

Two Higgs Doublet Model III  
Lagrangian, Rotations and Interactions for eigenstates 'EWSB'  
including Renormalization Group Equations  
including one-loop Self-Energies

SARAH 4.6.0

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References: [arXiv: 1309.7223](#) , [Comput.Phys.Commun.184:1792-1809,2011 \(1207.0906\)](#) , [Comput.Phys.Commun.182:833,2011 \(1002.0840\)](#) , [Comput.Phys.Commun.181:1077-1086,2010 \(0909.2863\)](#) , [arXiv: 0806.0538](#)

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by **Florian Staub**, [florian.staub@cern.ch](mailto:florian.staub@cern.ch)

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# 1 Fields

## 1.1 Gauge Fields

Name	$SU(N)$	Coupling	Name
$B$	$U(1)$	$g_1$	hypercharge
$W$	$SU(2)$	$g_2$	left
$g$	$SU(3)$	$g_3$	color

## 1.2 Matter Superfields

Name	Spin	Generations	$(U(1) \otimes SU(2) \otimes SU(3))$
H1	0	1	$(\frac{1}{2}, \mathbf{2}, \mathbf{1})$
H2	0	1	$(\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$q$	$\frac{1}{2}$	3	$(\frac{1}{6}, \mathbf{2}, \mathbf{3})$
$l$	$\frac{1}{2}$	3	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$d$	$\frac{1}{2}$	3	$(\frac{1}{3}, \mathbf{1}, \mathbf{\bar{3}})$
$u$	$\frac{1}{2}$	3	$(-\frac{2}{3}, \mathbf{1}, \mathbf{\bar{3}})$
$e$	$\frac{1}{2}$	3	$(1, \mathbf{1}, \mathbf{1})$

# 2 Lagrangian

## 2.1 Input Lagrangian for Eigenstates GaugeES

$$L = 0 \tag{1}$$

## 2.2 Gauge fixing terms

### 2.2.1 Gauge fixing terms for eigenstates 'GaugeES'

$$L_{GF} = -\frac{1}{2}|\partial_\mu B|^2 \xi_B^{-1} - \frac{1}{2}|\partial_\mu g|^2 \xi_g^{-1} - \frac{1}{2}|\partial_\mu W|^2 \xi_W^{-1} \tag{2}$$

### 2.2.2 Gauge fixing terms for eigenstates 'EWSB'

$$L_{GF} = -\frac{1}{2}|\partial_\mu g|^2 \xi_g^{-1} - \frac{1}{2}|\partial_\mu \gamma|^2 \xi_\gamma^{-1} - \frac{i}{2}g_2(v_1 H_1^{+,*} + v_2 H_2^{+,*})\xi_{W^-} + \partial_\mu W^-|^2 \xi_{W^-}^{-1} - \frac{1}{2}\frac{1}{2}\left(2\partial_\mu Z - (\sigma_1 v_1 + \sigma_2 v_2)\xi_Z\left(g_1 \sin \Theta_W + g_2 \cos \Theta_W\right)\right)^2 \xi_Z^{-1} \tag{3}$$

### 2.3 Fields integrated out

None

## 3 Renormalization Group Equations

### 3.1 Gauge Couplings

$$\beta_{g_1}^{(1)} = \frac{21}{5}g_1^3 \quad (4)$$

$$\begin{aligned} \beta_{g_1}^{(2)} = & \frac{1}{50}g_1^3 \left( 208g_1^2 + 180g_2^2 + 440g_3^2 - 25\text{Tr}(\epsilon_d\epsilon_d^\dagger) - 75\text{Tr}(\epsilon_e\epsilon_e^\dagger) - 85\text{Tr}(\epsilon_u\epsilon_u^\dagger) - 25\text{Tr}(Y_dY_d^\dagger) \right. \\ & \left. - 75\text{Tr}(Y_eY_e^\dagger) - 85\text{Tr}(Y_uY_u^\dagger) \right) \end{aligned} \quad (5)$$

$$\beta_{g_2}^{(1)} = -3g_2^3 \quad (6)$$

$$\begin{aligned} \beta_{g_2}^{(2)} = & \frac{1}{10}g_2^3 \left( 12g_1^2 + 80g_2^2 + 120g_3^2 - 15\text{Tr}(\epsilon_d\epsilon_d^\dagger) - 5\text{Tr}(\epsilon_e\epsilon_e^\dagger) - 15\text{Tr}(\epsilon_u\epsilon_u^\dagger) - 15\text{Tr}(Y_dY_d^\dagger) \right. \\ & \left. - 5\text{Tr}(Y_eY_e^\dagger) - 15\text{Tr}(Y_uY_u^\dagger) \right) \end{aligned} \quad (7)$$

$$\beta_{g_3}^{(1)} = -7g_3^3 \quad (8)$$

$$\beta_{g_3}^{(2)} = -\frac{1}{10}g_3^3 \left( -11g_1^2 + 20\text{Tr}(\epsilon_d\epsilon_d^\dagger) + 20\text{Tr}(\epsilon_u\epsilon_u^\dagger) + 20\text{Tr}(Y_dY_d^\dagger) + 20\text{Tr}(Y_uY_u^\dagger) + 260g_3^2 - 45g_2^2 \right) \quad (9)$$

