$$

$$\begin{aligned}
\Sigma_{i,j}^L(p^2) = & -\frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^3 B_1(p^2, m_{\nu_b}^2, m_{H_a^-}^2) \Gamma_{\tilde{e}_j, H_a^-, \nu_b}^{L*} \Gamma_{\tilde{e}_i, H_a^-, \nu_b}^L \\
& - \frac{1}{2} \sum_{a=1}^4 \sum_{b=1}^3 B_1(p^2, m_{e_b}^2, m_{h_a}^2) \Gamma_{\tilde{e}_j, h_a, e_b}^{L*} \Gamma_{\tilde{e}_i, h_a, e_b}^L - \sum_{b=1}^3 B_1(p^2, m_{e_b}^2, 0) \Gamma_{\tilde{e}_j, \gamma, e_b}^{R*} \Gamma_{\tilde{e}_i, \gamma, e_b}^R \\
& - \sum_{b=1}^3 B_1(p^2, m_{\nu_b}^2, m_{W^-}^2) \Gamma_{\tilde{e}_j, W^-, \nu_b}^{R*} \Gamma_{\tilde{e}_i, W^-, \nu_b}^R - \sum_{b=1}^3 B_1(p^2, m_{e_b}^2, m_Z^2) \Gamma_{\tilde{e}_j, Z, e_b}^{R*} \Gamma_{\tilde{e}_i, Z, e_b}^R \tag{104}
\end{aligned}$$

• **Self-Energy for Z-Boson** (Z)

$$\begin{aligned}
\Pi(p^2) = & + |\Gamma_{Z, \eta^-, \eta^-}|^2 B_{00}(p^2, m_{\eta^-}^2, m_{\eta^-}^2) + |\Gamma_{Z, \eta^+, \eta^+}|^2 B_{00}(p^2, m_{\eta^+}^2, m_{\eta^+}^2) \\
& - |\Gamma_{Z, W^+, W^-}|^2 \left(10 B_{00}(p^2, m_{W^-}^2, m_{W^-}^2) + 2 A_0(m_{W^-}^2) - 2 \text{rMS} \left(2 m_{W^-}^2 - \frac{1}{3} p^2 \right) + B_0(p^2, m_{W^-}^2, m_{W^-}^2) (2 m_{W^-}^2 + 4 p^2) \right) \\
& + \sum_{a=1}^2 A_0(m_{H_a^-}^2) \Gamma_{Z, Z, H_a^+, H_a^-} - 4 \sum_{a=1}^2 \sum_{b=1}^2 |\Gamma_{Z, H_a^+, H_b^-}|^2 B_{00}(p^2, m_{H_a^-}^2, m_{H_b^-}^2) \\
& + 3 \sum_{a=1}^3 \sum_{b=1}^3 \left[(|\Gamma_{Z, \bar{d}_a, d_b}^L|^2 + |\Gamma_{Z, \bar{d}_a, d_b}^R|^2) H_0(p^2, m_{\bar{d}_a}^2, m_{d_b}^2) \right]
\end{aligned}$$

$$\begin{aligned}
& + 4B_0(p^2, m_{d_a}^2, m_{d_b}^2) m_{d_a} m_{d_b} \Re\left(\Gamma_{Z, \bar{d}_a, d_b}^{L*} \Gamma_{Z, \bar{d}_a, d_b}^R\right) \\
& + \sum_{a=1}^3 \sum_{b=1}^3 \left[\left(|\Gamma_{Z, \bar{e}_a, e_b}^L|^2 + |\Gamma_{Z, \bar{e}_a, e_b}^R|^2 \right) H_0(p^2, m_{e_a}^2, m_{e_b}^2) \right. \\
& + 4B_0(p^2, m_{e_a}^2, m_{e_b}^2) m_{e_a} m_{e_b} \Re\left(\Gamma_{Z, \bar{e}_a, e_b}^{L*} \Gamma_{Z, \bar{e}_a, e_b}^R\right) \\
& + 3 \sum_{a=1}^3 \sum_{b=1}^3 \left[\left(|\Gamma_{Z, \bar{u}_a, u_b}^L|^2 + |\Gamma_{Z, \bar{u}_a, u_b}^R|^2 \right) H_0(p^2, m_{u_a}^2, m_{u_b}^2) \right. \\
& + 4B_0(p^2, m_{u_a}^2, m_{u_b}^2) m_{u_a} m_{u_b} \Re\left(\Gamma_{Z, \bar{u}_a, u_b}^{L*} \Gamma_{Z, \bar{u}_a, u_b}^R\right) \\
& + \sum_{a=1}^3 \sum_{b=1}^3 \left[\left(|\Gamma_{Z, \bar{\nu}_a, \nu_b}^L|^2 + |\Gamma_{Z, \bar{\nu}_a, \nu_b}^R|^2 \right) H_0(p^2, m_{\nu_a}^2, m_{\nu_b}^2) \right. \\
& + 4B_0(p^2, m_{\nu_a}^2, m_{\nu_b}^2) m_{\nu_a} m_{\nu_b} \Re\left(\Gamma_{Z, \bar{\nu}_a, \nu_b}^{L*} \Gamma_{Z, \bar{\nu}_a, \nu_b}^R\right) \\
& + \frac{1}{2} \sum_{a=1}^4 A_0(m_{h_a}^2) \Gamma_{Z, Z, h_a, h_a} - 2 \sum_{a=1}^4 \sum_{b=1}^4 |\Gamma_{Z, h_a, h_b}|^2 B_{00}(p^2, m_{h_a}^2, m_{h_b}^2) \\
& + 2 \sum_{b=1}^2 |\Gamma_{Z, W^+, H_b^-}|^2 B_0(p^2, m_{W^-}^2, m_{H_b^-}^2) + \sum_{b=1}^4 |\Gamma_{Z, Z, h_b}|^2 B_0(p^2, m_Z^2, m_{h_b}^2) + 2\text{rMS} m_{W^-}^2 \Gamma_{Z, Z, W^+, W^-}^1 \\
& - A_0(m_{W^-}^2) \left(4\Gamma_{Z, Z, W^+, W^-}^1 + \Gamma_{Z, Z, W^+, W^-}^2 + \Gamma_{Z, Z, W^+, W^-}^3 \right)
\end{aligned} \tag{105}$$

• **Self-Energy for W-Boson (W^-)**

$$\begin{aligned}
\Pi(p^2) & = 2\text{rMS} m_{W^-}^2 \Gamma_{W^-, W^+, W^+, W^-}^1 + 3 \sum_{a=1}^3 \sum_{b=1}^3 \left[\left(|\Gamma_{W^+, \bar{u}_a, d_b}^L|^2 + |\Gamma_{W^+, \bar{u}_a, d_b}^R|^2 \right) H_0(p^2, m_{u_a}^2, m_{d_b}^2) \right. \\
& + 4B_0(p^2, m_{u_a}^2, m_{d_b}^2) m_{d_b} m_{u_a} \Re\left(\Gamma_{W^+, \bar{u}_a, d_b}^{L*} \Gamma_{W^+, \bar{u}_a, d_b}^R\right) \left. - 4 \sum_{a=1}^2 \sum_{b=1}^4 |\Gamma_{W^+, H_a^-, h_b}|^2 B_{00}(p^2, m_{h_b}^2, m_{H_a^-}^2) - A_0(m_{W^-}^2) \left(4\Gamma_{W^+}^1 \right. \right. \\
& + 4B_0(p^2, m_{\nu_a}^2, m_{e_b}^2) m_{e_b} m_{\nu_a} \Re\left(\Gamma_{W^+, \bar{\nu}_a, e_b}^{L*} \Gamma_{W^+, \bar{\nu}_a, e_b}^R\right) \left. + \sum_{b=1}^2 |\Gamma_{W^+, \gamma, H_b^-}|^2 B_0(p^2, 0, m_{H_b^-}^2) + \sum_{b=1}^2 |\Gamma_{W^+, Z, H_b^-}|^2 B_0(p^2, m_Z^2, \right. \\
& \left. \left. \right) \right]
\end{aligned} \tag{106}$$

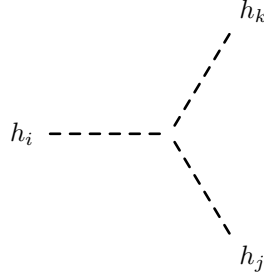
8.2 Tadpoles

$$\begin{aligned}
\delta t_h^{(1)} & = + A_0(m_{\eta^-}^2) \Gamma_{\tilde{h}_i, \eta^-, \eta^-} + A_0(m_{\eta^+}^2) \Gamma_{\tilde{h}_i, \eta^+, \eta^+} + A_0(m_{\eta^z}^2) \Gamma_{\tilde{h}_i, \eta^z, \eta^z} \\
& + 4\Gamma_{\tilde{h}_i, W^+, W^-} \left(-\frac{1}{2} \text{rMS} m_{W^-}^2 + A_0(m_{W^-}^2) \right) + 2\Gamma_{\tilde{h}_i, Z, Z} \left(-\frac{1}{2} \text{rMS} m_Z^2 + A_0(m_Z^2) \right) - \sum_{a=1}^2 A_0(m_{H_a^-}^2) \Gamma_{\tilde{h}_i, H_a^+, H_a^-}
\end{aligned}$$

$$\begin{aligned}
& + 6 \sum_{a=1}^3 A_0(m_{\bar{d}_a}^2) m_{d_a} \left(\Gamma_{\tilde{h}_i, \bar{d}_a, d_a}^L + \Gamma_{\tilde{h}_i, \bar{d}_a, d_a}^R \right) \\
& + 2 \sum_{a=1}^3 A_0(m_{e_a}^2) m_{e_a} \left(\Gamma_{\tilde{h}_i, \bar{e}_a, e_a}^L + \Gamma_{\tilde{h}_i, \bar{e}_a, e_a}^R \right) \\
& + 6 \sum_{a=1}^3 A_0(m_{u_a}^2) m_{u_a} \left(\Gamma_{\tilde{h}_i, \bar{u}_a, u_a}^L + \Gamma_{\tilde{h}_i, \bar{u}_a, u_a}^R \right) - \frac{1}{2} \sum_{a=1}^4 A_0(m_{h_a}^2) \Gamma_{\tilde{h}_i, h_a, h_a}
\end{aligned} \tag{107}$$

9 Interactions for eigenstates 'EWSB'

9.1 Three Scalar-Interaction

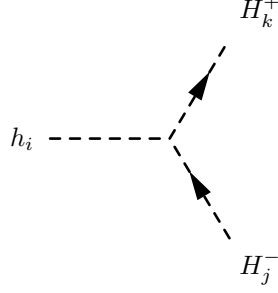


$$\begin{aligned}
& \frac{1}{2} e^{-2i\eta} \left(- e^{i\eta} \lambda_6 v_2 Z_{i3}^H Z_{j1}^H Z_{k1}^H + e^{3i\eta} v_2 \lambda_6^* Z_{i3}^H Z_{j1}^H Z_{k1}^H + 3e^{i\eta} \lambda_6 v_1 Z_{i4}^H Z_{j1}^H Z_{k1}^H \right. \\
& - e^{4i\eta} \lambda_5 v_2 Z_{i4}^H Z_{j1}^H Z_{k1}^H + v_2 \lambda_5^* Z_{i4}^H Z_{j1}^H Z_{k1}^H - 3e^{3i\eta} v_1 \lambda_6^* Z_{i4}^H Z_{j1}^H Z_{k1}^H \\
& - e^{i\eta} \lambda_6 v_1 Z_{i3}^H Z_{j2}^H Z_{k1}^H + e^{4i\eta} \lambda_5 v_2 Z_{i3}^H Z_{j2}^H Z_{k1}^H - v_2 \lambda_5^* Z_{i3}^H Z_{j2}^H Z_{k1}^H \\
& + e^{3i\eta} v_1 \lambda_6^* Z_{i3}^H Z_{j2}^H Z_{k1}^H - e^{4i\eta} \lambda_5 v_1 Z_{i4}^H Z_{j2}^H Z_{k1}^H + e^{i\eta} \lambda_7 v_2 Z_{i4}^H Z_{j2}^H Z_{k1}^H \\
& + v_1 \lambda_5^* Z_{i4}^H Z_{j2}^H Z_{k1}^H - e^{3i\eta} v_2 \lambda_7^* Z_{i4}^H Z_{j2}^H Z_{k1}^H - 4ie^{2i\eta} \lambda_1 v_1 Z_{i3}^H Z_{j3}^H Z_{k1}^H \\
& - ie^{i\eta} \lambda_6 v_2 Z_{i3}^H Z_{j3}^H Z_{k1}^H - ie^{3i\eta} v_2 \lambda_6^* Z_{i3}^H Z_{j3}^H Z_{k1}^H - ie^{i\eta} \lambda_6 v_1 Z_{i4}^H Z_{j3}^H Z_{k1}^H \\
& - ie^{4i\eta} \lambda_5 v_2 Z_{i4}^H Z_{j3}^H Z_{k1}^H - iv_2 \lambda_5^* Z_{i4}^H Z_{j3}^H Z_{k1}^H - ie^{3i\eta} v_1 \lambda_6^* Z_{i4}^H Z_{j3}^H Z_{k1}^H \\
& - ie^{i\eta} \lambda_6 v_1 Z_{i3}^H Z_{j4}^H Z_{k1}^H - ie^{4i\eta} \lambda_5 v_2 Z_{i3}^H Z_{j4}^H Z_{k1}^H - iv_2 \lambda_5^* Z_{i3}^H Z_{j4}^H Z_{k1}^H \\
& - ie^{3i\eta} v_1 \lambda_6^* Z_{i3}^H Z_{j4}^H Z_{k1}^H - 2ie^{2i\eta} \lambda_3 v_1 Z_{i4}^H Z_{j4}^H Z_{k1}^H - 2ie^{2i\eta} \lambda_4 v_1 Z_{i4}^H Z_{j4}^H Z_{k1}^H \\
& + ie^{4i\eta} \lambda_5 v_1 Z_{i4}^H Z_{j4}^H Z_{k1}^H - ie^{i\eta} \lambda_7 v_2 Z_{i4}^H Z_{j4}^H Z_{k1}^H + iv_1 \lambda_5^* Z_{i4}^H Z_{j4}^H Z_{k1}^H \\
& - ie^{3i\eta} v_2 \lambda_7^* Z_{i4}^H Z_{j4}^H Z_{k1}^H - e^{i\eta} \lambda_6 v_1 Z_{i3}^H Z_{j1}^H Z_{k2}^H + e^{4i\eta} \lambda_5 v_2 Z_{i3}^H Z_{j1}^H Z_{k2}^H \\
& - v_2 \lambda_5^* Z_{i3}^H Z_{j1}^H Z_{k2}^H + e^{3i\eta} v_1 \lambda_6^* Z_{i3}^H Z_{j1}^H Z_{k2}^H - e^{4i\eta} \lambda_5 v_1 Z_{i4}^H Z_{j1}^H Z_{k2}^H \\
& + e^{i\eta} \lambda_7 v_2 Z_{i4}^H Z_{j1}^H Z_{k2}^H + v_1 \lambda_5^* Z_{i4}^H Z_{j1}^H Z_{k2}^H - e^{3i\eta} v_2 \lambda_7^* Z_{i4}^H Z_{j1}^H Z_{k2}^H \\
& + e^{4i\eta} \lambda_5 v_1 Z_{i3}^H Z_{j2}^H Z_{k2}^H - 3e^{i\eta} \lambda_7 v_2 Z_{i3}^H Z_{j2}^H Z_{k2}^H - v_1 \lambda_5^* Z_{i3}^H Z_{j2}^H Z_{k2}^H \\
& \left. + 3e^{3i\eta} v_2 \lambda_7^* Z_{i3}^H Z_{j2}^H Z_{k2}^H + e^{i\eta} \lambda_7 v_1 Z_{i4}^H Z_{j2}^H Z_{k2}^H - e^{3i\eta} v_1 \lambda_7^* Z_{i4}^H Z_{j2}^H Z_{k2}^H \right)
\end{aligned}$$