### 3.2 Quartic scalar couplings

$$\begin{aligned} \beta_{\lambda_6}^{(1)} = & -\frac{9}{5}g_1^2\lambda_6 - 9g_2^2\lambda_6 + 24\lambda_1\lambda_6 + 6\lambda_3\lambda_6 + 8\lambda_4\lambda_6 + 10\lambda_5\lambda_6 + 6\lambda_3\lambda_7 + 4\lambda_4\lambda_7 + 2\lambda_5\lambda_7 + 3\lambda_6\text{Tr}(\epsilon_d\epsilon_d^\dagger) \\ & + 3(\lambda_3 + \lambda_4 + \lambda_5)\text{Tr}(\epsilon_dY_d^\dagger) + \lambda_6\text{Tr}(\epsilon_e\epsilon_e^\dagger) + \lambda_3\text{Tr}(\epsilon_eY_e^\dagger) + \lambda_4\text{Tr}(\epsilon_eY_e^\dagger) + \lambda_5\text{Tr}(\epsilon_eY_e^\dagger) \\ & + 9\lambda_6\text{Tr}(\epsilon_u\epsilon_u^\dagger) + 6\lambda_1\text{Tr}(\epsilon_uY_u^\dagger) + 6\lambda_1\text{Tr}(Y_d\epsilon_d^\dagger) + 9\lambda_6\text{Tr}(Y_dY_d^\dagger) + 2\lambda_1\text{Tr}(Y_e\epsilon_e^\dagger) \\ & + 3\lambda_6\text{Tr}(Y_eY_e^\dagger) + 3\lambda_3\text{Tr}(Y_u\epsilon_u^\dagger) + 3\lambda_4\text{Tr}(Y_u\epsilon_u^\dagger) + 3\lambda_5\text{Tr}(Y_u\epsilon_u^\dagger) + 3\lambda_6\text{Tr}(Y_uY_u^\dagger) \\ & - 6\text{Tr}(\epsilon_dY_d^\dagger Y_dY_d^\dagger) - 2\text{Tr}(\epsilon_eY_e^\dagger Y_eY_e^\dagger) - 3\text{Tr}(\epsilon_u\epsilon_u^\dagger\epsilon_uY_u^\dagger) - 6\text{Tr}(\epsilon_u\epsilon_u^\dagger Y_u\epsilon_u^\dagger) \\ & - 3\text{Tr}(\epsilon_uY_u^\dagger\epsilon_u\epsilon_u^\dagger) - 3\text{Tr}(Y_d\epsilon_d^\dagger Y_dY_d^\dagger) - 3\text{Tr}(Y_dY_d^\dagger Y_d\epsilon_d^\dagger) - \text{Tr}(Y_e\epsilon_e^\dagger Y_eY_e^\dagger) \\ & - \text{Tr}(Y_eY_e^\dagger Y_e\epsilon_e^\dagger) \end{aligned} \quad (10)$$

$$\begin{aligned} \beta_{\lambda_6}^{(2)} = & +\frac{1683}{200}g_1^4\lambda_6 + \frac{87}{20}g_1^2g_2^2\lambda_6 - \frac{141}{8}g_2^4\lambda_6 + \frac{108}{5}g_1^2\lambda_1\lambda_6 + 108g_2^2\lambda_1\lambda_6 - 318\lambda_1^2\lambda_6 + 6\lambda_2^2\lambda_6 \\ & + \frac{18}{5}g_1^2\lambda_3\lambda_6 + 18g_2^2\lambda_3\lambda_6 - 132\lambda_1\lambda_3\lambda_6 - 36\lambda_2\lambda_3\lambda_6 - 32\lambda_3^2\lambda_6 + 6g_1^2\lambda_4\lambda_6 + 36g_2^2\lambda_4\lambda_6 \\ & - 140\lambda_1\lambda_4\lambda_6 - 28\lambda_2\lambda_4\lambda_6 - 68\lambda_3\lambda_4\lambda_6 - 34\lambda_4^2\lambda_6 + 12g_1^2\lambda_5\lambda_6 + 54g_2^2\lambda_5\lambda_6 - 148\lambda_1\lambda_5\lambda_6 \end{aligned}$$

$$\begin{aligned}
& -20\lambda_2\lambda_5\lambda_6 - 72\lambda_3\lambda_5\lambda_6 - 76\lambda_4\lambda_5\lambda_6 - 36\lambda_5^2\lambda_6 - 111\lambda_6^3 + \frac{27}{20}g_1^4\lambda_7 + \frac{3}{2}g_1^2g_2^2\lambda_7 + \frac{45}{4}g_2^4\lambda_7 \\
& + \frac{36}{5}g_1^2\lambda_3\lambda_7 + 36g_2^2\lambda_3\lambda_7 - 36\lambda_1\lambda_3\lambda_7 - 36\lambda_2\lambda_3\lambda_7 - 36\lambda_3^2\lambda_7 + \frac{24}{5}g_1^2\lambda_4\lambda_7 + 18g_2^2\lambda_4\lambda_7 \\
& - 28\lambda_1\lambda_4\lambda_7 - 28\lambda_2\lambda_4\lambda_7 - 56\lambda_3\lambda_4\lambda_7 - 34\lambda_4^2\lambda_7 - \frac{6}{5}g_1^2\lambda_5\lambda_7 - 20\lambda_1\lambda_5\lambda_7 - 20\lambda_2\lambda_5\lambda_7 \\
& - 40\lambda_3\lambda_5\lambda_7 - 44\lambda_4\lambda_5\lambda_7 - 42\lambda_5^2\lambda_7 - 126\lambda_6^2\lambda_7 - 33\lambda_6\lambda_7^2 - 42\lambda_7^3 \\
& + \frac{1}{8}\left(16\left(10g_3^2\lambda_6 - 3\left(3\lambda_3\left(2\lambda_7 + \lambda_6\right) + 4\lambda_4\left(\lambda_6 + \lambda_7\right) + \lambda_5\left(2\lambda_7 + 5\lambda_6\right)\right)\right) + 45g_2^2\lambda_6 + 5g_1^2\lambda_6\right)\text{Tr}\left(\epsilon_d\epsilon_d^\dagger\right) \\
& + \frac{3}{40}\left(3g_1^4 + 18g_1^2g_2^2 \right. \\
& \left. - 5\left(3g_2^4 + 8\left(10\lambda_1\lambda_5 + 12\lambda_6^2 + 12\lambda_6\lambda_7 + 3\lambda_3\lambda_4 + 3\lambda_4\lambda_5 + 4\lambda_3\lambda_5 + 6\lambda_1\lambda_3 + 8\lambda_1\lambda_4 + \lambda_3^2 + \lambda_4^2\right)\right)\right)\text{Tr}\left(\epsilon_dY_d^\dagger\right) \\
& + \frac{15}{8}g_1^2\lambda_6\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) + \frac{15}{8}g_2^2\lambda_6\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - 6\lambda_3\lambda_6\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - 8\lambda_4\lambda_6\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) \\
& - 10\lambda_5\lambda_6\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - 12\lambda_3\lambda_7\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - 8\lambda_4\lambda_7\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - 4\lambda_5\lambda_7\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) \\
& - \frac{9}{8}g_1^4\text{Tr}\left(\epsilon_eY_e^\dagger\right) + \frac{33}{20}g_1^2g_2^2\text{Tr}\left(\epsilon_eY_e^\dagger\right) - \frac{3}{8}g_2^4\text{Tr}\left(\epsilon_eY_e^\dagger\right) - 6\lambda_1\lambda_3\text{Tr}\left(\epsilon_eY_e^\dagger\right) \\
& - \lambda_3^2\text{Tr}\left(\epsilon_eY_e^\dagger\right) - 8\lambda_1\lambda_4\text{Tr}\left(\epsilon_eY_e^\dagger\right) - 3\lambda_3\lambda_4\text{Tr}\left(\epsilon_eY_e^\dagger\right) - \lambda_4^2\text{Tr}\left(\epsilon_eY_e^\dagger\right) \\
& - 10\lambda_1\lambda_5\text{Tr}\left(\epsilon_eY_e^\dagger\right) - 4\lambda_3\lambda_5\text{Tr}\left(\epsilon_eY_e^\dagger\right) - 3\lambda_4\lambda_5\text{Tr}\left(\epsilon_eY_e^\dagger\right) - 12\lambda_6^2\text{Tr}\left(\epsilon_eY_e^\dagger\right) \\
& - 12\lambda_6\lambda_7\text{Tr}\left(\epsilon_eY_e^\dagger\right) + \frac{51}{8}g_1^2\lambda_6\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) + \frac{135}{8}g_2^2\lambda_6\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) + 60g_3^2\lambda_6\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) \\
& - 144\lambda_1\lambda_6\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) - 18\lambda_3\lambda_6\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) - 24\lambda_4\lambda_6\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) - 30\lambda_5\lambda_6\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) \\
& - \frac{171}{200}g_1^4\text{Tr}\left(\epsilon_uY_u^\dagger\right) + \frac{63}{20}g_1^2g_2^2\text{Tr}\left(\epsilon_uY_u^\dagger\right) - \frac{9}{8}g_2^4\text{Tr}\left(\epsilon_uY_u^\dagger\right) - 18\lambda_1\lambda_3\text{Tr}\left(\epsilon_uY_u^\dagger\right) \\
& - 3\lambda_3^2\text{Tr}\left(\epsilon_uY_u^\dagger\right) - 24\lambda_1\lambda_4\text{Tr}\left(\epsilon_uY_u^\dagger\right) - 9\lambda_3\lambda_4\text{Tr}\left(\epsilon_uY_u^\dagger\right) - 3\lambda_4^2\text{Tr}\left(\epsilon_uY_u^\dagger\right) \\
& - 30\lambda_1\lambda_5\text{Tr}\left(\epsilon_uY_u^\dagger\right) - 12\lambda_3\lambda_5\text{Tr}\left(\epsilon_uY_u^\dagger\right) - 9\lambda_4\lambda_5\text{Tr}\left(\epsilon_uY_u^\dagger\right) - 36\lambda_6^2\text{Tr}\left(\epsilon_uY_u^\dagger\right) \\
& - 36\lambda_6\lambda_7\text{Tr}\left(\epsilon_uY_u^\dagger\right) + \frac{9}{40}g_1^4\text{Tr}\left(Y_d\epsilon_d^\dagger\right) + \frac{27}{20}g_1^2g_2^2\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - \frac{9}{8}g_2^4\text{Tr}\left(Y_d\epsilon_d^\dagger\right) \\
& - 18\lambda_1\lambda_3\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - 3\lambda_3^2\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - 24\lambda_1\lambda_4\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - 9\lambda_3\lambda_4\text{Tr}\left(Y_d\epsilon_d^\dagger\right) \\
& - 3\lambda_4^2\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - 30\lambda_1\lambda_5\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - 12\lambda_3\lambda_5\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - 9\lambda_4\lambda_5\text{Tr}\left(Y_d\epsilon_d^\dagger\right) \\
& - 36\lambda_6^2\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - 36\lambda_6\lambda_7\text{Tr}\left(Y_d\epsilon_d^\dagger\right) + \frac{15}{8}g_1^2\lambda_6\text{Tr}\left(Y_dY_d^\dagger\right) + \frac{135}{8}g_2^2\lambda_6\text{Tr}\left(Y_dY_d^\dagger\right) \\
& + 60g_3^2\lambda_6\text{Tr}\left(Y_dY_d^\dagger\right) - 144\lambda_1\lambda_6\text{Tr}\left(Y_dY_d^\dagger\right) - 18\lambda_3\lambda_6\text{Tr}\left(Y_dY_d^\dagger\right) - 24\lambda_4\lambda_6\text{Tr}\left(Y_dY_d^\dagger\right) \\
& - 30\lambda_5\lambda_6\text{Tr}\left(Y_dY_d^\dagger\right) - \frac{9}{8}g_1^4\text{Tr}\left(Y_e\epsilon_e^\dagger\right) + \frac{33}{20}g_1^2g_2^2\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - \frac{3}{8}g_2^4\text{Tr}\left(Y_e\epsilon_e^\dagger\right) \\
& - 6\lambda_1\lambda_3\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - \lambda_3^2\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - 8\lambda_1\lambda_4\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - 3\lambda_3\lambda_4\text{Tr}\left(Y_e\epsilon_e^\dagger\right)
\end{aligned}$$