$$\begin{aligned}
& -ie^{i\eta}\lambda_6v_1Z_{i3}^HZ_{j3}^HZ_{k2}^H - 2ie^{2i\eta}\lambda_3v_2Z_{i3}^HZ_{j3}^HZ_{k2}^H - 2ie^{2i\eta}\lambda_4v_2Z_{i3}^HZ_{j3}^HZ_{k2}^H \\
& + ie^{4i\eta}\lambda_5v_2Z_{i3}^HZ_{j3}^HZ_{k2}^H + iv_2\lambda_5^*Z_{i3}^HZ_{j3}^HZ_{k2}^H - ie^{3i\eta}v_1\lambda_6^*Z_{i3}^HZ_{j3}^HZ_{k2}^H \\
& - ie^{4i\eta}\lambda_5v_1Z_{i4}^HZ_{j3}^HZ_{k2}^H - ie^{i\eta}\lambda_7v_2Z_{i4}^HZ_{j3}^HZ_{k2}^H - iv_1\lambda_5^*Z_{i4}^HZ_{j3}^HZ_{k2}^H \\
& - ie^{3i\eta}v_2\lambda_7^*Z_{i4}^HZ_{j3}^HZ_{k2}^H - ie^{4i\eta}\lambda_5v_1Z_{i3}^HZ_{j4}^HZ_{k2}^H - ie^{i\eta}\lambda_7v_2Z_{i3}^HZ_{j4}^HZ_{k2}^H \\
& - iv_1\lambda_5^*Z_{i3}^HZ_{j4}^HZ_{k2}^H - ie^{3i\eta}v_2\lambda_7^*Z_{i3}^HZ_{j4}^HZ_{k2}^H - ie^{i\eta}\lambda_7v_1Z_{i4}^HZ_{j4}^HZ_{k2}^H \\
& - 4ie^{2i\eta}\lambda_2v_2Z_{i4}^HZ_{j4}^HZ_{k2}^H - ie^{3i\eta}v_1\lambda_7^*Z_{i4}^HZ_{j4}^HZ_{k2}^H - 4ie^{2i\eta}\lambda_1v_1Z_{i3}^HZ_{j1}^HZ_{k3}^H \\
& - ie^{i\eta}\lambda_6v_2Z_{i3}^HZ_{j1}^HZ_{k3}^H - ie^{3i\eta}v_2\lambda_6^*Z_{i3}^HZ_{j1}^HZ_{k3}^H - ie^{i\eta}\lambda_6v_1Z_{i4}^HZ_{j1}^HZ_{k3}^H \\
& - ie^{4i\eta}\lambda_5v_2Z_{i4}^HZ_{j1}^HZ_{k3}^H - iv_2\lambda_5^*Z_{i4}^HZ_{j1}^HZ_{k3}^H - ie^{3i\eta}v_1\lambda_6^*Z_{i4}^HZ_{j1}^HZ_{k3}^H \\
& - ie^{i\eta}\lambda_6v_1Z_{i3}^HZ_{j2}^HZ_{k3}^H - 2ie^{2i\eta}\lambda_3v_2Z_{i3}^HZ_{j2}^HZ_{k3}^H - 2ie^{2i\eta}\lambda_4v_2Z_{i3}^HZ_{j2}^HZ_{k3}^H \\
& + ie^{4i\eta}\lambda_5v_2Z_{i3}^HZ_{j2}^HZ_{k3}^H + iv_2\lambda_5^*Z_{i3}^HZ_{j2}^HZ_{k3}^H - ie^{3i\eta}v_1\lambda_6^*Z_{i3}^HZ_{j2}^HZ_{k3}^H \\
& - ie^{4i\eta}\lambda_5v_1Z_{i4}^HZ_{j2}^HZ_{k3}^H - ie^{i\eta}\lambda_7v_2Z_{i4}^HZ_{j2}^HZ_{k3}^H - iv_1\lambda_5^*Z_{i4}^HZ_{j2}^HZ_{k3}^H \\
& - ie^{3i\eta}v_2\lambda_7^*Z_{i4}^HZ_{j2}^HZ_{k3}^H - 3e^{i\eta}\lambda_6v_2Z_{i3}^HZ_{j3}^HZ_{k3}^H + 3e^{3i\eta}v_2\lambda_6^*Z_{i3}^HZ_{j3}^HZ_{k3}^H \\
& + e^{i\eta}\lambda_6v_1Z_{i4}^HZ_{j3}^HZ_{k3}^H + e^{4i\eta}\lambda_5v_2Z_{i4}^HZ_{j3}^HZ_{k3}^H - v_2\lambda_5^*Z_{i4}^HZ_{j3}^HZ_{k3}^H \\
& - e^{3i\eta}v_1\lambda_6^*Z_{i4}^HZ_{j3}^HZ_{k3}^H + e^{i\eta}\lambda_6v_1Z_{i3}^HZ_{j4}^HZ_{k3}^H + e^{4i\eta}\lambda_5v_2Z_{i3}^HZ_{j4}^HZ_{k3}^H \\
& - v_2\lambda_5^*Z_{i3}^HZ_{j4}^HZ_{k3}^H - e^{3i\eta}v_1\lambda_6^*Z_{i3}^HZ_{j4}^HZ_{k3}^H - e^{4i\eta}\lambda_5v_1Z_{i4}^HZ_{j4}^HZ_{k3}^H \\
& - e^{i\eta}\lambda_7v_2Z_{i4}^HZ_{j4}^HZ_{k3}^H + v_1\lambda_5^*Z_{i4}^HZ_{j4}^HZ_{k3}^H + e^{3i\eta}v_2\lambda_7^*Z_{i4}^HZ_{j4}^HZ_{k3}^H \\
& - ie^{i\eta}\lambda_6v_1Z_{i3}^HZ_{j1}^HZ_{k4}^H - ie^{4i\eta}\lambda_5v_2Z_{i3}^HZ_{j1}^HZ_{k4}^H - iv_2\lambda_5^*Z_{i3}^HZ_{j1}^HZ_{k4}^H \\
& - ie^{3i\eta}v_1\lambda_6^*Z_{i3}^HZ_{j1}^HZ_{k4}^H - 2ie^{2i\eta}\lambda_3v_1Z_{i4}^HZ_{j1}^HZ_{k4}^H - 2ie^{2i\eta}\lambda_4v_1Z_{i4}^HZ_{j1}^HZ_{k4}^H \\
& + ie^{4i\eta}\lambda_5v_1Z_{i4}^HZ_{j1}^HZ_{k4}^H - ie^{i\eta}\lambda_7v_2Z_{i4}^HZ_{j1}^HZ_{k4}^H + iv_1\lambda_5^*Z_{i4}^HZ_{j1}^HZ_{k4}^H \\
& - ie^{3i\eta}v_2\lambda_7^*Z_{i4}^HZ_{j1}^HZ_{k4}^H - ie^{4i\eta}\lambda_5v_1Z_{i3}^HZ_{j2}^HZ_{k4}^H - ie^{i\eta}\lambda_7v_2Z_{i3}^HZ_{j2}^HZ_{k4}^H \\
& - iv_1\lambda_5^*Z_{i3}^HZ_{j2}^HZ_{k4}^H - ie^{3i\eta}v_2\lambda_7^*Z_{i3}^HZ_{j2}^HZ_{k4}^H - ie^{i\eta}\lambda_7v_1Z_{i4}^HZ_{j2}^HZ_{k4}^H \\
& - 4ie^{2i\eta}\lambda_2v_2Z_{i4}^HZ_{j2}^HZ_{k4}^H - ie^{3i\eta}v_1\lambda_7^*Z_{i4}^HZ_{j2}^HZ_{k4}^H + e^{i\eta}\lambda_6v_1Z_{i3}^HZ_{j3}^HZ_{k4}^H \\
& + e^{4i\eta}\lambda_5v_2Z_{i3}^HZ_{j3}^HZ_{k4}^H - v_2\lambda_5^*Z_{i3}^HZ_{j3}^HZ_{k4}^H - e^{3i\eta}v_1\lambda_6^*Z_{i3}^HZ_{j3}^HZ_{k4}^H \\
& - e^{4i\eta}\lambda_5v_1Z_{i4}^HZ_{j3}^HZ_{k4}^H - e^{i\eta}\lambda_7v_2Z_{i4}^HZ_{j3}^HZ_{k4}^H + v_1\lambda_5^*Z_{i4}^HZ_{j3}^HZ_{k4}^H \\
& + e^{3i\eta}v_2\lambda_7^*Z_{i4}^HZ_{j3}^HZ_{k4}^H - e^{4i\eta}\lambda_5v_1Z_{i3}^HZ_{j4}^HZ_{k4}^H - e^{i\eta}\lambda_7v_2Z_{i3}^HZ_{j4}^HZ_{k4}^H \\
& + v_1\lambda_5^*Z_{i3}^HZ_{j4}^HZ_{k4}^H + e^{3i\eta}v_2\lambda_7^*Z_{i3}^HZ_{j4}^HZ_{k4}^H + 3e^{i\eta}\lambda_7v_1Z_{i4}^HZ_{j4}^HZ_{k4}^H \\
& - 3e^{3i\eta}v_1\lambda_7^*Z_{i4}^HZ_{j4}^HZ_{k4}^H \\
& + Z_{i2}^H \left(-e^{i\eta}\lambda_6v_1Z_{j3}^HZ_{k1}^H + e^{4i\eta}\lambda_5v_2Z_{j3}^HZ_{k1}^H - v_2\lambda_5^*Z_{j3}^HZ_{k1}^H \right. \\
& + e^{3i\eta}v_1\lambda_6^*Z_{j3}^HZ_{k1}^H - e^{4i\eta}\lambda_5v_1Z_{j4}^HZ_{k1}^H + e^{i\eta}\lambda_7v_2Z_{j4}^HZ_{k1}^H + v_1\lambda_5^*Z_{j4}^HZ_{k1}^H \\
& - e^{3i\eta}v_2\lambda_7^*Z_{j4}^HZ_{k1}^H + e^{4i\eta}\lambda_5v_1Z_{j3}^HZ_{k2}^H - 3e^{i\eta}\lambda_7v_2Z_{j3}^HZ_{k2}^H \\
& - v_1\lambda_5^*Z_{j3}^HZ_{k2}^H + 3e^{3i\eta}v_2\lambda_7^*Z_{j3}^HZ_{k2}^H + e^{i\eta}\lambda_7v_1Z_{j4}^HZ_{k2}^H \\
& \left. - e^{3i\eta}v_1\lambda_7^*Z_{j4}^HZ_{k2}^H - ie^{i\eta}\lambda_6v_1Z_{j3}^HZ_{k3}^H - 2ie^{2i\eta}\lambda_3v_2Z_{j3}^HZ_{k3}^H \right)
\end{aligned}$$

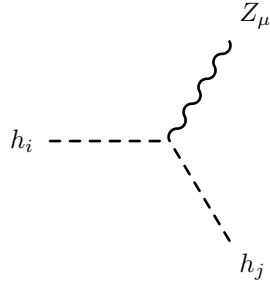
$$\begin{aligned}
& -2ie^{2i\eta}\lambda_4v_2Z_{j_3}^HZ_{k_3}^H + ie^{4i\eta}\lambda_5v_2Z_{j_3}^HZ_{k_3}^H + iv_2\lambda_5^*Z_{j_3}^HZ_{k_3}^H \\
& -ie^{3i\eta}v_1\lambda_6^*Z_{j_3}^HZ_{k_3}^H - ie^{4i\eta}\lambda_5v_1Z_{j_4}^HZ_{k_3}^H - ie^{i\eta}\lambda_7v_2Z_{j_4}^HZ_{k_3}^H \\
& -iv_1\lambda_5^*Z_{j_4}^HZ_{k_3}^H - ie^{3i\eta}v_2\lambda_7^*Z_{j_4}^HZ_{k_3}^H - ie^{4i\eta}\lambda_5v_1Z_{j_3}^HZ_{k_4}^H \\
& -ie^{i\eta}\lambda_7v_2Z_{j_3}^HZ_{k_4}^H - iv_1\lambda_5^*Z_{j_3}^HZ_{k_4}^H - ie^{3i\eta}v_2\lambda_7^*Z_{j_3}^HZ_{k_4}^H \\
& -ie^{i\eta}\lambda_7v_1Z_{j_4}^HZ_{k_4}^H - 4ie^{2i\eta}\lambda_2v_2Z_{j_4}^HZ_{k_4}^H - ie^{3i\eta}v_1\lambda_7^*Z_{j_4}^HZ_{k_4}^H \\
& + Z_{j_2}^H\left(-i\left(2e^{2i\eta}\lambda_3v_1 + 2e^{2i\eta}\lambda_4v_1 + 3e^{3i\eta}v_2\lambda_7^* + 3e^{i\eta}\lambda_7v_2 + e^{4i\eta}\lambda_5v_1 + v_1\lambda_5^*\right)Z_{k_1}^H\right. \\
& - 3ie^{i\eta}\left(4e^{i\eta}\lambda_2v_2 + e^{2i\eta}v_1\lambda_7^* + \lambda_7v_1\right)Z_{k_2}^H + e^{4i\eta}\lambda_5v_1Z_{k_3}^H - 3e^{i\eta}\lambda_7v_2Z_{k_3}^H \\
& \left.- v_1\lambda_5^*Z_{k_3}^H + 3e^{3i\eta}v_2\lambda_7^*Z_{k_3}^H + e^{i\eta}\lambda_7v_1Z_{k_4}^H - e^{3i\eta}v_1\lambda_7^*Z_{k_4}^H\right) \\
& + Z_{j_1}^H\left(-i\left(2e^{2i\eta}\lambda_3v_2 + 2e^{2i\eta}\lambda_4v_2 + 3e^{3i\eta}v_1\lambda_6^* + 3e^{i\eta}\lambda_6v_1 + e^{4i\eta}\lambda_5v_2 + v_2\lambda_5^*\right)Z_{k_1}^H\right. \\
& - i\left(2e^{2i\eta}\lambda_3v_1 + 2e^{2i\eta}\lambda_4v_1 + 3e^{3i\eta}v_2\lambda_7^* + 3e^{i\eta}\lambda_7v_2 + e^{4i\eta}\lambda_5v_1 + v_1\lambda_5^*\right)Z_{k_2}^H \\
& \left.- e^{i\eta}\lambda_6v_1Z_{k_3}^H + e^{4i\eta}\lambda_5v_2Z_{k_3}^H - v_2\lambda_5^*Z_{k_3}^H + e^{3i\eta}v_1\lambda_6^*Z_{k_3}^H\right. \\
& \left.- e^{4i\eta}\lambda_5v_1Z_{k_4}^H + e^{i\eta}\lambda_7v_2Z_{k_4}^H + v_1\lambda_5^*Z_{k_4}^H - e^{3i\eta}v_2\lambda_7^*Z_{k_4}^H\right) \\
& + Z_{i1}^H\left(-e^{i\eta}\lambda_6v_2Z_{j_3}^HZ_{k_1}^H + e^{3i\eta}v_2\lambda_6^*Z_{j_3}^HZ_{k_1}^H + 3e^{i\eta}\lambda_6v_1Z_{j_4}^HZ_{k_1}^H\right. \\
& - e^{4i\eta}\lambda_5v_2Z_{j_4}^HZ_{k_1}^H + v_2\lambda_5^*Z_{j_4}^HZ_{k_1}^H - 3e^{3i\eta}v_1\lambda_6^*Z_{j_4}^HZ_{k_1}^H \\
& - e^{i\eta}\lambda_6v_1Z_{j_3}^HZ_{k_2}^H + e^{4i\eta}\lambda_5v_2Z_{j_3}^HZ_{k_2}^H - v_2\lambda_5^*Z_{j_3}^HZ_{k_2}^H \\
& + e^{3i\eta}v_1\lambda_6^*Z_{j_3}^HZ_{k_2}^H - e^{4i\eta}\lambda_5v_1Z_{j_4}^HZ_{k_2}^H + e^{i\eta}\lambda_7v_2Z_{j_4}^HZ_{k_2}^H + v_1\lambda_5^*Z_{j_4}^HZ_{k_2}^H \\
& - e^{3i\eta}v_2\lambda_7^*Z_{j_4}^HZ_{k_2}^H - 4ie^{2i\eta}\lambda_1v_1Z_{j_3}^HZ_{k_3}^H - ie^{i\eta}\lambda_6v_2Z_{j_3}^HZ_{k_3}^H \\
& - ie^{3i\eta}v_2\lambda_6^*Z_{j_3}^HZ_{k_3}^H - ie^{i\eta}\lambda_6v_1Z_{j_4}^HZ_{k_3}^H - ie^{4i\eta}\lambda_5v_2Z_{j_4}^HZ_{k_3}^H \\
& - iv_2\lambda_5^*Z_{j_4}^HZ_{k_3}^H - ie^{3i\eta}v_1\lambda_6^*Z_{j_4}^HZ_{k_3}^H - ie^{i\eta}\lambda_6v_1Z_{j_3}^HZ_{k_4}^H \\
& - ie^{4i\eta}\lambda_5v_2Z_{j_3}^HZ_{k_4}^H - iv_2\lambda_5^*Z_{j_3}^HZ_{k_4}^H - ie^{3i\eta}v_1\lambda_6^*Z_{j_3}^HZ_{k_4}^H \\
& - 2ie^{2i\eta}\lambda_3v_1Z_{j_4}^HZ_{k_4}^H - 2ie^{2i\eta}\lambda_4v_1Z_{j_4}^HZ_{k_4}^H + ie^{4i\eta}\lambda_5v_1Z_{j_4}^HZ_{k_4}^H \\
& \left.- ie^{i\eta}\lambda_7v_2Z_{j_4}^HZ_{k_4}^H + iv_1\lambda_5^*Z_{j_4}^HZ_{k_4}^H - ie^{3i\eta}v_2\lambda_7^*Z_{j_4}^HZ_{k_4}^H\right) \\
& + Z_{j_1}^H\left(-3ie^{i\eta}\left(4e^{i\eta}\lambda_1v_1 + e^{2i\eta}v_2\lambda_6^* + \lambda_6v_2\right)Z_{k_1}^H\right. \\
& - i\left(2e^{2i\eta}\lambda_3v_2 + 2e^{2i\eta}\lambda_4v_2 + 3e^{3i\eta}v_1\lambda_6^* + 3e^{i\eta}\lambda_6v_1 + e^{4i\eta}\lambda_5v_2 + v_2\lambda_5^*\right)Z_{k_2}^H \\
& \left.- e^{i\eta}\lambda_6v_2Z_{k_3}^H + e^{3i\eta}v_2\lambda_6^*Z_{k_3}^H + 3e^{i\eta}\lambda_6v_1Z_{k_4}^H - e^{4i\eta}\lambda_5v_2Z_{k_4}^H + v_2\lambda_5^*Z_{k_4}^H\right. \\
& \left.- 3e^{3i\eta}v_1\lambda_6^*Z_{k_4}^H\right) \\
& + Z_{j_2}^H\left(-i\left(2e^{2i\eta}\lambda_3v_2 + 2e^{2i\eta}\lambda_4v_2 + 3e^{3i\eta}v_1\lambda_6^* + 3e^{i\eta}\lambda_6v_1 + e^{4i\eta}\lambda_5v_2 + v_2\lambda_5^*\right)Z_{k_1}^H\right. \\
& - i\left(2e^{2i\eta}\lambda_3v_1 + 2e^{2i\eta}\lambda_4v_1 + 3e^{3i\eta}v_2\lambda_7^* + 3e^{i\eta}\lambda_7v_2 + e^{4i\eta}\lambda_5v_1 + v_1\lambda_5^*\right)Z_{k_2}^H \\
& \left.- e^{i\eta}\lambda_6v_1Z_{k_3}^H + e^{4i\eta}\lambda_5v_2Z_{k_3}^H - v_2\lambda_5^*Z_{k_3}^H + e^{3i\eta}v_1\lambda_6^*Z_{k_3}^H\right)
\end{aligned}$$

$$- e^{4i\eta}\lambda_5 v_1 Z_{k4}^H + e^{i\eta}\lambda_7 v_2 Z_{k4}^H + v_1 \lambda_5^* Z_{k4}^H - e^{3i\eta} v_2 \lambda_7^* Z_{k4}^H \Big) \Big) \Big) \quad (108)$$

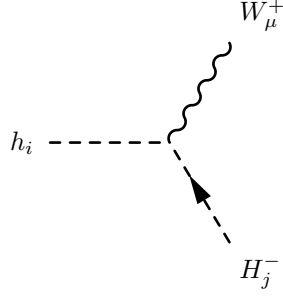


$$\begin{aligned} & \frac{1}{2} e^{-2i\eta} \left(Z_{j1}^{+,*} \left(\left(-v_1 Z_{i4}^H + v_2 Z_{i3}^H \right) \left(\left(e^{2i\eta} \lambda_4 - \lambda_5^* \right) Z_{k2}^+ + e^{i\eta} \left(e^{2i\eta} \lambda_6^* - \lambda_6 \right) Z_{k1}^+ \right) \right. \right. \\ & - i Z_{i2}^H \left(\left(2e^{i\eta} \lambda_7 v_2 + e^{2i\eta} \lambda_4 v_1 + v_1 \lambda_5^* \right) Z_{k2}^+ + e^{i\eta} \left(2e^{i\eta} \lambda_3 v_2 + e^{2i\eta} v_1 \lambda_6^* + \lambda_6 v_1 \right) Z_{k1}^+ \right) \\ & - i Z_{i1}^H \left(\left(2e^{i\eta} \lambda_6 v_1 + e^{2i\eta} \lambda_4 v_2 + v_2 \lambda_5^* \right) Z_{k2}^+ + e^{i\eta} \left(4e^{i\eta} \lambda_1 v_1 + e^{2i\eta} v_2 \lambda_6^* + \lambda_6 v_2 \right) Z_{k1}^+ \right) \\ & + e^{i\eta} Z_{j2}^{+,*} \left(\left(-v_1 Z_{i4}^H + v_2 Z_{i3}^H \right) \left(- \left(-e^{2i\eta} \lambda_7^* + \lambda_7 \right) Z_{k2}^+ + e^{i\eta} \left(e^{2i\eta} \lambda_5 - \lambda_4 \right) Z_{k1}^+ \right) \right. \\ & - i Z_{i2}^H \left(\left(4e^{i\eta} \lambda_2 v_2 + e^{2i\eta} v_1 \lambda_7^* + \lambda_7 v_1 \right) Z_{k2}^+ + e^{i\eta} \left(2e^{i\eta} v_2 \lambda_7^* + e^{2i\eta} \lambda_5 v_1 + \lambda_4 v_1 \right) Z_{k1}^+ \right) \\ & \left. \left. - i Z_{i1}^H \left(\left(2e^{i\eta} \lambda_3 v_1 + e^{2i\eta} v_2 \lambda_7^* + \lambda_7 v_2 \right) Z_{k2}^+ + e^{i\eta} \left(2e^{i\eta} v_1 \lambda_6^* + e^{2i\eta} \lambda_5 v_2 + \lambda_4 v_2 \right) Z_{k1}^+ \right) \right) \right) \Big) \Big) \quad (109) \end{aligned}$$