$$\begin{aligned}
& -\lambda_4^2 \text{Tr}(Y_e \epsilon_e^\dagger) - 10\lambda_1 \lambda_5 \text{Tr}(Y_e \epsilon_e^\dagger) - 4\lambda_3 \lambda_5 \text{Tr}(Y_e \epsilon_e^\dagger) - 3\lambda_4 \lambda_5 \text{Tr}(Y_e \epsilon_e^\dagger) \\
& - 12\lambda_6^2 \text{Tr}(Y_e \epsilon_e^\dagger) - 12\lambda_6 \lambda_7 \text{Tr}(Y_e \epsilon_e^\dagger) + \frac{45}{8} g_1^2 \lambda_6 \text{Tr}(Y_e Y_e^\dagger) + \frac{45}{8} g_2^2 \lambda_6 \text{Tr}(Y_e Y_e^\dagger) \\
& - 48\lambda_1 \lambda_6 \text{Tr}(Y_e Y_e^\dagger) - 6\lambda_3 \lambda_6 \text{Tr}(Y_e Y_e^\dagger) - 8\lambda_4 \lambda_6 \text{Tr}(Y_e Y_e^\dagger) - 10\lambda_5 \lambda_6 \text{Tr}(Y_e Y_e^\dagger) \\
& - \frac{171}{200} g_1^4 \text{Tr}(Y_u \epsilon_u^\dagger) + \frac{63}{20} g_1^2 g_2^2 \text{Tr}(Y_u \epsilon_u^\dagger) - \frac{9}{8} g_2^4 \text{Tr}(Y_u \epsilon_u^\dagger) - 18\lambda_1 \lambda_3 \text{Tr}(Y_u \epsilon_u^\dagger) \\
& - 3\lambda_3^2 \text{Tr}(Y_u \epsilon_u^\dagger) - 24\lambda_1 \lambda_4 \text{Tr}(Y_u \epsilon_u^\dagger) - 9\lambda_3 \lambda_4 \text{Tr}(Y_u \epsilon_u^\dagger) - 3\lambda_4^2 \text{Tr}(Y_u \epsilon_u^\dagger) \\
& - 30\lambda_1 \lambda_5 \text{Tr}(Y_u \epsilon_u^\dagger) - 12\lambda_3 \lambda_5 \text{Tr}(Y_u \epsilon_u^\dagger) - 9\lambda_4 \lambda_5 \text{Tr}(Y_u \epsilon_u^\dagger) - 36\lambda_6^2 \text{Tr}(Y_u \epsilon_u^\dagger) \\
& - 36\lambda_6 \lambda_7 \text{Tr}(Y_u \epsilon_u^\dagger) + \frac{17}{8} g_1^2 \lambda_6 \text{Tr}(Y_u Y_u^\dagger) + \frac{45}{8} g_2^2 \lambda_6 \text{Tr}(Y_u Y_u^\dagger) + 20g_3^2 \lambda_6 \text{Tr}(Y_u Y_u^\dagger) \\
& - 18\lambda_3 \lambda_6 \text{Tr}(Y_u Y_u^\dagger) - 24\lambda_4 \lambda_6 \text{Tr}(Y_u Y_u^\dagger) - 30\lambda_5 \lambda_6 \text{Tr}(Y_u Y_u^\dagger) - 36\lambda_3 \lambda_7 \text{Tr}(Y_u Y_u^\dagger) \\
& - 24\lambda_4 \lambda_7 \text{Tr}(Y_u Y_u^\dagger) - 12\lambda_5 \lambda_7 \text{Tr}(Y_u Y_u^\dagger) - \frac{27}{4} \lambda_6 \text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger) + 3\lambda_5 \text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d Y_d^\dagger) \\
& + 6\lambda_5 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger) - 6\lambda_6 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger) - 21\lambda_6 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) \\
& - 12\lambda_1 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_d^\dagger) - 6\lambda_4 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_d^\dagger) - 6\lambda_5 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_d^\dagger) \\
& - 6\lambda_3 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u \epsilon_d^\dagger) + 12\lambda_6 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) - 12\lambda_7 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) \\
& + 3\lambda_5 \text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger) + 6\lambda_7 \text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d Y_d^\dagger) + 3\lambda_6 \text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger) + \frac{4}{5} g_1^2 \text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger) \\
& - 32g_3^2 \text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger) + 12\lambda_1 \text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger) + 6\lambda_3 \text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger) + 6\lambda_4 \text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger) \\
& - 6\lambda_3 \text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) - 12\lambda_6 \text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u Y_d^\dagger) + \frac{3}{2} \lambda_6 \text{Tr}(\epsilon_d Y_u^\dagger Y_u \epsilon_d^\dagger) \\
& - 6\lambda_3 \text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger) - \frac{9}{4} \lambda_6 \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger) + \lambda_5 \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger) + 2\lambda_5 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) \\
& - 2\lambda_6 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger) + \lambda_5 \text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger) + 2\lambda_7 \text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e Y_e^\dagger) + \lambda_6 \text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger) \\
& - \frac{12}{5} g_1^2 \text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger) + 4\lambda_1 \text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger) + 2\lambda_3 \text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger) + 2\lambda_4 \text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger) \\
& - 12\lambda_1 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d \epsilon_u^\dagger) - 6\lambda_4 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d \epsilon_u^\dagger) - 6\lambda_5 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d \epsilon_u^\dagger) \\
& + 12\lambda_6 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) - 12\lambda_7 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) - \frac{33}{4} \lambda_6 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger) \\
& - \frac{4}{5} g_1^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger) - 16g_3^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger) + 6\lambda_1 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger) \\
& + 3\lambda_3 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger) + 3\lambda_4 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger) - \frac{8}{5} g_1^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) \\
& - 32g_3^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) + 12\lambda_1 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) + 6\lambda_3 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) + 6\lambda_4 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) \\
& - 6\lambda_6 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger) - \frac{39}{2} \lambda_6 \text{Tr}(\epsilon_u Y_d^\dagger Y_d \epsilon_u^\dagger) - 12\lambda_1 \text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger)
\end{aligned}$$