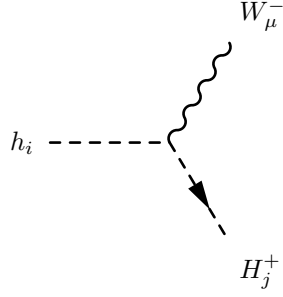
9.2 Two Scalar-One Vector Boson-Interaction



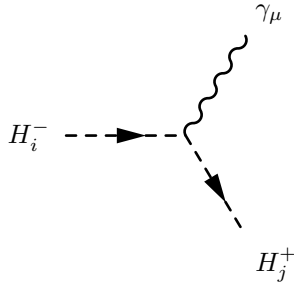
$$- \frac{1}{2} \left(g_1 \sin \Theta_W + g_2 \cos \Theta_W \right) \left(- Z_{i1}^H Z_{j3}^H - Z_{i2}^H Z_{j4}^H + Z_{i3}^H Z_{j1}^H + Z_{i4}^H Z_{j2}^H \right) \left(- p_\mu^{h_j} + p_\mu^{h_i} \right) \quad (110)$$



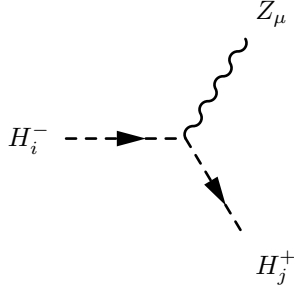
$$\frac{1}{2}g_2 \left(Z_{j1}^{+,*} \left(-iZ_{i1}^H + Z_{i3}^H \right) + Z_{j2}^{+,*} \left(-iZ_{i2}^H + Z_{i4}^H \right) \right) \left(-p_\mu^{H_j^-} + p_\mu^{h_i} \right) \quad (111)$$



$$\frac{1}{2}g_2 \left(iZ_{i1}^H Z_{j1}^+ + \left(iZ_{i2}^H + Z_{i4}^H \right) Z_{j2}^+ + Z_{i3}^H Z_{j1}^+ \right) \left(-p_\mu^{H_j^+} + p_\mu^{h_i} \right) \quad (112)$$

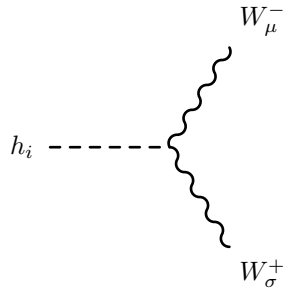


$$\frac{i}{2} \left(g_1 \cos \Theta_W + g_2 \sin \Theta_W \right) \left(Z_{i1}^{+,*} Z_{j1}^+ + Z_{i2}^{+,*} Z_{j2}^+ \right) \left(-p_\mu^{H_j^+} + p_\mu^{H_i^-} \right) \quad (113)$$

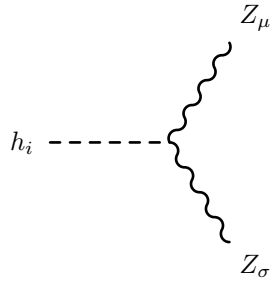


$$\frac{i}{2} \left(-g_1 \sin \Theta_W + g_2 \cos \Theta_W \right) \left(Z_{i1}^{+,*} Z_{j1}^+ + Z_{i2}^{+,*} Z_{j2}^+ \right) \left(-p_\mu^{H_j^+} + p_\mu^{H_i^-} \right) \quad (114)$$

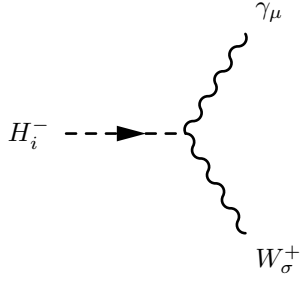
9.3 One Scalar-Two Vector Boson-Interaction



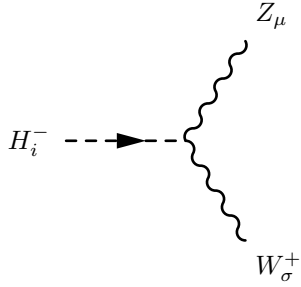
$$\frac{i}{2} g_2^2 \left(v_1 Z_{i1}^H + v_2 Z_{i2}^H \right) \left(g_{\sigma\mu} \right) \quad (115)$$



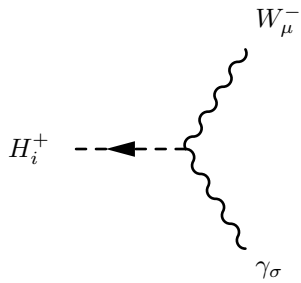
$$\frac{i}{2} \left(g_1 \sin \Theta_W + g_2 \cos \Theta_W \right)^2 \left(v_1 Z_{i1}^H + v_2 Z_{i2}^H \right) \left(g_{\sigma\mu} \right) \quad (116)$$



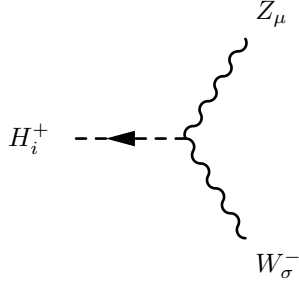
$$\frac{i}{2} g_1 g_2 (v_1 Z_{i1}^{+,*} + v_2 Z_{i2}^{+,*}) \cos \Theta_W (g_{\sigma\mu}) \quad (117)$$



$$-\frac{i}{2} g_1 g_2 (v_1 Z_{i1}^{+,*} + v_2 Z_{i2}^{+,*}) \sin \Theta_W (g_{\sigma\mu}) \quad (118)$$

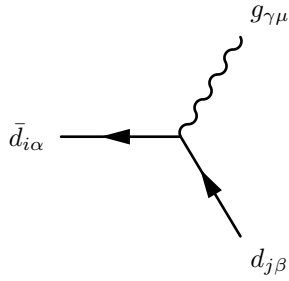


$$\frac{i}{2} g_1 g_2 \cos \Theta_W (v_1 Z_{i1}^+ + v_2 Z_{i2}^+) (g_{\sigma\mu}) \quad (119)$$



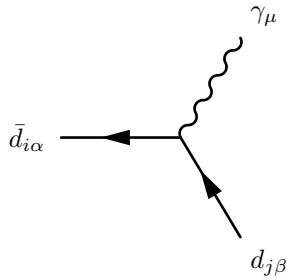
$$-\frac{i}{2}g_1g_2\sin\Theta_W\left(v_1Z_{i1}^+ + v_2Z_{i2}^+\right)\left(g_{\sigma\mu}\right) \quad (120)$$

9.4 Two Fermion-One Vector Boson-Interaction



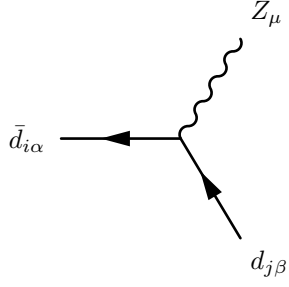
$$-\frac{i}{2}g_3\delta_{ij}\lambda_{\alpha,\beta}^\gamma\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (121)$$

$$+\frac{i}{2}g_3\delta_{ij}\lambda_{\alpha,\beta}^\gamma\left(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\right) \quad (122)$$



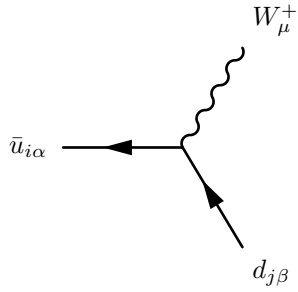
$$-\frac{i}{6}\delta_{\alpha\beta}\delta_{ij}\left(-3g_2\sin\Theta_W + g_1\cos\Theta_W\right)\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (123)$$

$$+\frac{i}{3}g_1\cos\Theta_W\delta_{\alpha\beta}\delta_{ij}\left(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\right) \quad (124)$$

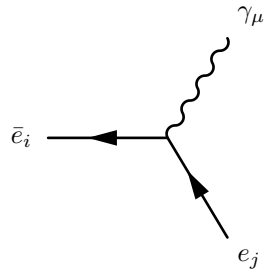


$$\frac{i}{6}\delta_{\alpha\beta}\delta_{ij}\left(3g_2\cos\Theta_W + g_1\sin\Theta_W\right)\left(\gamma_\mu \cdot \frac{1-\gamma_5}{2}\right) \quad (125)$$

$$+ \frac{i}{3}g_1\delta_{\alpha\beta}\delta_{ij}\sin\Theta_W\left(\gamma_\mu \cdot \frac{1+\gamma_5}{2}\right) \quad (126)$$

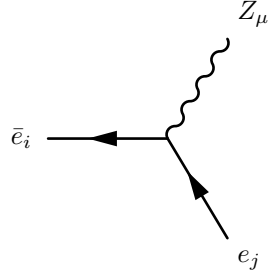


$$-i\frac{1}{\sqrt{2}}g_2\delta_{\alpha\beta}\sum_{a=1}^3U_{L,ja}^{d,*}U_{L,ia}^u\left(\gamma_\mu \cdot \frac{1-\gamma_5}{2}\right) \quad (127)$$



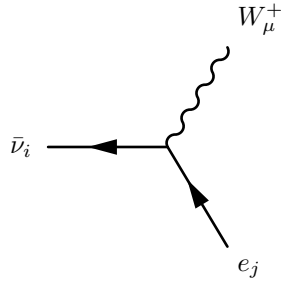
$$\frac{i}{2}\delta_{ij}\left(g_1\cos\Theta_W + g_2\sin\Theta_W\right)\left(\gamma_\mu \cdot \frac{1-\gamma_5}{2}\right) \quad (128)$$

$$+ ig_1\cos\Theta_W\delta_{ij}\left(\gamma_\mu \cdot \frac{1+\gamma_5}{2}\right) \quad (129)$$

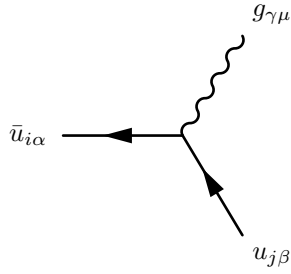


$$\frac{i}{2}\delta_{ij}\left(-g_1\sin\Theta_W+g_2\cos\Theta_W\right)\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (130)$$

$$+ -ig_1\delta_{ij}\sin\Theta_W\left(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\right) \quad (131)$$

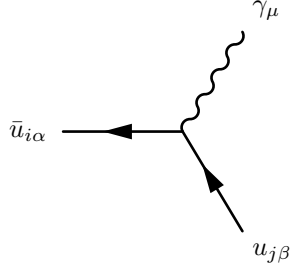


$$-i\frac{1}{\sqrt{2}}g_2U_{L,ji}^{e,*}\Theta_{i,3}\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (132)$$



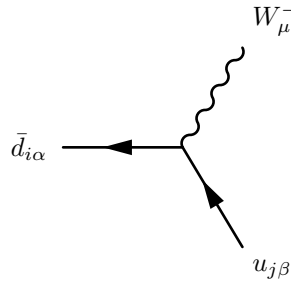
$$-\frac{i}{2}g_3\delta_{ij}\lambda_{\alpha,\beta}^\gamma\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (133)$$

$$+\frac{i}{2}g_3\delta_{ij}\lambda_{\alpha,\beta}^\gamma\left(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\right) \quad (134)$$

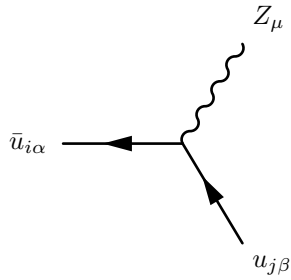


$$-\frac{i}{6}\delta_{\alpha\beta}\delta_{ij}\left(3g_2\sin\Theta_W+g_1\cos\Theta_W\right)\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (135)$$

$$+\frac{2i}{3}g_1\cos\Theta_W\delta_{\alpha\beta}\delta_{ij}\left(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\right) \quad (136)$$

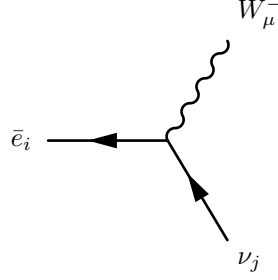


$$-i\frac{1}{\sqrt{2}}g_2\delta_{\alpha\beta}\sum_{a=1}^3U_{L,ja}^{u,*}U_{L,ia}^d\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (137)$$

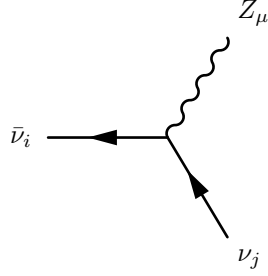


$$-\frac{i}{6}\delta_{\alpha\beta}\delta_{ij}\left(3g_2\cos\Theta_W-g_1\sin\Theta_W\right)\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (138)$$

$$+\frac{2i}{3}g_1\delta_{\alpha\beta}\delta_{ij}\sin\Theta_W\left(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\right) \quad (139)$$

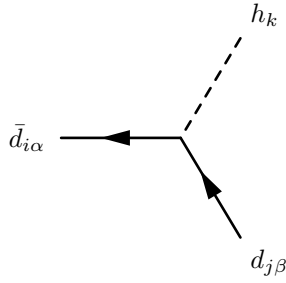


$$-i \frac{1}{\sqrt{2}} g_2 \Theta_{j,3} U_{L,ij}^e \left(\gamma_\mu \cdot \frac{1-\gamma_5}{2} \right) \quad (140)$$



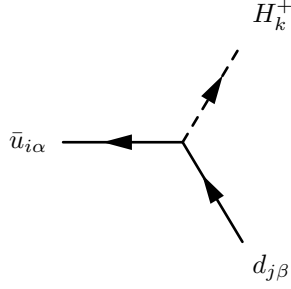
$$-\frac{i}{2} \delta_{ij} \left(g_1 \sin \Theta_W + g_2 \cos \Theta_W \right) \left(\gamma_\mu \cdot \frac{1-\gamma_5}{2} \right) \quad (141)$$

9.5 Two Fermion-One Scalar Boson-Interaction



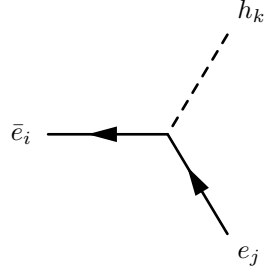
$$-i \frac{1}{\sqrt{2}} e^{-i\eta} \delta_{\alpha\beta} \left(e^{i\eta} \sum_{b=1}^3 U_{L,jb}^{d,*} \sum_{a=1}^3 U_{R,ia}^{d,*} \epsilon_{d,ab} \left(-iZ_{k4}^H + Z_{k2}^H \right) + \sum_{b=1}^3 U_{L,jb}^{d,*} \sum_{a=1}^3 U_{R,ia}^{d,*} Y_{d,ab} \left(-iZ_{k3}^H + Z_{k1}^H \right) \right) \left(\frac{1-\gamma_5}{2} \right) \quad (142)$$

$$+ \frac{1}{\sqrt{2}} \delta_{\alpha\beta} \left(e^{i\eta} \sum_{b=1}^3 \sum_{a=1}^3 Y_{d,ab}^* U_{R,ja}^d U_{L,ib}^d \left(-iZ_{k1}^H + Z_{k3}^H \right) + \sum_{b=1}^3 \sum_{a=1}^3 \epsilon_{d,ab}^* U_{R,ja}^d U_{L,ib}^d \left(-iZ_{k2}^H + Z_{k4}^H \right) \right) \left(\frac{1+\gamma_5}{2} \right) \quad (143)$$



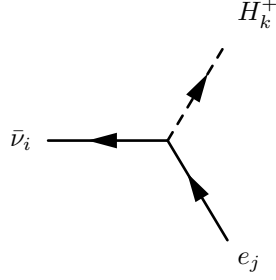
$$- i\delta_{\alpha\beta} \left(e^{i\eta} \sum_{b=1}^3 U_{L,jb}^{d,*} \sum_{a=1}^3 U_{R,ia}^{u,*} \epsilon_{u,ab} Z_{k1}^+ + \sum_{b=1}^3 U_{L,jb}^{d,*} \sum_{a=1}^3 U_{R,ia}^{u,*} Y_{u,ab} Z_{k2}^+ \right) \left(\frac{1-\gamma_5}{2} \right) \quad (144)$$

$$+ -i\delta_{\alpha\beta} \left(e^{i\eta} \sum_{b=1}^3 \sum_{a=1}^3 Y_{d,ab}^* U_{R,ja}^d U_{L,ib}^u Z_{k1}^+ + \sum_{b=1}^3 \sum_{a=1}^3 \epsilon_{d,ab}^* U_{R,ja}^d U_{L,ib}^u Z_{k2}^+ \right) \left(\frac{1+\gamma_5}{2} \right) \quad (145)$$



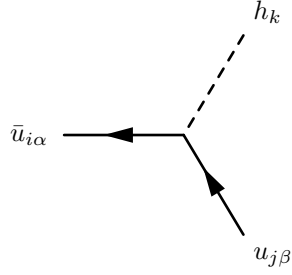
$$\frac{1}{\sqrt{2}} e^{-i\eta} \left(-ie^{i\eta} \sum_{b=1}^3 U_{L,jb}^{e,*} \sum_{a=1}^3 U_{R,ia}^{e,*} \epsilon_{e,ab} \left(-iZ_{k4}^H + Z_{k2}^H \right) + \sum_{b=1}^3 U_{L,jb}^{e,*} \sum_{a=1}^3 U_{R,ia}^{e,*} Y_{e,ab} \left(-iZ_{k1}^H - Z_{k3}^H \right) \right) \left(\frac{1-\gamma_5}{2} \right) \quad (146)$$

$$+ \frac{1}{\sqrt{2}} \left(e^{i\eta} \sum_{b=1}^3 \sum_{a=1}^3 Y_{e,ab}^* U_{R,ja}^e U_{L,ib}^e \left(-iZ_{k1}^H + Z_{k3}^H \right) + \sum_{b=1}^3 \sum_{a=1}^3 \epsilon_{e,ab}^* U_{R,ja}^e U_{L,ib}^e \left(-iZ_{k2}^H + Z_{k4}^H \right) \right) \left(\frac{1+\gamma_5}{2} \right) \quad (147)$$