$$\begin{aligned}
& -6\lambda_4 \text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger) - 6\lambda_5 \text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger) - \frac{4}{5} g_1^2 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger) \\
& - 16g_3^2 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger) + 6\lambda_1 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger) + 3\lambda_3 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger) \\
& + 3\lambda_4 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger) + 6\lambda_7 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u Y_u^\dagger) + 3\lambda_6 \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) + 6\lambda_5 \text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger) \\
& + 6\lambda_7 \text{Tr}(Y_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger) + \frac{2}{5} g_1^2 \text{Tr}(Y_d \epsilon_d^\dagger Y_d Y_d^\dagger) - 16g_3^2 \text{Tr}(Y_d \epsilon_d^\dagger Y_d Y_d^\dagger) \\
& + 6\lambda_1 \text{Tr}(Y_d \epsilon_d^\dagger Y_d Y_d^\dagger) + 3\lambda_3 \text{Tr}(Y_d \epsilon_d^\dagger Y_d Y_d^\dagger) + 3\lambda_4 \text{Tr}(Y_d \epsilon_d^\dagger Y_d Y_d^\dagger) \\
& - 12\lambda_6 \text{Tr}(Y_d \epsilon_u^\dagger Y_u \epsilon_u^\dagger) - 12\lambda_1 \text{Tr}(Y_d \epsilon_u^\dagger Y_u Y_d^\dagger) - 6\lambda_4 \text{Tr}(Y_d \epsilon_u^\dagger Y_u Y_d^\dagger) \\
& - 6\lambda_5 \text{Tr}(Y_d \epsilon_u^\dagger Y_u Y_d^\dagger) + \frac{2}{5} g_1^2 \text{Tr}(Y_d Y_d^\dagger Y_d \epsilon_d^\dagger) - 16g_3^2 \text{Tr}(Y_d Y_d^\dagger Y_d \epsilon_d^\dagger) \\
& + 6\lambda_1 \text{Tr}(Y_d Y_d^\dagger Y_d \epsilon_d^\dagger) + 3\lambda_3 \text{Tr}(Y_d Y_d^\dagger Y_d \epsilon_d^\dagger) + 3\lambda_4 \text{Tr}(Y_d Y_d^\dagger Y_d \epsilon_d^\dagger) - \frac{33}{4} \lambda_6 \text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) \\
& - 6\lambda_3 \text{Tr}(Y_d Y_u^\dagger Y_u \epsilon_d^\dagger) - 21\lambda_6 \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) + 2\lambda_7 \text{Tr}(Y_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) \\
& - \frac{6}{5} g_1^2 \text{Tr}(Y_e \epsilon_e^\dagger Y_e Y_e^\dagger) + 2\lambda_1 \text{Tr}(Y_e \epsilon_e^\dagger Y_e Y_e^\dagger) + \lambda_3 \text{Tr}(Y_e \epsilon_e^\dagger Y_e Y_e^\dagger) \\
& + \lambda_4 \text{Tr}(Y_e \epsilon_e^\dagger Y_e Y_e^\dagger) - \frac{6}{5} g_1^2 \text{Tr}(Y_e Y_e^\dagger Y_e \epsilon_e^\dagger) + 2\lambda_1 \text{Tr}(Y_e Y_e^\dagger Y_e \epsilon_e^\dagger) \\
& + \lambda_3 \text{Tr}(Y_e Y_e^\dagger Y_e \epsilon_e^\dagger) + \lambda_4 \text{Tr}(Y_e Y_e^\dagger Y_e \epsilon_e^\dagger) - \frac{11}{4} \lambda_6 \text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) + 6\lambda_7 \text{Tr}(Y_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) \\
& + 3\lambda_5 \text{Tr}(Y_u \epsilon_u^\dagger Y_u Y_u^\dagger) + 3\lambda_5 \text{Tr}(Y_u Y_u^\dagger Y_u \epsilon_u^\dagger) - \frac{27}{4} \lambda_6 \text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger) + 3\text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d Y_d^\dagger Y_d Y_d^\dagger) \\
& + 12\text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger Y_d Y_d^\dagger) + 9\text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger Y_d \epsilon_d^\dagger) + 3\text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger \epsilon_d Y_d^\dagger) \\
& + 6\text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger Y_d \epsilon_d^\dagger) - 3\text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger Y_u \epsilon_d^\dagger) - 3\text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_d^\dagger Y_d Y_d^\dagger) \\
& + 3\text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_u^\dagger \epsilon_u \epsilon_d^\dagger) - 6\text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_u^\dagger Y_u Y_d^\dagger) + 3\text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) \\
& - 6\text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u \epsilon_u^\dagger Y_u Y_d^\dagger) - 6\text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger \epsilon_d Y_d^\dagger) - 6\text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger Y_d \epsilon_d^\dagger) \\
& + 12\text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger) + 3\text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) + 6\text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger) \\
& + 3\text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger \epsilon_d Y_d^\dagger) + 9\text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger) - 3\text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_u^\dagger \epsilon_u Y_d^\dagger) \\
& + 6\text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger) + 30\text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger Y_d Y_d^\dagger) - 6\text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) \\
& + 3\text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_u^\dagger Y_u Y_d^\dagger) - 3\text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) - 3\text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger Y_d Y_d^\dagger) \\
& + \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger Y_e Y_e^\dagger) + 4\text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger Y_e Y_e^\dagger) + 3\text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger Y_e \epsilon_e^\dagger) \\
& + 4\text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger) + 2\text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger) + \text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger) \\
& + 3\text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) + 2\text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger) + 10\text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger Y_e Y_e^\dagger)
\end{aligned}$$



$$\begin{aligned}
& -6\text{Tr}\left(\epsilon_u\epsilon_d^\dagger Y_d\epsilon_d^\dagger Y_d Y_u^\dagger\right) - 3\text{Tr}\left(\epsilon_u\epsilon_d^\dagger Y_d\epsilon_u^\dagger\epsilon_u\epsilon_u^\dagger\right) - 3\text{Tr}\left(\epsilon_u\epsilon_d^\dagger Y_d Y_d^\dagger Y_d\epsilon_u^\dagger\right) \\
& -6\text{Tr}\left(\epsilon_u\epsilon_d^\dagger Y_d Y_u^\dagger\epsilon_u Y_u^\dagger\right) - 6\text{Tr}\left(\epsilon_u\epsilon_d^\dagger Y_d Y_u^\dagger Y_u\epsilon_u^\dagger\right) + 3\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger Y_d\epsilon_u^\dagger\right) \\
& +6\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\epsilon_u\epsilon_u^\dagger\epsilon_u Y_u^\dagger\right) + 21\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\epsilon_u\epsilon_u^\dagger Y_u\epsilon_u^\dagger\right) + 12\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\epsilon_u Y_u^\dagger\epsilon_u\epsilon_u^\dagger\right) + 6\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\epsilon_u Y_u^\dagger Y_u Y_u^\dagger\right) \\
& +9\text{Tr}\left(\epsilon_u\epsilon_u^\dagger Y_u\epsilon_u^\dagger\epsilon_u\epsilon_u^\dagger\right) + 12\text{Tr}\left(\epsilon_u\epsilon_u^\dagger Y_u\epsilon_u^\dagger Y_u Y_u^\dagger\right) - 3\text{Tr}\left(\epsilon_u\epsilon_u^\dagger Y_u Y_d^\dagger Y_d\epsilon_u^\dagger\right) \\
& +3\text{Tr}\left(\epsilon_u\epsilon_u^\dagger Y_u Y_u^\dagger\epsilon_u Y_u^\dagger\right) + 9\text{Tr}\left(\epsilon_u\epsilon_u^\dagger Y_u Y_u^\dagger Y_u\epsilon_u^\dagger\right) - 3\text{Tr}\left(\epsilon_u Y_d^\dagger Y_d\epsilon_d^\dagger Y_d\epsilon_u^\dagger\right) \\
& -6\text{Tr}\left(\epsilon_u Y_d^\dagger Y_d\epsilon_u^\dagger\epsilon_u Y_u^\dagger\right) - 3\text{Tr}\left(\epsilon_u Y_d^\dagger Y_d\epsilon_u^\dagger Y_u\epsilon_u^\dagger\right) - 3\text{Tr}\left(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger\epsilon_u\epsilon_u^\dagger\right) \\
& +12\text{Tr}\left(\epsilon_u Y_u^\dagger\epsilon_u\epsilon_u^\dagger\epsilon_u\epsilon_u^\dagger\right) + 9\text{Tr}\left(\epsilon_u Y_u^\dagger\epsilon_u\epsilon_u^\dagger Y_u Y_u^\dagger\right) + 3\text{Tr}\left(\epsilon_u Y_u^\dagger\epsilon_u Y_d^\dagger Y_d\epsilon_u^\dagger\right) \\
& +3\text{Tr}\left(\epsilon_u Y_u^\dagger\epsilon_u Y_u^\dagger Y_u\epsilon_u^\dagger\right) + 6\text{Tr}\left(\epsilon_u Y_u^\dagger Y_u\epsilon_u^\dagger\epsilon_u Y_u^\dagger\right) + 9\text{Tr}\left(\epsilon_u Y_u^\dagger Y_u\epsilon_u^\dagger Y_u\epsilon_u^\dagger\right) \\
& +6\text{Tr}\left(\epsilon_u Y_u^\dagger Y_u Y_d^\dagger Y_d Y_u^\dagger\right) + 3\text{Tr}\left(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger\epsilon_u\epsilon_u^\dagger\right) + 12\text{Tr}\left(Y_d\epsilon_d^\dagger Y_d Y_d^\dagger Y_d Y_d^\dagger\right) \\
& -3\text{Tr}\left(Y_d\epsilon_d^\dagger Y_u Y_d^\dagger Y_d Y_d^\dagger\right) + 12\text{Tr}\left(Y_d Y_d^\dagger Y_d\epsilon_d^\dagger Y_d Y_d^\dagger\right) + 3\text{Tr}\left(Y_d Y_d^\dagger Y_d\epsilon_u^\dagger Y_u Y_d^\dagger\right) \\
& +6\text{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger Y_d\epsilon_d^\dagger\right) - 3\text{Tr}\left(Y_d Y_u^\dagger Y_u\epsilon_d^\dagger Y_d Y_d^\dagger\right) + 6\text{Tr}\left(Y_d Y_u^\dagger Y_u\epsilon_u^\dagger Y_u Y_d^\dagger\right) \\
& +3\text{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger Y_d\epsilon_d^\dagger\right) + 4\text{Tr}\left(Y_e\epsilon_e^\dagger Y_e Y_e^\dagger Y_e Y_e^\dagger\right) + 4\text{Tr}\left(Y_e Y_e^\dagger Y_e\epsilon_e^\dagger Y_e Y_e^\dagger\right) \\
& +2\text{Tr}\left(Y_e Y_e^\dagger Y_e Y_e^\dagger Y_e\epsilon_e^\dagger\right)
\end{aligned} \tag{11}$$