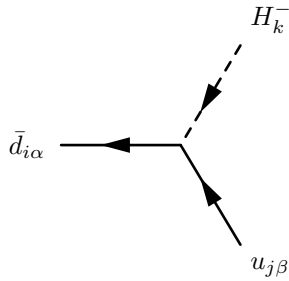
(148)

$$+ -i \left(e^{i\eta} \sum_{a=1}^3 Y_{e,ai}^* U_{R,ja}^e Z_{k1}^+ + \sum_{a=1}^3 \epsilon_{e,ai}^* U_{R,ja}^e Z_{k2}^+ \right) \left(\frac{1 + \gamma_5}{2} \right) \quad (149)$$



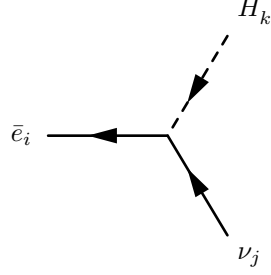
$$i \frac{1}{\sqrt{2}} \delta_{\alpha\beta} \left(e^{i\eta} \sum_{b=1}^3 U_{L,jb}^{u,*} \sum_{a=1}^3 U_{R,ia}^{u,*} \epsilon_{u,ab} (iZ_{k3}^H + Z_{k1}^H) + \sum_{b=1}^3 U_{L,jb}^{u,*} \sum_{a=1}^3 U_{R,ia}^{u,*} Y_{u,ab} (iZ_{k4}^H + Z_{k2}^H) \right) \left(\frac{1 - \gamma_5}{2} \right) \quad (150)$$

$$+ \frac{1}{\sqrt{2}} e^{-i\eta} \delta_{\alpha\beta} \left(e^{i\eta} \sum_{b=1}^3 \sum_{a=1}^3 Y_{u,ab}^* U_{R,ja}^u U_{L,ib}^u (iZ_{k2}^H + Z_{k4}^H) + \sum_{b=1}^3 \sum_{a=1}^3 \epsilon_{u,ab}^* U_{R,ja}^u U_{L,ib}^u (iZ_{k1}^H + Z_{k3}^H) \right) \left(\frac{1 + \gamma_5}{2} \right) \quad (151)$$



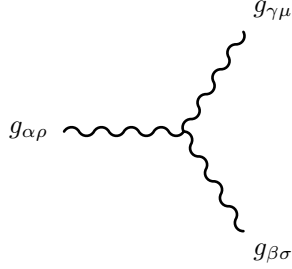
$$-i\delta_{\alpha\beta}\left(e^{-i\eta}Z_{k1}^{+,*}\sum_{b=1}^3U_{L,jb}^{u,*}\sum_{a=1}^3U_{R,ia}^{d,*}Y_{d,ab}+Z_{k2}^{+,*}\sum_{b=1}^3U_{L,jb}^{u,*}\sum_{a=1}^3U_{R,ia}^{d,*}\epsilon_{d,ab}\right)\left(\frac{1-\gamma_5}{2}\right) \quad (152)$$

$$+ -i\delta_{\alpha\beta}\left(e^{-i\eta}Z_{k1}^{+,*}\sum_{b=1}^3\sum_{a=1}^3\epsilon_{u,ab}^*U_{R,ja}^uU_{L,ib}^d+Z_{k2}^{+,*}\sum_{b=1}^3\sum_{a=1}^3Y_{u,ab}^*U_{R,ja}^uU_{L,ib}^d\right)\left(\frac{1+\gamma_5}{2}\right) \quad (153)$$

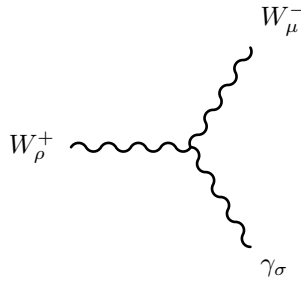


$$-i\left(e^{-i\eta}Z_{k1}^{+,*}\sum_{a=1}^3U_{R,ia}^{e,*}Y_{e,aj}+Z_{k2}^{+,*}\sum_{a=1}^3U_{R,ia}^{e,*}\epsilon_{e,aj}\right)\left(\frac{1-\gamma_5}{2}\right) \quad (154)$$

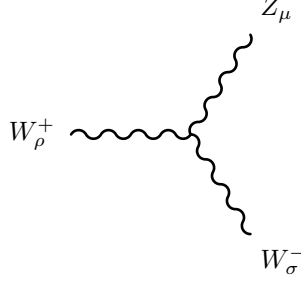
9.6 Three Vector Boson-Interaction



$$g_3f_{\alpha,\beta,\gamma}\left(g_{\rho\mu}\left(-p_\sigma^{g\gamma\mu}+p_\sigma^{g\alpha\rho}\right)+g_{\rho\sigma}\left(-p_\mu^{g\alpha\rho}+p_\mu^{g\beta\sigma}\right)+g_{\sigma\mu}\left(-p_\rho^{g\beta\sigma}+p_\rho^{g\gamma\mu}\right)\right) \quad (155)$$

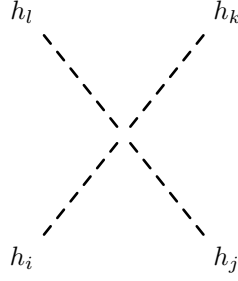


$$ig_2 \sin \Theta_W \left(g_{\rho\mu} \left(-p_\sigma^{W^-} + p_\sigma^{W^+} \right) + g_{\rho\sigma} \left(-p_\mu^{W^+} + p_\mu^{\gamma\sigma} \right) + g_{\sigma\mu} \left(-p_\rho^{\gamma\sigma} + p_\rho^{W^-} \right) \right) \quad (156)$$



$$-ig_2 \cos \Theta_W \left(g_{\rho\mu} \left(-p_\sigma^{Z^0} + p_\sigma^{W^+} \right) + g_{\rho\sigma} \left(-p_\mu^{W^+} + p_\mu^{W^-} \right) + g_{\sigma\mu} \left(-p_\rho^{W^-} + p_\rho^{Z^0} \right) \right) \quad (157)$$

9.7 Four Scalar-Interaction



$$\begin{aligned} & \frac{1}{2} e^{-2i\eta} \left(3e^{i\eta} \lambda_6 Z_{i4}^H Z_{j1}^H Z_{k1}^H Z_{l1}^H - 3e^{3i\eta} \lambda_6^* Z_{i4}^H Z_{j1}^H Z_{k1}^H Z_{l1}^H \right. \\ & - e^{i\eta} \lambda_6 Z_{i3}^H Z_{j2}^H Z_{k1}^H Z_{l1}^H + e^{3i\eta} \lambda_6^* Z_{i3}^H Z_{j2}^H Z_{k1}^H Z_{l1}^H \\ & - e^{4i\eta} \lambda_5 Z_{i4}^H Z_{j2}^H Z_{k1}^H Z_{l1}^H + \lambda_5^* Z_{i4}^H Z_{j2}^H Z_{k1}^H Z_{l1}^H - 4ie^{2i\eta} \lambda_1 Z_{i3}^H Z_{j3}^H Z_{k1}^H Z_{l1}^H \\ & - ie^{i\eta} \lambda_6 Z_{i4}^H Z_{j3}^H Z_{k1}^H Z_{l1}^H - ie^{3i\eta} \lambda_6^* Z_{i4}^H Z_{j3}^H Z_{k1}^H Z_{l1}^H \\ & - ie^{i\eta} \lambda_6 Z_{i3}^H Z_{j4}^H Z_{k1}^H Z_{l1}^H - ie^{3i\eta} \lambda_6^* Z_{i3}^H Z_{j4}^H Z_{k1}^H Z_{l1}^H \\ & - 2ie^{2i\eta} \lambda_3 Z_{i4}^H Z_{j4}^H Z_{k1}^H Z_{l1}^H - 2ie^{2i\eta} \lambda_4 Z_{i4}^H Z_{j4}^H Z_{k1}^H Z_{l1}^H \\ & + ie^{4i\eta} \lambda_5 Z_{i4}^H Z_{j4}^H Z_{k1}^H Z_{l1}^H + i\lambda_5^* Z_{i4}^H Z_{j4}^H Z_{k1}^H Z_{l1}^H - e^{i\eta} \lambda_6 Z_{i3}^H Z_{j1}^H Z_{k2}^H Z_{l1}^H \\ & + e^{3i\eta} \lambda_6^* Z_{i3}^H Z_{j1}^H Z_{k2}^H Z_{l1}^H - e^{4i\eta} \lambda_5 Z_{i4}^H Z_{j1}^H Z_{k2}^H Z_{l1}^H + \lambda_5^* Z_{i4}^H Z_{j1}^H Z_{k2}^H Z_{l1}^H \\ & + e^{4i\eta} \lambda_5 Z_{i3}^H Z_{j2}^H Z_{k2}^H Z_{l1}^H - \lambda_5^* Z_{i3}^H Z_{j2}^H Z_{k2}^H Z_{l1}^H + e^{i\eta} \lambda_7 Z_{i4}^H Z_{j2}^H Z_{k2}^H Z_{l1}^H \\ & - e^{3i\eta} \lambda_7^* Z_{i4}^H Z_{j2}^H Z_{k2}^H Z_{l1}^H - ie^{i\eta} \lambda_6 Z_{i3}^H Z_{j3}^H Z_{k2}^H Z_{l1}^H \\ & \left. - ie^{3i\eta} \lambda_6^* Z_{i3}^H Z_{j3}^H Z_{k2}^H Z_{l1}^H - ie^{4i\eta} \lambda_5 Z_{i4}^H Z_{j3}^H Z_{k2}^H Z_{l1}^H - i\lambda_5^* Z_{i4}^H Z_{j3}^H Z_{k2}^H Z_{l1}^H \right) \end{aligned}$$

$$\begin{aligned}
& -ie^{4i\eta}\lambda_5 Z_{i3}^H Z_{j4}^H Z_{k2}^H Z_{l1}^H - i\lambda_5^* Z_{i3}^H Z_{j4}^H Z_{k2}^H Z_{l1}^H - ie^{i\eta}\lambda_7 Z_{i4}^H Z_{j4}^H Z_{k2}^H Z_{l1}^H \\
& - ie^{3i\eta}\lambda_7^* Z_{i4}^H Z_{j4}^H Z_{k2}^H Z_{l1}^H - 4ie^{2i\eta}\lambda_1 Z_{i3}^H Z_{j1}^H Z_{k3}^H Z_{l1}^H \\
& - ie^{i\eta}\lambda_6 Z_{i4}^H Z_{j1}^H Z_{k3}^H Z_{l1}^H - ie^{3i\eta}\lambda_6^* Z_{i4}^H Z_{j1}^H Z_{k3}^H Z_{l1}^H \\
& - ie^{i\eta}\lambda_6 Z_{i3}^H Z_{j2}^H Z_{k3}^H Z_{l1}^H - ie^{3i\eta}\lambda_6^* Z_{i3}^H Z_{j2}^H Z_{k3}^H Z_{l1}^H \\
& - ie^{4i\eta}\lambda_5 Z_{i4}^H Z_{j2}^H Z_{k3}^H Z_{l1}^H - i\lambda_5^* Z_{i4}^H Z_{j2}^H Z_{k3}^H Z_{l1}^H + e^{i\eta}\lambda_6 Z_{i4}^H Z_{j3}^H Z_{k3}^H Z_{l1}^H \\
& - e^{3i\eta}\lambda_6^* Z_{i4}^H Z_{j3}^H Z_{k3}^H Z_{l1}^H + e^{i\eta}\lambda_6 Z_{i3}^H Z_{j4}^H Z_{k3}^H Z_{l1}^H \\
& - e^{3i\eta}\lambda_6^* Z_{i3}^H Z_{j4}^H Z_{k3}^H Z_{l1}^H - e^{4i\eta}\lambda_5 Z_{i4}^H Z_{j4}^H Z_{k3}^H Z_{l1}^H + \lambda_5^* Z_{i4}^H Z_{j4}^H Z_{k3}^H Z_{l1}^H \\
& - ie^{i\eta}\lambda_6 Z_{i3}^H Z_{j1}^H Z_{k4}^H Z_{l1}^H - ie^{3i\eta}\lambda_6^* Z_{i3}^H Z_{j1}^H Z_{k4}^H Z_{l1}^H \\
& - 2ie^{2i\eta}\lambda_3 Z_{i4}^H Z_{j1}^H Z_{k4}^H Z_{l1}^H - 2ie^{2i\eta}\lambda_4 Z_{i4}^H Z_{j1}^H Z_{k4}^H Z_{l1}^H \\
& + ie^{4i\eta}\lambda_5 Z_{i4}^H Z_{j1}^H Z_{k4}^H Z_{l1}^H + i\lambda_5^* Z_{i4}^H Z_{j1}^H Z_{k4}^H Z_{l1}^H - ie^{4i\eta}\lambda_5 Z_{i3}^H Z_{j2}^H Z_{k4}^H Z_{l1}^H \\
& - i\lambda_5^* Z_{i3}^H Z_{j2}^H Z_{k4}^H Z_{l1}^H - ie^{i\eta}\lambda_7 Z_{i4}^H Z_{j2}^H Z_{k4}^H Z_{l1}^H - ie^{3i\eta}\lambda_7^* Z_{i4}^H Z_{j2}^H Z_{k4}^H Z_{l1}^H \\
& + e^{i\eta}\lambda_6 Z_{i3}^H Z_{j3}^H Z_{k4}^H Z_{l1}^H - e^{3i\eta}\lambda_6^* Z_{i3}^H Z_{j3}^H Z_{k4}^H Z_{l1}^H \\
& - e^{4i\eta}\lambda_5 Z_{i4}^H Z_{j3}^H Z_{k4}^H Z_{l1}^H + \lambda_5^* Z_{i4}^H Z_{j3}^H Z_{k4}^H Z_{l1}^H - e^{4i\eta}\lambda_5 Z_{i3}^H Z_{j4}^H Z_{k4}^H Z_{l1}^H \\
& + \lambda_5^* Z_{i3}^H Z_{j4}^H Z_{k4}^H Z_{l1}^H + 3e^{i\eta}\lambda_7 Z_{i4}^H Z_{j4}^H Z_{k4}^H Z_{l1}^H - 3e^{3i\eta}\lambda_7^* Z_{i4}^H Z_{j4}^H Z_{k4}^H Z_{l1}^H \\
& - e^{i\eta}\lambda_6 Z_{i3}^H Z_{j1}^H Z_{k1}^H Z_{l2}^H + e^{3i\eta}\lambda_6^* Z_{i3}^H Z_{j1}^H Z_{k1}^H Z_{l2}^H \\
& - e^{4i\eta}\lambda_5 Z_{i4}^H Z_{j1}^H Z_{k1}^H Z_{l2}^H + \lambda_5^* Z_{i4}^H Z_{j1}^H Z_{k1}^H Z_{l2}^H + e^{4i\eta}\lambda_5 Z_{i3}^H Z_{j2}^H Z_{k1}^H Z_{l2}^H \\
& - \lambda_5^* Z_{i3}^H Z_{j2}^H Z_{k1}^H Z_{l2}^H + e^{i\eta}\lambda_7 Z_{i4}^H Z_{j2}^H Z_{k1}^H Z_{l2}^H - e^{3i\eta}\lambda_7^* Z_{i4}^H Z_{j2}^H Z_{k1}^H Z_{l2}^H \\
& - ie^{i\eta}\lambda_6 Z_{i3}^H Z_{j3}^H Z_{k1}^H Z_{l2}^H - ie^{3i\eta}\lambda_6^* Z_{i3}^H Z_{j3}^H Z_{k1}^H Z_{l2}^H \\
& - ie^{4i\eta}\lambda_5 Z_{i4}^H Z_{j3}^H Z_{k1}^H Z_{l2}^H - i\lambda_5^* Z_{i4}^H Z_{j3}^H Z_{k1}^H Z_{l2}^H - ie^{4i\eta}\lambda_5 Z_{i3}^H Z_{j4}^H Z_{k1}^H Z_{l2}^H \\
& - i\lambda_5^* Z_{i3}^H Z_{j4}^H Z_{k1}^H Z_{l2}^H - ie^{i\eta}\lambda_7 Z_{i4}^H Z_{j4}^H Z_{k1}^H Z_{l2}^H - ie^{3i\eta}\lambda_7^* Z_{i4}^H Z_{j4}^H Z_{k1}^H Z_{l2}^H \\
& + e^{4i\eta}\lambda_5 Z_{i3}^H Z_{j1}^H Z_{k2}^H Z_{l2}^H - \lambda_5^* Z_{i3}^H Z_{j1}^H Z_{k2}^H Z_{l2}^H + e^{i\eta}\lambda_7 Z_{i4}^H Z_{j1}^H Z_{k2}^H Z_{l2}^H \\
& - e^{3i\eta}\lambda_7^* Z_{i4}^H Z_{j1}^H Z_{k2}^H Z_{l2}^H - 3e^{i\eta}\lambda_7 Z_{i3}^H Z_{j2}^H Z_{k2}^H Z_{l2}^H \\
& + 3e^{3i\eta}\lambda_7^* Z_{i3}^H Z_{j2}^H Z_{k2}^H Z_{l2}^H - 2ie^{2i\eta}\lambda_3 Z_{i3}^H Z_{j3}^H Z_{k2}^H Z_{l2}^H \\
& - 2ie^{2i\eta}\lambda_4 Z_{i3}^H Z_{j3}^H Z_{k2}^H Z_{l2}^H + ie^{4i\eta}\lambda_5 Z_{i3}^H Z_{j3}^H Z_{k2}^H Z_{l2}^H + i\lambda_5^* Z_{i3}^H Z_{j3}^H Z_{k2}^H Z_{l2}^H \\
& - ie^{i\eta}\lambda_7 Z_{i4}^H Z_{j3}^H Z_{k2}^H Z_{l2}^H - ie^{3i\eta}\lambda_7^* Z_{i4}^H Z_{j3}^H Z_{k2}^H Z_{l2}^H \\
& - ie^{i\eta}\lambda_7 Z_{i3}^H Z_{j4}^H Z_{k2}^H Z_{l2}^H - ie^{3i\eta}\lambda_7^* Z_{i3}^H Z_{j4}^H Z_{k2}^H Z_{l2}^H \\
& - 4ie^{2i\eta}\lambda_2 Z_{i4}^H Z_{j4}^H Z_{k2}^H Z_{l2}^H - ie^{i\eta}\lambda_6 Z_{i3}^H Z_{j1}^H Z_{k3}^H Z_{l2}^H \\
& - ie^{3i\eta}\lambda_6^* Z_{i3}^H Z_{j1}^H Z_{k3}^H Z_{l2}^H - ie^{4i\eta}\lambda_5 Z_{i4}^H Z_{j1}^H Z_{k3}^H Z_{l2}^H - i\lambda_5^* Z_{i4}^H Z_{j1}^H Z_{k3}^H Z_{l2}^H \\
& - 2ie^{2i\eta}\lambda_3 Z_{i3}^H Z_{j2}^H Z_{k3}^H Z_{l2}^H - 2ie^{2i\eta}\lambda_4 Z_{i3}^H Z_{j2}^H Z_{k3}^H Z_{l2}^H \\
& + ie^{4i\eta}\lambda_5 Z_{i3}^H Z_{j2}^H Z_{k3}^H Z_{l2}^H + i\lambda_5^* Z_{i3}^H Z_{j2}^H Z_{k3}^H Z_{l2}^H - ie^{i\eta}\lambda_7 Z_{i4}^H Z_{j2}^H Z_{k3}^H Z_{l2}^H \\
& - ie^{3i\eta}\lambda_7^* Z_{i4}^H Z_{j2}^H Z_{k3}^H Z_{l2}^H - 3e^{i\eta}\lambda_6 Z_{i3}^H Z_{j3}^H Z_{k3}^H Z_{l2}^H \\
& + 3e^{3i\eta}\lambda_6^* Z_{i3}^H Z_{j3}^H Z_{k3}^H Z_{l2}^H + e^{4i\eta}\lambda_5 Z_{i4}^H Z_{j3}^H Z_{k3}^H Z_{l2}^H - \lambda_5^* Z_{i4}^H Z_{j3}^H Z_{k3}^H Z_{l2}^H \\
& + e^{4i\eta}\lambda_5 Z_{i3}^H Z_{j4}^H Z_{k3}^H Z_{l2}^H - \lambda_5^* Z_{i3}^H Z_{j4}^H Z_{k3}^H Z_{l2}^H - e^{i\eta}\lambda_7 Z_{i4}^H Z_{j4}^H Z_{k3}^H Z_{l2}^H
\end{aligned}$$

$$\begin{aligned}
& + e^{3i\eta} \lambda_7^* Z_{i4}^H Z_{j4}^H Z_{k3}^H Z_{l2}^H - ie^{4i\eta} \lambda_5 Z_{i3}^H Z_{j1}^H Z_{k4}^H Z_{l2}^H - i \lambda_5^* Z_{i3}^H Z_{j1}^H Z_{k4}^H Z_{l2}^H \\
& - ie^{i\eta} \lambda_7 Z_{i4}^H Z_{j1}^H Z_{k4}^H Z_{l2}^H - ie^{3i\eta} \lambda_7^* Z_{i4}^H Z_{j1}^H Z_{k4}^H Z_{l2}^H \\
& - ie^{i\eta} \lambda_7 Z_{i3}^H Z_{j2}^H Z_{k4}^H Z_{l2}^H - ie^{3i\eta} \lambda_7^* Z_{i3}^H Z_{j2}^H Z_{k4}^H Z_{l2}^H \\
& - 4ie^{2i\eta} \lambda_2 Z_{i4}^H Z_{j2}^H Z_{k4}^H Z_{l2}^H + e^{4i\eta} \lambda_5 Z_{i3}^H Z_{j3}^H Z_{k4}^H Z_{l2}^H - \lambda_5^* Z_{i3}^H Z_{j3}^H Z_{k4}^H Z_{l2}^H \\
& - e^{i\eta} \lambda_7 Z_{i4}^H Z_{j3}^H Z_{k4}^H Z_{l2}^H + e^{3i\eta} \lambda_7^* Z_{i4}^H Z_{j3}^H Z_{k4}^H Z_{l2}^H \\
& - e^{i\eta} \lambda_7 Z_{i3}^H Z_{j4}^H Z_{k4}^H Z_{l2}^H + e^{3i\eta} \lambda_7^* Z_{i3}^H Z_{j4}^H Z_{k4}^H Z_{l2}^H \\
& - 4ie^{2i\eta} \lambda_1 Z_{i3}^H Z_{j1}^H Z_{k1}^H Z_{l3}^H - ie^{i\eta} \lambda_6 Z_{i4}^H Z_{j1}^H Z_{k1}^H Z_{l3}^H \\
& - ie^{3i\eta} \lambda_6^* Z_{i4}^H Z_{j1}^H Z_{k1}^H Z_{l3}^H - ie^{i\eta} \lambda_6 Z_{i3}^H Z_{j2}^H Z_{k1}^H Z_{l3}^H \\
& - ie^{3i\eta} \lambda_6^* Z_{i3}^H Z_{j2}^H Z_{k1}^H Z_{l3}^H - ie^{4i\eta} \lambda_5 Z_{i4}^H Z_{j2}^H Z_{k1}^H Z_{l3}^H - i \lambda_5^* Z_{i4}^H Z_{j2}^H Z_{k1}^H Z_{l3}^H \\
& + e^{i\eta} \lambda_6 Z_{i4}^H Z_{j3}^H Z_{k1}^H Z_{l3}^H - e^{3i\eta} \lambda_6^* Z_{i4}^H Z_{j3}^H Z_{k1}^H Z_{l3}^H \\
& + e^{i\eta} \lambda_6 Z_{i3}^H Z_{j4}^H Z_{k1}^H Z_{l3}^H - e^{3i\eta} \lambda_6^* Z_{i3}^H Z_{j4}^H Z_{k1}^H Z_{l3}^H \\
& - e^{4i\eta} \lambda_5 Z_{i4}^H Z_{j4}^H Z_{k1}^H Z_{l3}^H + \lambda_5^* Z_{i4}^H Z_{j4}^H Z_{k1}^H Z_{l3}^H - ie^{i\eta} \lambda_6 Z_{i3}^H Z_{j1}^H Z_{k2}^H Z_{l3}^H \\
& - ie^{3i\eta} \lambda_6^* Z_{i3}^H Z_{j1}^H Z_{k2}^H Z_{l3}^H - ie^{4i\eta} \lambda_5 Z_{i4}^H Z_{j1}^H Z_{k2}^H Z_{l3}^H - i \lambda_5^* Z_{i4}^H Z_{j1}^H Z_{k2}^H Z_{l3}^H \\
& - 2ie^{2i\eta} \lambda_3 Z_{i3}^H Z_{j2}^H Z_{k2}^H Z_{l3}^H - 2ie^{2i\eta} \lambda_4 Z_{i3}^H Z_{j2}^H Z_{k2}^H Z_{l3}^H \\
& + ie^{4i\eta} \lambda_5 Z_{i3}^H Z_{j2}^H Z_{k2}^H Z_{l3}^H + i \lambda_5^* Z_{i3}^H Z_{j2}^H Z_{k2}^H Z_{l3}^H - ie^{i\eta} \lambda_7 Z_{i4}^H Z_{j2}^H Z_{k2}^H Z_{l3}^H \\
& - ie^{3i\eta} \lambda_7^* Z_{i4}^H Z_{j2}^H Z_{k2}^H Z_{l3}^H - 3e^{i\eta} \lambda_6 Z_{i3}^H Z_{j3}^H Z_{k2}^H Z_{l3}^H \\
& + 3e^{3i\eta} \lambda_6^* Z_{i3}^H Z_{j3}^H Z_{k2}^H Z_{l3}^H + e^{4i\eta} \lambda_5 Z_{i4}^H Z_{j3}^H Z_{k2}^H Z_{l3}^H - \lambda_5^* Z_{i4}^H Z_{j3}^H Z_{k2}^H Z_{l3}^H \\
& + e^{4i\eta} \lambda_5 Z_{i3}^H Z_{j4}^H Z_{k2}^H Z_{l3}^H - \lambda_5^* Z_{i3}^H Z_{j4}^H Z_{k2}^H Z_{l3}^H - e^{i\eta} \lambda_7 Z_{i4}^H Z_{j4}^H Z_{k2}^H Z_{l3}^H \\
& + e^{3i\eta} \lambda_7^* Z_{i4}^H Z_{j4}^H Z_{k2}^H Z_{l3}^H + e^{i\eta} \lambda_6 Z_{i4}^H Z_{j1}^H Z_{k3}^H Z_{l3}^H \\
& - e^{3i\eta} \lambda_6^* Z_{i4}^H Z_{j1}^H Z_{k3}^H Z_{l3}^H - 3e^{i\eta} \lambda_6 Z_{i3}^H Z_{j2}^H Z_{k3}^H Z_{l3}^H \\
& + 3e^{3i\eta} \lambda_6^* Z_{i3}^H Z_{j2}^H Z_{k3}^H Z_{l3}^H + e^{4i\eta} \lambda_5 Z_{i4}^H Z_{j2}^H Z_{k3}^H Z_{l3}^H - \lambda_5^* Z_{i4}^H Z_{j2}^H Z_{k3}^H Z_{l3}^H \\
& - 12ie^{2i\eta} \lambda_1 Z_{i3}^H Z_{j3}^H Z_{k3}^H Z_{l3}^H - 3ie^{i\eta} \lambda_6 Z_{i4}^H Z_{j3}^H Z_{k3}^H Z_{l3}^H \\
& - 3ie^{3i\eta} \lambda_6^* Z_{i4}^H Z_{j3}^H Z_{k3}^H Z_{l3}^H - 3ie^{i\eta} \lambda_6 Z_{i3}^H Z_{j4}^H Z_{k3}^H Z_{l3}^H \\
& - 3ie^{3i\eta} \lambda_6^* Z_{i3}^H Z_{j4}^H Z_{k3}^H Z_{l3}^H - 2ie^{2i\eta} \lambda_3 Z_{i4}^H Z_{j4}^H Z_{k3}^H Z_{l3}^H \\
& - 2ie^{2i\eta} \lambda_4 Z_{i4}^H Z_{j4}^H Z_{k3}^H Z_{l3}^H - ie^{4i\eta} \lambda_5 Z_{i4}^H Z_{j4}^H Z_{k3}^H Z_{l3}^H - i \lambda_5^* Z_{i4}^H Z_{j4}^H Z_{k3}^H Z_{l3}^H \\
& + e^{i\eta} \lambda_6 Z_{i3}^H Z_{j1}^H Z_{k4}^H Z_{l3}^H - e^{3i\eta} \lambda_6^* Z_{i3}^H Z_{j1}^H Z_{k4}^H Z_{l3}^H \\
& - e^{4i\eta} \lambda_5 Z_{i4}^H Z_{j1}^H Z_{k4}^H Z_{l3}^H + \lambda_5^* Z_{i4}^H Z_{j1}^H Z_{k4}^H Z_{l3}^H + e^{4i\eta} \lambda_5 Z_{i3}^H Z_{j2}^H Z_{k4}^H Z_{l3}^H \\
& - \lambda_5^* Z_{i3}^H Z_{j2}^H Z_{k4}^H Z_{l3}^H - e^{i\eta} \lambda_7 Z_{i4}^H Z_{j2}^H Z_{k4}^H Z_{l3}^H + e^{3i\eta} \lambda_7^* Z_{i4}^H Z_{j2}^H Z_{k4}^H Z_{l3}^H \\
& - 3ie^{i\eta} \lambda_6 Z_{i3}^H Z_{j3}^H Z_{k4}^H Z_{l3}^H - 3ie^{3i\eta} \lambda_6^* Z_{i3}^H Z_{j3}^H Z_{k4}^H Z_{l3}^H \\
& - 2ie^{2i\eta} \lambda_3 Z_{i4}^H Z_{j3}^H Z_{k4}^H Z_{l3}^H - 2ie^{2i\eta} \lambda_4 Z_{i4}^H Z_{j3}^H Z_{k4}^H Z_{l3}^H \\
& - ie^{4i\eta} \lambda_5 Z_{i4}^H Z_{j3}^H Z_{k4}^H Z_{l3}^H - i \lambda_5^* Z_{i4}^H Z_{j3}^H Z_{k4}^H Z_{l3}^H - 2ie^{2i\eta} \lambda_3 Z_{i3}^H Z_{j4}^H Z_{k4}^H Z_{l3}^H \\
& - 2ie^{2i\eta} \lambda_4 Z_{i3}^H Z_{j4}^H Z_{k4}^H Z_{l3}^H - ie^{4i\eta} \lambda_5 Z_{i3}^H Z_{j4}^H Z_{k4}^H Z_{l3}^H - i \lambda_5^* Z_{i3}^H Z_{j4}^H Z_{k4}^H Z_{l3}^H \\
& - 3ie^{i\eta} \lambda_7 Z_{i4}^H Z_{j4}^H Z_{k4}^H Z_{l3}^H - 3ie^{3i\eta} \lambda_7^* Z_{i4}^H Z_{j4}^H Z_{k4}^H Z_{l3}^H
\end{aligned}$$

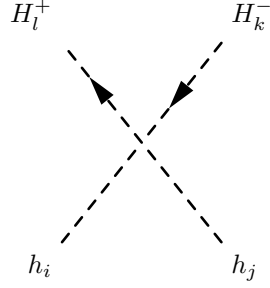
$$\begin{aligned}
& -ie^{i\eta}\lambda_6 Z_{i3}^H Z_{j1}^H Z_{k1}^H Z_{l4}^H - ie^{3i\eta}\lambda_6^* Z_{i3}^H Z_{j1}^H Z_{k1}^H Z_{l4}^H \\
& - 2ie^{2i\eta}\lambda_3 Z_{i4}^H Z_{j1}^H Z_{k1}^H Z_{l4}^H - 2ie^{2i\eta}\lambda_4 Z_{i4}^H Z_{j1}^H Z_{k1}^H Z_{l4}^H \\
& + ie^{4i\eta}\lambda_5 Z_{i4}^H Z_{j1}^H Z_{k1}^H Z_{l4}^H + i\lambda_5^* Z_{i4}^H Z_{j1}^H Z_{k1}^H Z_{l4}^H - ie^{4i\eta}\lambda_5 Z_{i3}^H Z_{j2}^H Z_{k1}^H Z_{l4}^H \\
& - i\lambda_5^* Z_{i3}^H Z_{j2}^H Z_{k1}^H Z_{l4}^H - ie^{i\eta}\lambda_7 Z_{i4}^H Z_{j2}^H Z_{k1}^H Z_{l4}^H - ie^{3i\eta}\lambda_7^* Z_{i4}^H Z_{j2}^H Z_{k1}^H Z_{l4}^H \\
& + e^{i\eta}\lambda_6 Z_{i3}^H Z_{j3}^H Z_{k1}^H Z_{l4}^H - e^{3i\eta}\lambda_6^* Z_{i3}^H Z_{j3}^H Z_{k1}^H Z_{l4}^H \\
& - e^{4i\eta}\lambda_5 Z_{i4}^H Z_{j3}^H Z_{k1}^H Z_{l4}^H + \lambda_5^* Z_{i4}^H Z_{j3}^H Z_{k1}^H Z_{l4}^H - e^{4i\eta}\lambda_5 Z_{i3}^H Z_{j4}^H Z_{k1}^H Z_{l4}^H \\
& + \lambda_5^* Z_{i3}^H Z_{j4}^H Z_{k1}^H Z_{l4}^H + 3e^{i\eta}\lambda_7 Z_{i4}^H Z_{j4}^H Z_{k1}^H Z_{l4}^H - 3e^{3i\eta}\lambda_7^* Z_{i4}^H Z_{j4}^H Z_{k1}^H Z_{l4}^H \\
& - ie^{4i\eta}\lambda_5 Z_{i3}^H Z_{j1}^H Z_{k2}^H Z_{l4}^H - i\lambda_5^* Z_{i3}^H Z_{j1}^H Z_{k2}^H Z_{l4}^H - ie^{i\eta}\lambda_7 Z_{i4}^H Z_{j1}^H Z_{k2}^H Z_{l4}^H \\
& - ie^{3i\eta}\lambda_7^* Z_{i4}^H Z_{j1}^H Z_{k2}^H Z_{l4}^H - ie^{i\eta}\lambda_7 Z_{i3}^H Z_{j2}^H Z_{k2}^H Z_{l4}^H \\
& - ie^{3i\eta}\lambda_7^* Z_{i3}^H Z_{j2}^H Z_{k2}^H Z_{l4}^H - 4ie^{2i\eta}\lambda_2 Z_{i4}^H Z_{j2}^H Z_{k2}^H Z_{l4}^H \\
& + e^{4i\eta}\lambda_5 Z_{i3}^H Z_{j3}^H Z_{k2}^H Z_{l4}^H - \lambda_5^* Z_{i3}^H Z_{j3}^H Z_{k2}^H Z_{l4}^H - e^{i\eta}\lambda_7 Z_{i4}^H Z_{j3}^H Z_{k2}^H Z_{l4}^H \\
& + e^{3i\eta}\lambda_7^* Z_{i4}^H Z_{j3}^H Z_{k2}^H Z_{l4}^H - e^{i\eta}\lambda_7 Z_{i3}^H Z_{j4}^H Z_{k2}^H Z_{l4}^H \\
& + e^{3i\eta}\lambda_7^* Z_{i3}^H Z_{j4}^H Z_{k2}^H Z_{l4}^H + e^{i\eta}\lambda_6 Z_{i3}^H Z_{j1}^H Z_{k3}^H Z_{l4}^H \\
& - e^{3i\eta}\lambda_6^* Z_{i3}^H Z_{j1}^H Z_{k3}^H Z_{l4}^H - e^{4i\eta}\lambda_5 Z_{i4}^H Z_{j1}^H Z_{k3}^H Z_{l4}^H + \lambda_5^* Z_{i4}^H Z_{j1}^H Z_{k3}^H Z_{l4}^H \\
& + e^{4i\eta}\lambda_5 Z_{i3}^H Z_{j2}^H Z_{k3}^H Z_{l4}^H - \lambda_5^* Z_{i3}^H Z_{j2}^H Z_{k3}^H Z_{l4}^H - e^{i\eta}\lambda_7 Z_{i4}^H Z_{j2}^H Z_{k3}^H Z_{l4}^H \\
& + e^{3i\eta}\lambda_7^* Z_{i4}^H Z_{j2}^H Z_{k3}^H Z_{l4}^H - 3ie^{i\eta}\lambda_6 Z_{i3}^H Z_{j3}^H Z_{k3}^H Z_{l4}^H \\
& - 3ie^{3i\eta}\lambda_6^* Z_{i3}^H Z_{j3}^H Z_{k3}^H Z_{l4}^H - 2ie^{2i\eta}\lambda_3 Z_{i4}^H Z_{j3}^H Z_{k3}^H Z_{l4}^H \\
& - 2ie^{2i\eta}\lambda_4 Z_{i4}^H Z_{j3}^H Z_{k3}^H Z_{l4}^H - ie^{4i\eta}\lambda_5 Z_{i4}^H Z_{j3}^H Z_{k3}^H Z_{l4}^H - i\lambda_5^* Z_{i4}^H Z_{j3}^H Z_{k3}^H Z_{l4}^H \\
& - 2ie^{2i\eta}\lambda_3 Z_{i3}^H Z_{j4}^H Z_{k3}^H Z_{l4}^H - 2ie^{2i\eta}\lambda_4 Z_{i3}^H Z_{j4}^H Z_{k3}^H Z_{l4}^H \\
& - ie^{4i\eta}\lambda_5 Z_{i3}^H Z_{j4}^H Z_{k3}^H Z_{l4}^H - i\lambda_5^* Z_{i3}^H Z_{j4}^H Z_{k3}^H Z_{l4}^H - 3ie^{i\eta}\lambda_7 Z_{i4}^H Z_{j4}^H Z_{k3}^H Z_{l4}^H \\
& - 3ie^{3i\eta}\lambda_7^* Z_{i4}^H Z_{j4}^H Z_{k3}^H Z_{l4}^H - e^{4i\eta}\lambda_5 Z_{i3}^H Z_{j1}^H Z_{k4}^H Z_{l4}^H + \lambda_5^* Z_{i3}^H Z_{j1}^H Z_{k4}^H Z_{l4}^H \\
& + 3e^{i\eta}\lambda_7 Z_{i4}^H Z_{j1}^H Z_{k4}^H Z_{l4}^H - 3e^{3i\eta}\lambda_7^* Z_{i4}^H Z_{j1}^H Z_{k4}^H Z_{l4}^H \\
& - e^{i\eta}\lambda_7 Z_{i3}^H Z_{j2}^H Z_{k4}^H Z_{l4}^H + e^{3i\eta}\lambda_7^* Z_{i3}^H Z_{j2}^H Z_{k4}^H Z_{l4}^H
\end{aligned}$$