$$\begin{aligned}
\beta_{\lambda_5}^{(1)} &= -\frac{9}{5}g_1^2\lambda_5 - 9g_2^2\lambda_5 + 4\lambda_1\lambda_5 + 4\lambda_2\lambda_5 + 8\lambda_3\lambda_5 + 12\lambda_4\lambda_5 + 10\lambda_6^2 + 4\lambda_6\lambda_7 + 10\lambda_7^2 + 6\lambda_5\text{Tr}\left(\epsilon_d\epsilon_d^\dagger\right) \\
& +6\lambda_7\text{Tr}\left(\epsilon_d Y_d^\dagger\right) + 2\lambda_5\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) + 2\lambda_7\text{Tr}\left(\epsilon_e Y_e^\dagger\right) + 6\lambda_5\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) + 6\lambda_6\text{Tr}\left(\epsilon_u Y_u^\dagger\right) \\
& +6\lambda_6\text{Tr}\left(Y_d\epsilon_d^\dagger\right) + 6\lambda_5\text{Tr}\left(Y_d Y_d^\dagger\right) + 2\lambda_6\text{Tr}\left(Y_e\epsilon_e^\dagger\right) + 2\lambda_5\text{Tr}\left(Y_e Y_e^\dagger\right) + 6\lambda_7\text{Tr}\left(Y_u\epsilon_u^\dagger\right) \\
& +6\lambda_5\text{Tr}\left(Y_u Y_u^\dagger\right) - 6\text{Tr}\left(\epsilon_d Y_d^\dagger\epsilon_d Y_d^\dagger\right) - 2\text{Tr}\left(\epsilon_e Y_e^\dagger\epsilon_e Y_e^\dagger\right) - 6\text{Tr}\left(\epsilon_u Y_u^\dagger\epsilon_u Y_u^\dagger\right) - 6\text{Tr}\left(Y_d\epsilon_d^\dagger Y_d\epsilon_d^\dagger\right) \\
& -2\text{Tr}\left(Y_e\epsilon_e^\dagger Y_e\epsilon_e^\dagger\right) - 6\text{Tr}\left(Y_u\epsilon_u^\dagger Y_u\epsilon_u^\dagger\right)
\end{aligned} \tag{12}$$

$$\begin{aligned}
\beta_{\lambda_5}^{(2)} &= +\frac{1413}{200}g_1^4\lambda_5 + \frac{57}{20}g_1^2g_2^2\lambda_5 - \frac{231}{8}g_2^4\lambda_5 - \frac{12}{5}g_1^2\lambda_1\lambda_5 - 28\lambda_1^2\lambda_5 - \frac{12}{5}g_1^2\lambda_2\lambda_5 - 28\lambda_2^2\lambda_5 \\
& +\frac{48}{5}g_1^2\lambda_3\lambda_5 + 36g_2^2\lambda_3\lambda_5 - 80\lambda_1\lambda_3\lambda_5 - 80\lambda_2\lambda_3\lambda_5 - 28\lambda_3^2\lambda_5 + \frac{72}{5}g_1^2\lambda_4\lambda_5 + 72g_2^2\lambda_4\lambda_5 \\
& -88\lambda_1\lambda_4\lambda_5 - 88\lambda_2\lambda_4\lambda_5 - 76\lambda_3\lambda_4\lambda_5 - 32\lambda_4^2\lambda_5 + 6\lambda_5^3 + 12g_1^2\lambda_6^2 + 54g_2^2\lambda_6^2 - 148\lambda_1\lambda_6^2 \\
& -20\lambda_2\lambda_6^2 - 72\lambda_3\lambda_6^2 - 76\lambda_4\lambda_6^2 - 72\lambda_5\lambda_6^2 - \frac{12}{5}g_1^2\lambda_6\lambda_7 - 40\lambda_1\lambda_6\lambda_7 - 40\lambda_2\lambda_6\lambda_7 \\
& -80\lambda_3\lambda_6\lambda_7 - 88\lambda_4\lambda_6\lambda_7 - 168\lambda_5\lambda_6\lambda_7 + 12g_1^2\lambda_7^2 + 54g_2^2\lambda_7^2 - 20\lambda_1\lambda_7^2 - 148\lambda_2\lambda_7^2 \\
& -72\lambda_3\lambda_7^2 - 76\lambda_4\lambda_7^2 - 72\lambda_5\lambda_7^2 \\
& +\frac{1}{4}\left(16\left(10g_3^2\lambda_5 - 3\left(2\lambda_2\lambda_5 + 2\lambda_3\lambda_5 + 3\lambda_4\lambda_5 + 5\lambda_7^2 + \lambda_6\lambda_7\right)\right) + 45g_2^2\lambda_5 + 5g_1^2\lambda_5\right)\text{Tr}\left(\epsilon_d\epsilon_d^\dagger\right)
\end{aligned}$$