$$\begin{aligned}
& - 2ie^{2i\eta}\lambda_3 Z_{i3}^H Z_{j3}^H Z_{k4}^H Z_{l4}^H - 2ie^{2i\eta}\lambda_4 Z_{i3}^H Z_{j3}^H Z_{k4}^H Z_{l4}^H \\
& - ie^{4i\eta}\lambda_5 Z_{i3}^H Z_{j3}^H Z_{k4}^H Z_{l4}^H - i\lambda_5^* Z_{i3}^H Z_{j3}^H Z_{k4}^H Z_{l4}^H - 3ie^{i\eta}\lambda_7 Z_{i4}^H Z_{j3}^H Z_{k4}^H Z_{l4}^H \\
& - 3ie^{3i\eta}\lambda_7^* Z_{i4}^H Z_{j3}^H Z_{k4}^H Z_{l4}^H - 3ie^{i\eta}\lambda_7 Z_{i3}^H Z_{j4}^H Z_{k4}^H Z_{l4}^H \\
& - 3ie^{3i\eta}\lambda_7^* Z_{i3}^H Z_{j4}^H Z_{k4}^H Z_{l4}^H - 12ie^{2i\eta}\lambda_2 Z_{i4}^H Z_{j4}^H Z_{k4}^H Z_{l4}^H \\
& + Z_{i2}^H \left(-e^{i\eta}\lambda_6 Z_{j3}^H Z_{k1}^H Z_{l1}^H + e^{3i\eta}\lambda_6^* Z_{j3}^H Z_{k1}^H Z_{l1}^H - e^{4i\eta}\lambda_5 Z_{j4}^H Z_{k1}^H Z_{l1}^H \right. \\
& + \lambda_5^* Z_{j4}^H Z_{k1}^H Z_{l1}^H + e^{4i\eta}\lambda_5 Z_{j3}^H Z_{k2}^H Z_{l1}^H - \lambda_5^* Z_{j3}^H Z_{k2}^H Z_{l1}^H + e^{i\eta}\lambda_7 Z_{j4}^H Z_{k2}^H Z_{l1}^H \\
& - e^{3i\eta}\lambda_7^* Z_{j4}^H Z_{k2}^H Z_{l1}^H - ie^{i\eta}\lambda_6 Z_{j3}^H Z_{k3}^H Z_{l1}^H - ie^{3i\eta}\lambda_6^* Z_{j3}^H Z_{k3}^H Z_{l1}^H \\
& - ie^{4i\eta}\lambda_5 Z_{j4}^H Z_{k3}^H Z_{l1}^H - i\lambda_5^* Z_{j4}^H Z_{k3}^H Z_{l1}^H - ie^{4i\eta}\lambda_5 Z_{j3}^H Z_{k4}^H Z_{l1}^H \\
& \left. - i\lambda_5^* Z_{j3}^H Z_{k4}^H Z_{l1}^H - ie^{i\eta}\lambda_7 Z_{j4}^H Z_{k4}^H Z_{l1}^H - ie^{3i\eta}\lambda_7^* Z_{j4}^H Z_{k4}^H Z_{l1}^H \right)
\end{aligned}$$

$$\begin{aligned}
& + e^{4i\eta} \lambda_5 Z_{j_3}^H Z_{k_1}^H Z_{l_2}^H - \lambda_5^* Z_{j_3}^H Z_{k_1}^H Z_{l_2}^H + e^{i\eta} \lambda_7 Z_{j_4}^H Z_{k_1}^H Z_{l_2}^H \\
& - e^{3i\eta} \lambda_7^* Z_{j_4}^H Z_{k_1}^H Z_{l_2}^H - 3e^{i\eta} \lambda_7 Z_{j_3}^H Z_{k_2}^H Z_{l_2}^H + 3e^{3i\eta} \lambda_7^* Z_{j_3}^H Z_{k_2}^H Z_{l_2}^H \\
& - 2ie^{2i\eta} \lambda_3 Z_{j_3}^H Z_{k_3}^H Z_{l_2}^H - 2ie^{2i\eta} \lambda_4 Z_{j_3}^H Z_{k_3}^H Z_{l_2}^H + ie^{4i\eta} \lambda_5 Z_{j_3}^H Z_{k_3}^H Z_{l_2}^H \\
& + i\lambda_5^* Z_{j_3}^H Z_{k_3}^H Z_{l_2}^H - ie^{i\eta} \lambda_7 Z_{j_4}^H Z_{k_3}^H Z_{l_2}^H - ie^{3i\eta} \lambda_7^* Z_{j_4}^H Z_{k_3}^H Z_{l_2}^H \\
& - ie^{i\eta} \lambda_7 Z_{j_3}^H Z_{k_4}^H Z_{l_2}^H - ie^{3i\eta} \lambda_7^* Z_{j_3}^H Z_{k_4}^H Z_{l_2}^H - 4ie^{2i\eta} \lambda_2 Z_{j_4}^H Z_{k_4}^H Z_{l_2}^H \\
& - ie^{i\eta} \lambda_6 Z_{j_3}^H Z_{k_1}^H Z_{l_3}^H - ie^{3i\eta} \lambda_6^* Z_{j_3}^H Z_{k_1}^H Z_{l_3}^H - ie^{4i\eta} \lambda_5 Z_{j_4}^H Z_{k_1}^H Z_{l_3}^H \\
& - i\lambda_5^* Z_{j_4}^H Z_{k_1}^H Z_{l_3}^H - 2ie^{2i\eta} \lambda_3 Z_{j_3}^H Z_{k_2}^H Z_{l_3}^H - 2ie^{2i\eta} \lambda_4 Z_{j_3}^H Z_{k_2}^H Z_{l_3}^H \\
& + ie^{4i\eta} \lambda_5 Z_{j_3}^H Z_{k_2}^H Z_{l_3}^H + i\lambda_5^* Z_{j_3}^H Z_{k_2}^H Z_{l_3}^H - ie^{i\eta} \lambda_7 Z_{j_4}^H Z_{k_2}^H Z_{l_3}^H \\
& - ie^{3i\eta} \lambda_7^* Z_{j_4}^H Z_{k_2}^H Z_{l_3}^H - 3e^{i\eta} \lambda_6 Z_{j_3}^H Z_{k_3}^H Z_{l_3}^H + 3e^{3i\eta} \lambda_6^* Z_{j_3}^H Z_{k_3}^H Z_{l_3}^H \\
& + e^{4i\eta} \lambda_5 Z_{j_4}^H Z_{k_3}^H Z_{l_3}^H - \lambda_5^* Z_{j_4}^H Z_{k_3}^H Z_{l_3}^H + e^{4i\eta} \lambda_5 Z_{j_3}^H Z_{k_4}^H Z_{l_3}^H \\
& - \lambda_5^* Z_{j_3}^H Z_{k_4}^H Z_{l_3}^H - e^{i\eta} \lambda_7 Z_{j_4}^H Z_{k_4}^H Z_{l_3}^H + e^{3i\eta} \lambda_7^* Z_{j_4}^H Z_{k_4}^H Z_{l_3}^H \\
& - ie^{4i\eta} \lambda_5 Z_{j_3}^H Z_{k_1}^H Z_{l_4}^H - i\lambda_5^* Z_{j_3}^H Z_{k_1}^H Z_{l_4}^H - ie^{i\eta} \lambda_7 Z_{j_4}^H Z_{k_1}^H Z_{l_4}^H \\
& - ie^{3i\eta} \lambda_7^* Z_{j_4}^H Z_{k_1}^H Z_{l_4}^H - ie^{i\eta} \lambda_7 Z_{j_3}^H Z_{k_2}^H Z_{l_4}^H - ie^{3i\eta} \lambda_7^* Z_{j_3}^H Z_{k_2}^H Z_{l_4}^H \\
& - 4ie^{2i\eta} \lambda_2 Z_{j_4}^H Z_{k_2}^H Z_{l_4}^H + e^{4i\eta} \lambda_5 Z_{j_3}^H Z_{k_3}^H Z_{l_4}^H - \lambda_5^* Z_{j_3}^H Z_{k_3}^H Z_{l_4}^H \\
& - e^{i\eta} \lambda_7 Z_{j_4}^H Z_{k_3}^H Z_{l_4}^H + e^{3i\eta} \lambda_7^* Z_{j_4}^H Z_{k_3}^H Z_{l_4}^H - e^{i\eta} \lambda_7 Z_{j_3}^H Z_{k_4}^H Z_{l_4}^H \\
& + e^{3i\eta} \lambda_7^* Z_{j_3}^H Z_{k_4}^H Z_{l_4}^H \\
& + Z_{j_2}^H \left(e^{4i\eta} \lambda_5 Z_{k_3}^H Z_{l_1}^H - \lambda_5^* Z_{k_3}^H Z_{l_1}^H + e^{i\eta} \lambda_7 Z_{k_4}^H Z_{l_1}^H - e^{3i\eta} \lambda_7^* Z_{k_4}^H Z_{l_1}^H \right. \\
& - 3e^{i\eta} \lambda_7 Z_{k_3}^H Z_{l_2}^H + 3e^{3i\eta} \lambda_7^* Z_{k_3}^H Z_{l_2}^H - 2ie^{2i\eta} \lambda_3 Z_{k_3}^H Z_{l_3}^H - 2ie^{2i\eta} \lambda_4 Z_{k_3}^H Z_{l_3}^H \\
& + ie^{4i\eta} \lambda_5 Z_{k_3}^H Z_{l_3}^H + i\lambda_5^* Z_{k_3}^H Z_{l_3}^H - ie^{i\eta} \lambda_7 Z_{k_4}^H Z_{l_3}^H - ie^{3i\eta} \lambda_7^* Z_{k_4}^H Z_{l_3}^H \\
& + 3e^{i\eta} Z_{k_2}^H \left(-4ie^{i\eta} \lambda_2 Z_{l_2}^H - \left(-e^{2i\eta} \lambda_7^* + \lambda_7 \right) Z_{l_3}^H - i \left(e^{2i\eta} \lambda_7^* + \lambda_7 \right) Z_{l_1}^H \right) \\
& - ie^{i\eta} \lambda_7 Z_{k_3}^H Z_{l_4}^H - ie^{3i\eta} \lambda_7^* Z_{k_3}^H Z_{l_4}^H - 4ie^{2i\eta} \lambda_2 Z_{k_4}^H Z_{l_4}^H \\
& \left. + Z_{k_1}^H \left(-i \left(e^{2i\eta} \left(2\lambda_3 + 2\lambda_4 + e^{2i\eta} \lambda_5 \right) + \lambda_5^* \right) Z_{l_1}^H - 3ie^{i\eta} \left(e^{2i\eta} \lambda_7^* + \lambda_7 \right) Z_{l_2}^H + e^{4i\eta} \lambda_5 Z_{l_3}^H \right. \right. \\
& \left. \left. - \lambda_5^* Z_{l_3}^H + e^{i\eta} \lambda_7 Z_{l_4}^H - e^{3i\eta} \lambda_7^* Z_{l_4}^H \right) \right) \\
& + Z_{j_1}^H \left(-e^{i\eta} \lambda_6 Z_{k_3}^H Z_{l_1}^H + e^{3i\eta} \lambda_6^* Z_{k_3}^H Z_{l_1}^H - e^{4i\eta} \lambda_5 Z_{k_4}^H Z_{l_1}^H + \lambda_5^* Z_{k_4}^H Z_{l_1}^H \right. \\
& + e^{4i\eta} \lambda_5 Z_{k_3}^H Z_{l_2}^H - \lambda_5^* Z_{k_3}^H Z_{l_2}^H + e^{i\eta} \lambda_7 Z_{k_4}^H Z_{l_2}^H - e^{3i\eta} \lambda_7^* Z_{k_4}^H Z_{l_2}^H \\
& - ie^{i\eta} \lambda_6 Z_{k_3}^H Z_{l_3}^H - ie^{3i\eta} \lambda_6^* Z_{k_3}^H Z_{l_3}^H - ie^{4i\eta} \lambda_5 Z_{k_4}^H Z_{l_3}^H - i\lambda_5^* Z_{k_4}^H Z_{l_3}^H \\
& - ie^{4i\eta} \lambda_5 Z_{k_3}^H Z_{l_4}^H - i\lambda_5^* Z_{k_3}^H Z_{l_4}^H - ie^{i\eta} \lambda_7 Z_{k_4}^H Z_{l_4}^H - ie^{3i\eta} \lambda_7^* Z_{k_4}^H Z_{l_4}^H \\
& \left. + Z_{k_1}^H \left(-3ie^{i\eta} \left(e^{2i\eta} \lambda_6^* + \lambda_6 \right) Z_{l_1}^H - i \left(e^{2i\eta} \left(2\lambda_3 + 2\lambda_4 + e^{2i\eta} \lambda_5 \right) + \lambda_5^* \right) Z_{l_2}^H - e^{i\eta} \lambda_6 Z_{l_3}^H \right. \right. \\
& \left. \left. + e^{3i\eta} \lambda_6^* Z_{l_3}^H - e^{4i\eta} \lambda_5 Z_{l_4}^H + \lambda_5^* Z_{l_4}^H \right) \right) \\
& + Z_{k_2}^H \left(-i \left(e^{2i\eta} \left(2\lambda_3 + 2\lambda_4 + e^{2i\eta} \lambda_5 \right) + \lambda_5^* \right) Z_{l_1}^H - 3ie^{i\eta} \left(e^{2i\eta} \lambda_7^* + \lambda_7 \right) Z_{l_2}^H + e^{4i\eta} \lambda_5 Z_{l_3}^H \right.
\end{aligned}$$

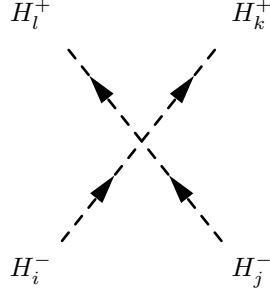
$$\begin{aligned}
& - \lambda_5^* Z_{l3}^H + e^{i\eta} \lambda_7 Z_{l4}^H - e^{3i\eta} \lambda_7^* Z_{l4}^H \Big) \Big) \\
& + Z_{i1}^H \left(3e^{i\eta} \lambda_6 Z_{j4}^H Z_{k1}^H Z_{l1}^H - 3e^{3i\eta} \lambda_6^* Z_{j4}^H Z_{k1}^H Z_{l1}^H - e^{i\eta} \lambda_6 Z_{j3}^H Z_{k2}^H Z_{l1}^H \right. \\
& + e^{3i\eta} \lambda_6^* Z_{j3}^H Z_{k2}^H Z_{l1}^H - e^{4i\eta} \lambda_5 Z_{j4}^H Z_{k2}^H Z_{l1}^H + \lambda_5^* Z_{j4}^H Z_{k2}^H Z_{l1}^H \\
& - 4ie^{2i\eta} \lambda_1 Z_{j3}^H Z_{k3}^H Z_{l1}^H - ie^{i\eta} \lambda_6 Z_{j4}^H Z_{k3}^H Z_{l1}^H - ie^{3i\eta} \lambda_6^* Z_{j4}^H Z_{k3}^H Z_{l1}^H \\
& - ie^{i\eta} \lambda_6 Z_{j3}^H Z_{k4}^H Z_{l1}^H - ie^{3i\eta} \lambda_6^* Z_{j3}^H Z_{k4}^H Z_{l1}^H - 2ie^{2i\eta} \lambda_3 Z_{j4}^H Z_{k4}^H Z_{l1}^H \\
& - 2ie^{2i\eta} \lambda_4 Z_{j4}^H Z_{k4}^H Z_{l1}^H + ie^{4i\eta} \lambda_5 Z_{j4}^H Z_{k4}^H Z_{l1}^H + i\lambda_5^* Z_{j4}^H Z_{k4}^H Z_{l1}^H \\
& - e^{i\eta} \lambda_6 Z_{j3}^H Z_{k1}^H Z_{l2}^H + e^{3i\eta} \lambda_6^* Z_{j3}^H Z_{k1}^H Z_{l2}^H - e^{4i\eta} \lambda_5 Z_{j4}^H Z_{k1}^H Z_{l2}^H \\
& + \lambda_5^* Z_{j4}^H Z_{k1}^H Z_{l2}^H + e^{4i\eta} \lambda_5 Z_{j3}^H Z_{k2}^H Z_{l2}^H - \lambda_5^* Z_{j3}^H Z_{k2}^H Z_{l2}^H + e^{i\eta} \lambda_7 Z_{j4}^H Z_{k2}^H Z_{l2}^H \\
& - e^{3i\eta} \lambda_7^* Z_{j4}^H Z_{k2}^H Z_{l2}^H - ie^{i\eta} \lambda_6 Z_{j3}^H Z_{k3}^H Z_{l2}^H - ie^{3i\eta} \lambda_6^* Z_{j3}^H Z_{k3}^H Z_{l2}^H \\
& - ie^{4i\eta} \lambda_5 Z_{j4}^H Z_{k3}^H Z_{l2}^H - i\lambda_5^* Z_{j4}^H Z_{k3}^H Z_{l2}^H - ie^{4i\eta} \lambda_5 Z_{j3}^H Z_{k4}^H Z_{l2}^H \\
& - i\lambda_5^* Z_{j3}^H Z_{k4}^H Z_{l2}^H - ie^{i\eta} \lambda_7 Z_{j4}^H Z_{k4}^H Z_{l2}^H - ie^{3i\eta} \lambda_7^* Z_{j4}^H Z_{k4}^H Z_{l2}^H \\
& - 4ie^{2i\eta} \lambda_1 Z_{j3}^H Z_{k1}^H Z_{l3}^H - ie^{i\eta} \lambda_6 Z_{j4}^H Z_{k1}^H Z_{l3}^H - ie^{3i\eta} \lambda_6^* Z_{j4}^H Z_{k1}^H Z_{l3}^H \\
& - ie^{i\eta} \lambda_6 Z_{j3}^H Z_{k2}^H Z_{l3}^H - ie^{3i\eta} \lambda_6^* Z_{j3}^H Z_{k2}^H Z_{l3}^H - ie^{4i\eta} \lambda_5 Z_{j4}^H Z_{k2}^H Z_{l3}^H \\
& - i\lambda_5^* Z_{j4}^H Z_{k2}^H Z_{l3}^H + e^{i\eta} \lambda_6 Z_{j4}^H Z_{k3}^H Z_{l3}^H - e^{3i\eta} \lambda_6^* Z_{j4}^H Z_{k3}^H Z_{l3}^H \\
& + e^{i\eta} \lambda_6 Z_{j3}^H Z_{k4}^H Z_{l3}^H - e^{3i\eta} \lambda_6^* Z_{j3}^H Z_{k4}^H Z_{l3}^H - e^{4i\eta} \lambda_5 Z_{j4}^H Z_{k4}^H Z_{l3}^H \\
& + \lambda_5^* Z_{j4}^H Z_{k4}^H Z_{l3}^H - ie^{i\eta} \lambda_6 Z_{j3}^H Z_{k1}^H Z_{l4}^H - ie^{3i\eta} \lambda_6^* Z_{j3}^H Z_{k1}^H Z_{l4}^H \\
& - 2ie^{2i\eta} \lambda_3 Z_{j4}^H Z_{k1}^H Z_{l4}^H - 2ie^{2i\eta} \lambda_4 Z_{j4}^H Z_{k1}^H Z_{l4}^H + ie^{4i\eta} \lambda_5 Z_{j4}^H Z_{k1}^H Z_{l4}^H \\
& + i\lambda_5^* Z_{j4}^H Z_{k1}^H Z_{l4}^H - ie^{4i\eta} \lambda_5 Z_{j3}^H Z_{k2}^H Z_{l4}^H - i\lambda_5^* Z_{j3}^H Z_{k2}^H Z_{l4}^H \\
& - ie^{i\eta} \lambda_7 Z_{j4}^H Z_{k2}^H Z_{l4}^H - ie^{3i\eta} \lambda_7^* Z_{j4}^H Z_{k2}^H Z_{l4}^H + e^{i\eta} \lambda_6 Z_{j3}^H Z_{k3}^H Z_{l4}^H \\
& - e^{3i\eta} \lambda_6^* Z_{j3}^H Z_{k3}^H Z_{l4}^H - e^{4i\eta} \lambda_5 Z_{j4}^H Z_{k3}^H Z_{l4}^H + \lambda_5^* Z_{j4}^H Z_{k3}^H Z_{l4}^H \\
& - e^{4i\eta} \lambda_5 Z_{j3}^H Z_{k4}^H Z_{l4}^H + \lambda_5^* Z_{j3}^H Z_{k4}^H Z_{l4}^H + 3e^{i\eta} \lambda_7 Z_{j4}^H Z_{k4}^H Z_{l4}^H \\
& - 3e^{3i\eta} \lambda_7^* Z_{j4}^H Z_{k4}^H Z_{l4}^H \\
& + Z_{j1}^H \left(3e^{i\eta} \lambda_6 Z_{k4}^H Z_{l1}^H - 3e^{3i\eta} \lambda_6^* Z_{k4}^H Z_{l1}^H - e^{i\eta} \lambda_6 Z_{k3}^H Z_{l2}^H + e^{3i\eta} \lambda_6^* Z_{k3}^H Z_{l2}^H \right. \\
& - e^{4i\eta} \lambda_5 Z_{k4}^H Z_{l2}^H + \lambda_5^* Z_{k4}^H Z_{l2}^H - 4ie^{2i\eta} \lambda_1 Z_{k3}^H Z_{l3}^H - ie^{i\eta} \lambda_6 Z_{k4}^H Z_{l3}^H \\
& - ie^{3i\eta} \lambda_6^* Z_{k4}^H Z_{l3}^H - ie^{i\eta} \lambda_6 Z_{k3}^H Z_{l4}^H - ie^{3i\eta} \lambda_6^* Z_{k3}^H Z_{l4}^H - 2ie^{2i\eta} \lambda_3 Z_{k4}^H Z_{l4}^H \\
& - 2ie^{2i\eta} \lambda_4 Z_{k4}^H Z_{l4}^H + ie^{4i\eta} \lambda_5 Z_{k4}^H Z_{l4}^H + i\lambda_5^* Z_{k4}^H Z_{l4}^H \\
& + Z_{k2}^H \left(- 3ie^{i\eta} \left(e^{2i\eta} \lambda_6^* + \lambda_6 \right) Z_{l1}^H - i \left(e^{2i\eta} \left(2\lambda_3 + 2\lambda_4 + e^{2i\eta} \lambda_5 \right) + \lambda_5^* \right) Z_{l2}^H - e^{i\eta} \lambda_6 Z_{l3}^H \right. \\
& + e^{3i\eta} \lambda_6^* Z_{l3}^H - e^{4i\eta} \lambda_5 Z_{l4}^H + \lambda_5^* Z_{l4}^H \Big) \\
& + 3e^{i\eta} Z_{k1}^H \left(- 4ie^{i\eta} \lambda_1 Z_{l1}^H + \left(- e^{2i\eta} \lambda_6^* + \lambda_6 \right) Z_{l4}^H - i \left(e^{2i\eta} \lambda_6^* + \lambda_6 \right) Z_{l2}^H \right) \\
& + Z_{j2}^H \left(- e^{i\eta} \lambda_6 Z_{k3}^H Z_{l1}^H + e^{3i\eta} \lambda_6^* Z_{k3}^H Z_{l1}^H - e^{4i\eta} \lambda_5 Z_{k4}^H Z_{l1}^H + \lambda_5^* Z_{k4}^H Z_{l1}^H \right. \\
& + e^{4i\eta} \lambda_5 Z_{k3}^H Z_{l2}^H - \lambda_5^* Z_{k3}^H Z_{l2}^H + e^{i\eta} \lambda_7 Z_{k4}^H Z_{l2}^H - e^{3i\eta} \lambda_7^* Z_{k4}^H Z_{l2}^H
\end{aligned}$$

$$\begin{aligned}
& -ie^{i\eta}\lambda_6 Z_{k3}^H Z_{l3}^H - ie^{3i\eta}\lambda_6^* Z_{k3}^H Z_{l3}^H - ie^{4i\eta}\lambda_5 Z_{k4}^H Z_{l3}^H - i\lambda_5^* Z_{k4}^H Z_{l3}^H \\
& - ie^{4i\eta}\lambda_5 Z_{k3}^H Z_{l4}^H - i\lambda_5^* Z_{k3}^H Z_{l4}^H - ie^{i\eta}\lambda_7 Z_{k4}^H Z_{l4}^H - ie^{3i\eta}\lambda_7^* Z_{k4}^H Z_{l4}^H \\
& + Z_{k1}^H \left(-3ie^{i\eta} \left(e^{2i\eta}\lambda_6^* + \lambda_6 \right) Z_{l1}^H - i \left(e^{2i\eta} \left(2\lambda_3 + 2\lambda_4 + e^{2i\eta}\lambda_5 \right) + \lambda_5^* \right) Z_{l2}^H - e^{i\eta}\lambda_6 Z_{l3}^H \right. \\
& + e^{3i\eta}\lambda_6^* Z_{l3}^H - e^{4i\eta}\lambda_5 Z_{l4}^H + \lambda_5^* Z_{l4}^H \left. \right) \\
& + Z_{k2}^H \left(-i \left(e^{2i\eta} \left(2\lambda_3 + 2\lambda_4 + e^{2i\eta}\lambda_5 \right) + \lambda_5^* \right) Z_{l1}^H - 3ie^{i\eta} \left(e^{2i\eta}\lambda_7^* + \lambda_7 \right) Z_{l2}^H + e^{4i\eta}\lambda_5 Z_{l3}^H \right. \\
& \left. - \lambda_5^* Z_{l3}^H + e^{i\eta}\lambda_7 Z_{l4}^H - e^{3i\eta}\lambda_7^* Z_{l4}^H \right) \left. \right) \left. \right) \left. \right) \tag{158}
\end{aligned}$$



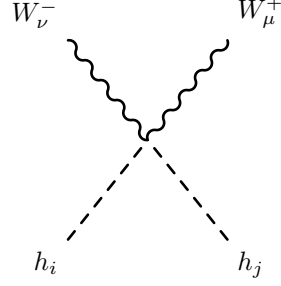
$$\begin{aligned}
& \frac{1}{2} e^{-2i\eta} \left(Z_{k1}^{+,*} \left(e^{i\eta}\lambda_6 Z_{i4}^H Z_{j1}^H Z_{l1}^+ - e^{3i\eta}\lambda_6^* Z_{i4}^H Z_{j1}^H Z_{l1}^+ - e^{i\eta}\lambda_6 Z_{i3}^H Z_{j2}^H Z_{l1}^+ \right. \right. \\
& + e^{3i\eta}\lambda_6^* Z_{i3}^H Z_{j2}^H Z_{l1}^+ - 4ie^{2i\eta}\lambda_1 Z_{i3}^H Z_{j3}^H Z_{l1}^+ - ie^{i\eta}\lambda_6 Z_{i4}^H Z_{j3}^H Z_{l1}^+ \\
& - ie^{3i\eta}\lambda_6 Z_{i4}^H Z_{j3}^H Z_{l1}^+ - ie^{i\eta}\lambda_6 Z_{i3}^H Z_{j4}^H Z_{l1}^+ - ie^{3i\eta}\lambda_6^* Z_{i3}^H Z_{j4}^H Z_{l1}^+ \\
& - 2ie^{2i\eta}\lambda_3 Z_{i4}^H Z_{j4}^H Z_{l1}^+ - e^{2i\eta}\lambda_4 Z_{i4}^H Z_{j1}^H Z_{l2}^+ + \lambda_5^* Z_{i4}^H Z_{j1}^H Z_{l2}^+ \\
& + e^{2i\eta}\lambda_4 Z_{i3}^H Z_{j2}^H Z_{l2}^+ - \lambda_5^* Z_{i3}^H Z_{j2}^H Z_{l2}^+ - 2ie^{i\eta}\lambda_6 Z_{i3}^H Z_{j3}^H Z_{l2}^+ \\
& - ie^{2i\eta}\lambda_4 Z_{i4}^H Z_{j3}^H Z_{l2}^+ - i\lambda_5^* Z_{i4}^H Z_{j3}^H Z_{l2}^+ - ie^{2i\eta}\lambda_4 Z_{i3}^H Z_{j4}^H Z_{l2}^+ \\
& \left. - i\lambda_5^* Z_{i3}^H Z_{j4}^H Z_{l2}^+ - 2ie^{i\eta}\lambda_7 Z_{i4}^H Z_{j4}^H Z_{l2}^+ \right. \\
& + Z_{i2}^H \left(-2ie^{i\eta} Z_{j2}^H \left(e^{i\eta}\lambda_3 Z_{l1}^+ + \lambda_7 Z_{l2}^+ \right) \right. \\
& + Z_{j3}^H \left(\left(e^{2i\eta}\lambda_4 - \lambda_5^* \right) Z_{l2}^+ + e^{i\eta} \left(e^{2i\eta}\lambda_6^* - \lambda_6 \right) Z_{l1}^+ \right) \\
& \left. - iZ_{j1}^H \left(\left(e^{2i\eta}\lambda_4 + \lambda_5^* \right) Z_{l2}^+ + e^{i\eta} \left(e^{2i\eta}\lambda_6^* + \lambda_6 \right) Z_{l1}^+ \right) \right) \\
& + Z_{i1}^H \left(-2ie^{i\eta} Z_{j1}^H \left(2e^{i\eta}\lambda_1 Z_{l1}^+ + \lambda_6 Z_{l2}^+ \right) \right. \\
& - Z_{j4}^H \left(\left(e^{2i\eta}\lambda_4 - \lambda_5^* \right) Z_{l2}^+ + e^{i\eta} \left(e^{2i\eta}\lambda_6^* - \lambda_6 \right) Z_{l1}^+ \right) \\
& \left. - iZ_{j2}^H \left(\left(e^{2i\eta}\lambda_4 + \lambda_5^* \right) Z_{l2}^+ + e^{i\eta} \left(e^{2i\eta}\lambda_6^* + \lambda_6 \right) Z_{l1}^+ \right) \right) \left. \right) \\
& + e^{i\eta} Z_{k2}^{+,*} \left(e^{i\eta}\lambda_4 Z_{i4}^H Z_{j1}^H Z_{l1}^+ - e^{3i\eta}\lambda_5 Z_{i4}^H Z_{j1}^H Z_{l1}^+ - ie^{i\eta}\lambda_4 Z_{i1}^H Z_{j2}^H Z_{l1}^+ \right.
\end{aligned}$$

$$\begin{aligned}
& -ie^{3i\eta}\lambda_5 Z_{i1}^H Z_{j2}^H Z_{l1}^+ - e^{i\eta}\lambda_4 Z_{i3}^H Z_{j2}^H Z_{l1}^+ + e^{3i\eta}\lambda_5 Z_{i3}^H Z_{j2}^H Z_{l1}^+ \\
& -ie^{i\eta}\lambda_4 Z_{i4}^H Z_{j3}^H Z_{l1}^+ - ie^{3i\eta}\lambda_5 Z_{i4}^H Z_{j3}^H Z_{l1}^+ \\
& -2ie^{2i\eta}\lambda_6^* \left(Z_{i1}^H Z_{j1}^H + Z_{i3}^H Z_{j3}^H \right) Z_{l1}^+ + e^{i\eta}\lambda_4 Z_{i1}^H Z_{j4}^H Z_{l1}^+ - e^{3i\eta}\lambda_5 Z_{i1}^H Z_{j4}^H Z_{l1}^+ \\
& -ie^{i\eta}\lambda_4 Z_{i3}^H Z_{j4}^H Z_{l1}^+ - ie^{3i\eta}\lambda_5 Z_{i3}^H Z_{j4}^H Z_{l1}^+ - 2ie^{2i\eta}\lambda_7^* Z_{i4}^H Z_{j4}^H Z_{l1}^+ \\
& -2ie^{i\eta}\lambda_3 Z_{i1}^H Z_{j1}^H Z_{l2}^+ + \lambda_7 Z_{i4}^H Z_{j1}^H Z_{l2}^+ - e^{2i\eta}\lambda_7^* Z_{i4}^H Z_{j1}^H Z_{l2}^+ \\
& -i\lambda_7 Z_{i1}^H Z_{j2}^H Z_{l2}^+ - ie^{2i\eta}\lambda_7^* Z_{i1}^H Z_{j2}^H Z_{l2}^+ - \lambda_7 Z_{i3}^H Z_{j2}^H Z_{l2}^+ \\
& + e^{2i\eta}\lambda_7^* Z_{i3}^H Z_{j2}^H Z_{l2}^+ - 2ie^{i\eta}\lambda_3 Z_{i3}^H Z_{j3}^H Z_{l2}^+ - i\lambda_7 Z_{i4}^H Z_{j3}^H Z_{l2}^+ \\
& -ie^{2i\eta}\lambda_7^* Z_{i4}^H Z_{j3}^H Z_{l2}^+ + \lambda_7 Z_{i1}^H Z_{j4}^H Z_{l2}^+ - e^{2i\eta}\lambda_7^* Z_{i1}^H Z_{j4}^H Z_{l2}^+ \\
& -i\lambda_7 Z_{i3}^H Z_{j4}^H Z_{l2}^+ - ie^{2i\eta}\lambda_7^* Z_{i3}^H Z_{j4}^H Z_{l2}^+ - 4ie^{i\eta}\lambda_2 Z_{i4}^H Z_{j4}^H Z_{l2}^+ \\
& + Z_{i2}^H \left(-e^{i\eta}\lambda_4 Z_{j3}^H Z_{l1}^+ + e^{3i\eta}\lambda_5 Z_{j3}^H Z_{l1}^+ - 4ie^{i\eta}\lambda_2 Z_{j2}^H Z_{l2}^+ - \lambda_7 Z_{j3}^H Z_{l2}^+ \right. \\
& \left. - iZ_{j1}^H \left(\left(e^{2i\eta}\lambda_7^* + \lambda_7 \right) Z_{l2}^+ + e^{i\eta} \left(e^{2i\eta}\lambda_5 + \lambda_4 \right) Z_{l1}^+ \right) + e^{2i\eta}\lambda_7^* \left(-2iZ_{j2}^H Z_{l1}^+ + Z_{j3}^H Z_{l2}^+ \right) \right) \quad (159)
\end{aligned}$$

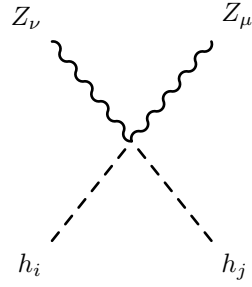


$$\begin{aligned}
& -ie^{-2i\eta} \left(Z_{i1}^{+,*} \left(e^{i\eta} Z_{j2}^{+,*} \left(2e^{2i\eta}\lambda_6^* Z_{k1}^+ Z_{l1}^+ + e^{i\eta} \left(\lambda_3 + \lambda_4 \right) Z_{k1}^+ Z_{l2}^+ + Z_{k2}^+ \left(2\lambda_7 Z_{l2}^+ + e^{i\eta} \left(\lambda_3 + \lambda_4 \right) Z_{l1}^+ \right) \right) \right. \right. \\
& \left. \left. + 2Z_{j1}^{+,*} \left(e^{i\eta} Z_{k1}^+ \left(2e^{i\eta}\lambda_1 Z_{l1}^+ + \lambda_6 Z_{l2}^+ \right) + Z_{k2}^+ \left(e^{i\eta}\lambda_6 Z_{l1}^+ + \lambda_5^* Z_{l2}^+ \right) \right) \right) \right. \\
& \left. + e^{i\eta} Z_{i2}^{+,*} \left(2e^{2i\eta}\lambda_6^* Z_{j1}^{+,*} Z_{k1}^+ Z_{l1}^+ + Z_{j1}^{+,*} \left(e^{i\eta} \left(\lambda_3 + \lambda_4 \right) Z_{k1}^+ Z_{l2}^+ + Z_{k2}^+ \left(2\lambda_7 Z_{l2}^+ + e^{i\eta} \left(\lambda_3 + \lambda_4 \right) Z_{l1}^+ \right) \right) \right) \right. \\
& \left. + 2e^{i\eta} Z_{j2}^{+,*} \left(e^{i\eta} Z_{k1}^+ \left(e^{i\eta}\lambda_5 Z_{l1}^+ + \lambda_7^* Z_{l2}^+ \right) + Z_{k2}^+ \left(2\lambda_2 Z_{l2}^+ + e^{i\eta}\lambda_7^* Z_{l1}^+ \right) \right) \right) \quad (160)
\end{aligned}$$

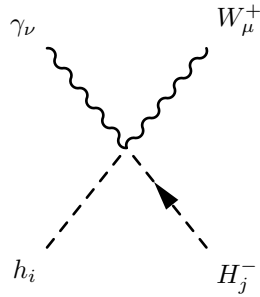
9.8 Two Scalar-Two Vector Boson-Interaction



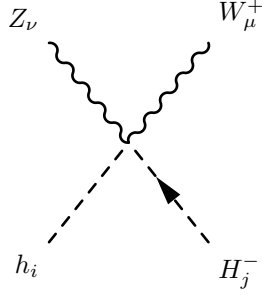
$$\frac{i}{2}g_2^2\left(Z_{i1}^H Z_{j1}^H + Z_{i2}^H Z_{j2}^H + Z_{i3}^H Z_{j3}^H + Z_{i4}^H Z_{j4}^H\right)\left(g_{\mu\nu}\right) \quad (161)$$



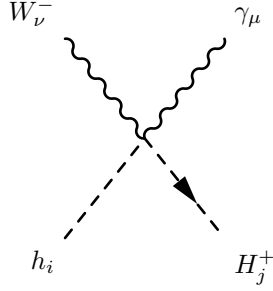
$$\frac{i}{2}\left(g_1 \sin \Theta_W + g_2 \cos \Theta_W\right)^2\left(Z_{i1}^H Z_{j1}^H + Z_{i2}^H Z_{j2}^H + Z_{i3}^H Z_{j3}^H + Z_{i4}^H Z_{j4}^H\right)\left(g_{\mu\nu}\right) \quad (162)$$



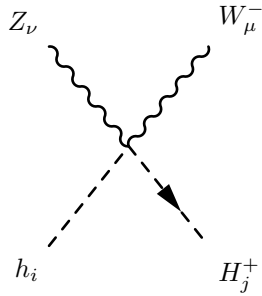
$$\frac{i}{2}g_1 g_2 \cos \Theta_W\left(Z_{j1}^{+,*}\left(iZ_{i3}^H + Z_{i1}^H\right) + Z_{j2}^{+,*}\left(iZ_{i4}^H + Z_{i2}^H\right)\right)\left(g_{\mu\nu}\right) \quad (163)$$



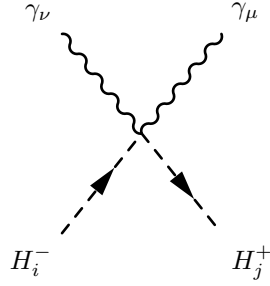
$$\frac{1}{2}g_1g_2 \sin \Theta_W \left(Z_{j1}^{+,*} \left(-iZ_{i1}^H + Z_{i3}^H \right) + Z_{j2}^{+,*} \left(-iZ_{i2}^H + Z_{i4}^H \right) \right) \left(g_{\mu\nu} \right) \quad (164)$$



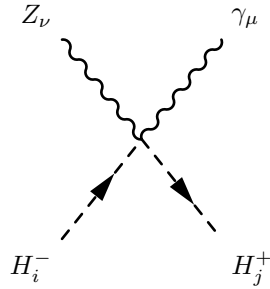
$$\frac{1}{2}g_1g_2 \cos \Theta_W \left(iZ_{i1}^H Z_{j1}^+ + \left(iZ_{i2}^H + Z_{i4}^H \right) Z_{j2}^+ + Z_{i3}^H Z_{j1}^+ \right) \left(g_{\mu\nu} \right) \quad (165)$$



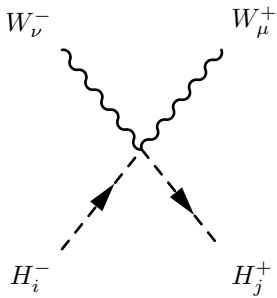
$$-\frac{i}{2}g_1g_2 \sin \Theta_W \left(-iZ_{i3}^H Z_{j1}^+ + \left(-iZ_{i4}^H + Z_{i2}^H \right) Z_{j2}^+ + Z_{i1}^H Z_{j1}^+ \right) \left(g_{\mu\nu} \right) \quad (166)$$



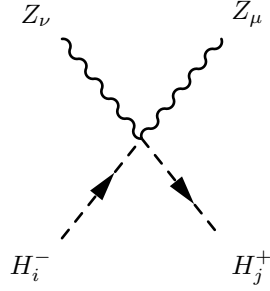
$$\frac{i}{2} (g_1 \cos \Theta_W + g_2 \sin \Theta_W)^2 (Z_{i1}^{+,*} Z_{j1}^+ + Z_{i2}^{+,*} Z_{j2}^+) (g_{\mu\nu}) \quad (167)$$



$$-\frac{i}{4} \left(-2g_1 g_2 \cos 2\Theta_W + (-g_2^2 + g_1^2) \sin 2\Theta_W \right) (Z_{i1}^{+,*} Z_{j1}^+ + Z_{i2}^{+,*} Z_{j2}^+) (g_{\mu\nu}) \quad (168)$$

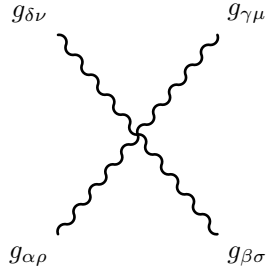


$$\frac{i}{2} g_2^2 (Z_{i1}^{+,*} Z_{j1}^+ + Z_{i2}^{+,*} Z_{j2}^+) (g_{\mu\nu}) \quad (169)$$



$$\frac{i}{2} \left(-g_1 \sin \Theta_W + g_2 \cos \Theta_W \right)^2 \left(Z_{i1}^{+,*} Z_{j1}^+ + Z_{i2}^{+,*} Z_{j2}^+ \right) (g_{\mu\nu}) \quad (170)$$

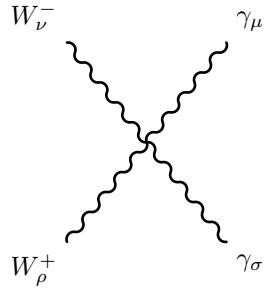
9.9 Four Vector Boson-Interaction



$$-ig_3^2 \left(\sum_{a=1}^8 f_{\alpha,\delta,a} f_{\beta,\gamma,a} + \sum_{a=1}^8 f_{\alpha,\gamma,a} f_{\beta,\delta,a} \right) (g_{\rho\sigma} g_{\mu\nu}) \quad (171)$$

$$+ ig_3^2 \left(-\sum_{a=1}^8 f_{\alpha,\beta,a} f_{\gamma,\delta,a} + \sum_{a=1}^8 f_{\alpha,\delta,a} f_{\beta,\gamma,a} \right) (g_{\rho\mu} g_{\sigma\nu}) \quad (172)$$

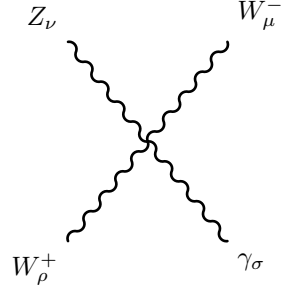
$$+ ig_3^2 \left(\sum_{a=1}^8 f_{\alpha,\gamma,a} f_{\beta,\delta,a} + \sum_{a=1}^8 f_{\alpha,\beta,a} f_{\gamma,\delta,a} \right) (g_{\rho\nu} g_{\sigma\mu}) \quad (173)$$



$$ig_2^2 \sin \Theta_W^2 (g_{\rho\sigma} g_{\mu\nu}) \quad (174)$$

$$+ ig_2^2 \sin \Theta_W^2 (g_{\rho\mu} g_{\sigma\nu}) \quad (175)$$

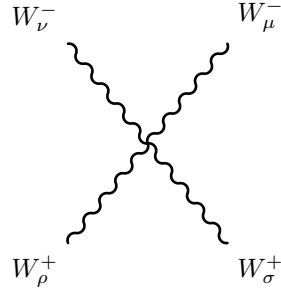
$$+ -2ig_2^2 \sin \Theta_W^2 (g_{\rho\nu} g_{\sigma\mu}) \quad (176)$$



$$ig_2^2 \cos \Theta_W \sin \Theta_W (g_{\rho\sigma} g_{\mu\nu}) \quad (177)$$

$$+ -ig_2^2 \sin 2\Theta_W (g_{\rho\mu} g_{\sigma\nu}) \quad (178)$$

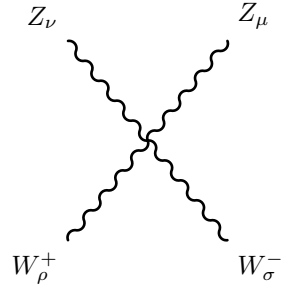
$$+ ig_2^2 \cos \Theta_W \sin \Theta_W (g_{\rho\nu} g_{\sigma\mu}) \quad (179)$$



$$2ig_2^2 (g_{\rho\sigma} g_{\mu\nu}) \quad (180)$$

$$+ -ig_2^2 (g_{\rho\mu} g_{\sigma\nu}) \quad (181)$$

$$+ -ig_2^2 (g_{\rho\nu} g_{\sigma\mu}) \quad (182)$$

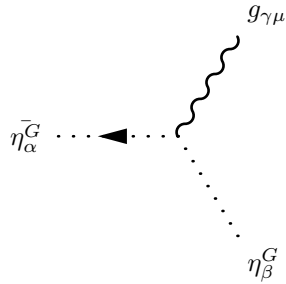


$$- 2ig_2^2 \cos^2 \Theta_W^2 (g_{\rho\sigma} g_{\mu\nu}) \quad (183)$$

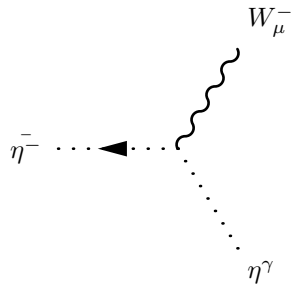
$$+ ig_2^2 \cos^2 \Theta_W^2 (g_{\rho\mu} g_{\sigma\nu}) \quad (184)$$

$$+ ig_2^2 \cos^2 \Theta_W^2 (g_{\rho\nu} g_{\sigma\mu}) \quad (185)$$

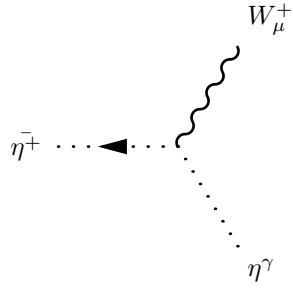
9.10 Two Ghosts-One Vector Boson-Interaction



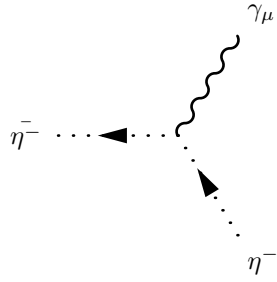
$$g_3 f_{\alpha,\beta,\gamma} (p_\mu^{\eta_\beta^G}) \quad (186)$$



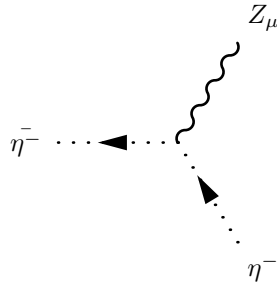
$$ig_2 \sin \Theta_W (p_\mu^{\eta^\gamma}) \quad (187)$$



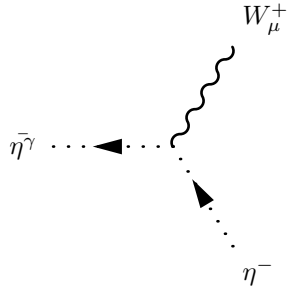
$$-ig_2 \sin \Theta_W (p_\mu^{\eta^\gamma}) \quad (188)$$



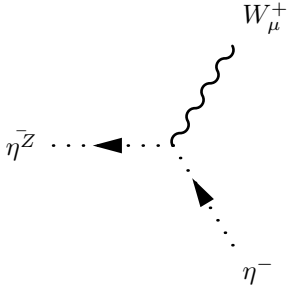
$$-ig_2 \sin \Theta_W (p_\mu^{\eta^-}) \quad (189)$$



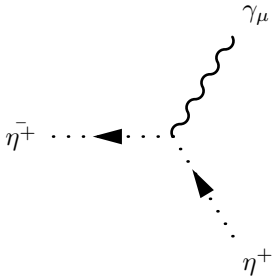
$$-ig_2 \cos \Theta_W (p_\mu^{\eta^-}) \quad (190)$$



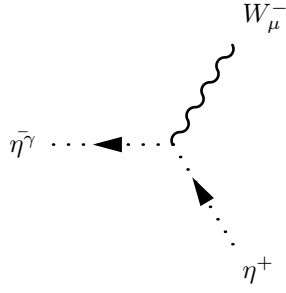
$$ig_2 \sin \Theta_W (p_\mu^{\eta^-}) \quad (191)$$



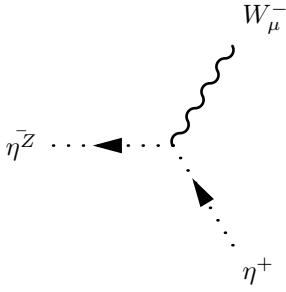
$$ig_2 \cos \Theta_W (p_\mu^{\eta^-}) \quad (192)$$



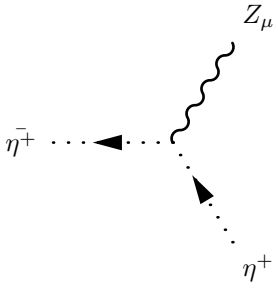
$$ig_2 \sin \Theta_W (p_\mu^{\eta^+}) \quad (193)$$



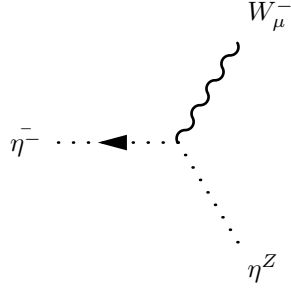
$$-ig_2 \sin \Theta_W (p_\mu^{\eta^+}) \quad (194)$$



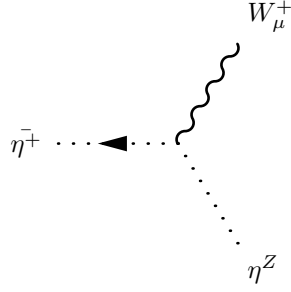
$$-ig_2 \cos \Theta_W (p_\mu^{\eta^+}) \quad (195)$$



$$ig_2 \cos \Theta_W (p_\mu^{\eta^+}) \quad (196)$$

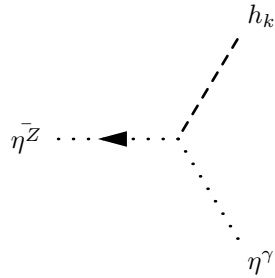


$$ig_2 \cos \Theta_W (p_\mu^{\eta^Z}) \quad (197)$$

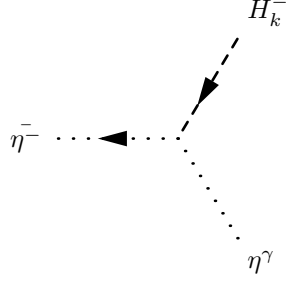


$$-ig_2 \cos \Theta_W (p_\mu^{\eta^Z}) \quad (198)$$

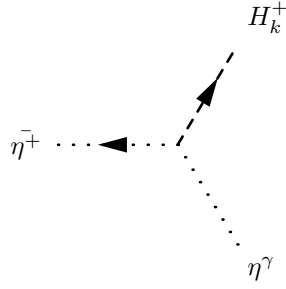
9.11 Two Ghosts-One Scalar-Interaction



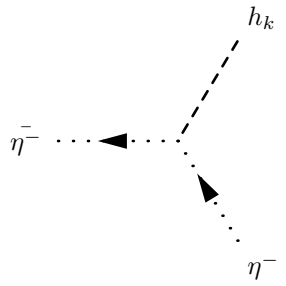
$$\frac{i}{8} \xi_Z \left(2g_1 g_2 \cos 2\Theta_W + (-g_2^2 + g_1^2) \sin 2\Theta_W \right) (v_1 Z_{k1}^H + v_2 Z_{k2}^H) \quad (199)$$



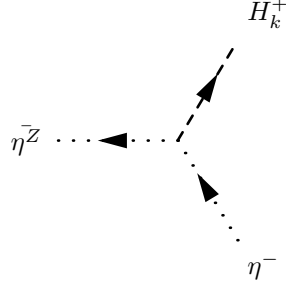
$$-\frac{i}{4}g_2(v_1Z_{k1}^{+,*} + v_2Z_{k2}^{+,*})\xi_{W^-}(g_1 \cos \Theta_W + g_2 \sin \Theta_W) \quad (200)$$



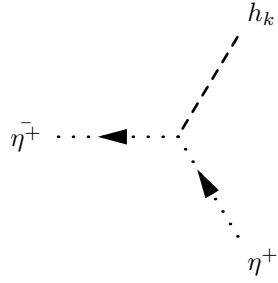
$$-\frac{i}{4}g_2\xi_{W^-}(g_1 \cos \Theta_W + g_2 \sin \Theta_W)(v_1Z_{k1}^+ + v_2Z_{k2}^+) \quad (201)$$



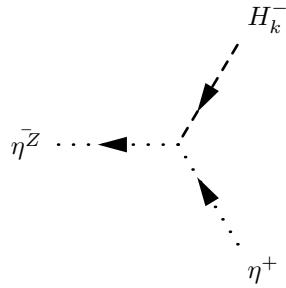
$$-\frac{i}{4}g_2^2\xi_{W^-}\left(-i(v_1Z_{k3}^H + v_2Z_{k4}^H) + v_1Z_{k1}^H + v_2Z_{k2}^H\right) \quad (202)$$



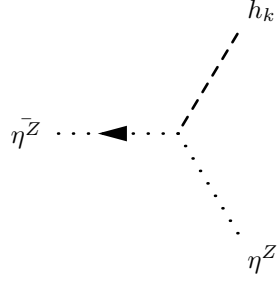
$$\frac{i}{4}g_2\xi_Z(g_1\sin\Theta_W + g_2\cos\Theta_W)(v_1Z_{k1}^+ + v_2Z_{k2}^+) \quad (203)$$



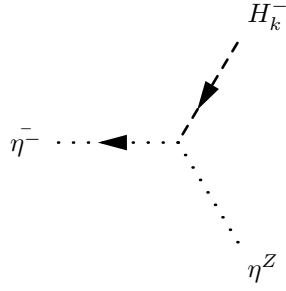
$$\frac{1}{4}g_2^2\xi_{W^-}(-iv_1Z_{k1}^H - iv_2Z_{k2}^H + v_1Z_{k3}^H + v_2Z_{k4}^H) \quad (204)$$



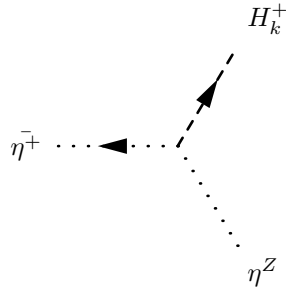
$$\frac{i}{4}g_2(v_1Z_{k1}^{+,*} + v_2Z_{k2}^{+,*})\xi_Z(g_1\sin\Theta_W + g_2\cos\Theta_W) \quad (205)$$



$$-\frac{i}{4}\xi_Z(g_1 \sin \Theta_W + g_2 \cos \Theta_W)^2(v_1 Z_{k1}^H + v_2 Z_{k2}^H) \quad (206)$$



$$-\frac{i}{4}g_2(v_1 Z_{k1}^{+,*} + v_2 Z_{k2}^{+,*})\xi_{W^-}(-g_1 \sin \Theta_W + g_2 \cos \Theta_W) \quad (207)$$



$$-\frac{i}{4}g_2\xi_{W^-}(-g_1 \sin \Theta_W + g_2 \cos \Theta_W)(v_1 Z_{k1}^+ + v_2 Z_{k2}^+) \quad (208)$$

10 Clebsch-Gordan Coefficients