$$\begin{aligned}
& -6\left(2\lambda_1\lambda_7 + 2\lambda_2\lambda_6 + 2\lambda_3(\lambda_6 + \lambda_7) + 3\lambda_4\lambda_6 + 3\lambda_4\lambda_7 + 6\lambda_5\lambda_6 + 6\lambda_5\lambda_7\right)\text{Tr}\left(\epsilon_d Y_d^\dagger\right) + \frac{15}{4}g_1^2\lambda_5\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) \\
& + \frac{15}{4}g_2^2\lambda_5\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - 8\lambda_2\lambda_5\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - 8\lambda_3\lambda_5\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - 12\lambda_4\lambda_5\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) \\
& - 4\lambda_6\lambda_7\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - 20\lambda_7^2\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - 4\lambda_2\lambda_6\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 4\lambda_3\lambda_6\text{Tr}\left(\epsilon_e Y_e^\dagger\right) \\
& - 6\lambda_4\lambda_6\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 12\lambda_5\lambda_6\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 4\lambda_1\lambda_7\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 4\lambda_3\lambda_7\text{Tr}\left(\epsilon_e Y_e^\dagger\right) \\
& - 6\lambda_4\lambda_7\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 12\lambda_5\lambda_7\text{Tr}\left(\epsilon_e Y_e^\dagger\right) + \frac{17}{4}g_1^2\lambda_5\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) + \frac{45}{4}g_2^2\lambda_5\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) \\
& + 40g_3^2\lambda_5\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) - 24\lambda_1\lambda_5\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) - 24\lambda_3\lambda_5\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) - 36\lambda_4\lambda_5\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) \\
& - 60\lambda_6^2\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) - 12\lambda_6\lambda_7\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) - 12\lambda_2\lambda_6\text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 12\lambda_3\lambda_6\text{Tr}\left(\epsilon_u Y_u^\dagger\right) \\
& - 18\lambda_4\lambda_6\text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 36\lambda_5\lambda_6\text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 12\lambda_1\lambda_7\text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 12\lambda_3\lambda_7\text{Tr}\left(\epsilon_u Y_u^\dagger\right) \\
& - 18\lambda_4\lambda_7\text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 36\lambda_5\lambda_7\text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 12\lambda_2\lambda_6\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - 12\lambda_3\lambda_6\text{Tr}\left(Y_d\epsilon_d^\dagger\right) \\
& - 18\lambda_4\lambda_6\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - 36\lambda_5\lambda_6\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - 12\lambda_1\lambda_7\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - 12\lambda_3\lambda_7\text{Tr}\left(Y_d\epsilon_d^\dagger\right) \\
& - 18\lambda_4\lambda_7\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - 36\lambda_5\lambda_7\text{Tr}\left(Y_d\epsilon_d^\dagger\right) + \frac{5}{4}g_1^2\lambda_5\text{Tr}\left(Y_d Y_d^\dagger\right) + \frac{45}{4}g_2^2\lambda_5\text{Tr}\left(Y_d Y_d^\dagger\right) \\
& + 40g_3^2\lambda_5\text{Tr}\left(Y_d Y_d^\dagger\right) - 24\lambda_1\lambda_5\text{Tr}\left(Y_d Y_d^\dagger\right) - 24\lambda_3\lambda_5\text{Tr}\left(Y_d Y_d^\dagger\right) - 36\lambda_4\lambda_5\text{Tr}\left(Y_d Y_d^\dagger\right) \\
& - 60\lambda_6^2\text{Tr}\left(Y_d Y_d^\dagger\right) - 12\lambda_6\lambda_7\text{Tr}\left(Y_d Y_d^\dagger\right) - 4\lambda_2\lambda_6\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - 4\lambda_3\lambda_6\text{Tr}\left(Y_e\epsilon_e^\dagger\right) \\
& - 6\lambda_4\lambda_6\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - 12\lambda_5\lambda_6\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - 4\lambda_1\lambda_7\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - 4\lambda_3\lambda_7\text{Tr}\left(Y_e\epsilon_e^\dagger\right) \\
& - 6\lambda_4\lambda_7\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - 12\lambda_5\lambda_7\text{Tr}\left(Y_e\epsilon_e^\dagger\right) + \frac{15}{4}g_1^2\lambda_5\text{Tr}\left(Y_e Y_e^\dagger\right) + \frac{15}{4}g_2^2\lambda_5\text{Tr}\left(Y_e Y_e^\dagger\right) \\
& - 8\lambda_1\lambda_5\text{Tr}\left(Y_e Y_e^\dagger\right) - 8\lambda_3\lambda_5\text{Tr}\left(Y_e Y_e^\dagger\right) - 12\lambda_4\lambda_5\text{Tr}\left(Y_e Y_e^\dagger\right) - 20\lambda_6^2\text{Tr}\left(Y_e Y_e^\dagger\right) \\
& - 4\lambda_6\lambda_7\text{Tr}\left(Y_e Y_e^\dagger\right) - 12\lambda_2\lambda_6\text{Tr}\left(Y_u\epsilon_u^\dagger\right) - 12\lambda_3\lambda_6\text{Tr}\left(Y_u\epsilon_u^\dagger\right) - 18\lambda_4\lambda_6\text{Tr}\left(Y_u\epsilon_u^\dagger\right) \\
& - 36\lambda_5\lambda_6\text{Tr}\left(Y_u\epsilon_u^\dagger\right) - 12\lambda_1\lambda_7\text{Tr}\left(Y_u\epsilon_u^\dagger\right) - 12\lambda_3\lambda_7\text{Tr}\left(Y_u\epsilon_u^\dagger\right) - 18\lambda_4\lambda_7\text{Tr}\left(Y_u\epsilon_u^\dagger\right) \\
& - 36\lambda_5\lambda_7\text{Tr}\left(Y_u\epsilon_u^\dagger\right) + \frac{17}{4}g_1^2\lambda_5\text{Tr}\left(Y_u Y_u^\dagger\right) + \frac{45}{4}g_2^2\lambda_5\text{Tr}\left(Y_u Y_u^\dagger\right) + 40g_3^2\lambda_5\text{Tr}\left(Y_u Y_u^\dagger\right) \\
& - 24\lambda_2\lambda_5\text{Tr}\left(Y_u Y_u^\dagger\right) - 24\lambda_3\lambda_5\text{Tr}\left(Y_u Y_u^\dagger\right) - 36\lambda_4\lambda_5\text{Tr}\left(Y_u Y_u^\dagger\right) - 12\lambda_6\lambda_7\text{Tr}\left(Y_u Y_u^\dagger\right) \\
& - 60\lambda_7^2\text{Tr}\left(Y_u Y_u^\dagger\right) - \frac{3}{2}\lambda_5\text{Tr}\left(\epsilon_d\epsilon_d^\dagger\epsilon_d\epsilon_d^\dagger\right) + 6\lambda_6\text{Tr}\left(\epsilon_d\epsilon_d^\dagger\epsilon_d Y_d^\dagger\right) + 12\lambda_6\text{Tr}\left(\epsilon_d\epsilon_d^\dagger Y_d\epsilon_d^\dagger\right) \\
& - 18\lambda_5\text{Tr}\left(\epsilon_d\epsilon_d^\dagger Y_d Y_d^\dagger\right) - 33\lambda_5\text{Tr}\left(\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger\right) - 12\lambda_6\text{Tr}\left(\epsilon_d\epsilon_u^\dagger\epsilon_u Y_d^\dagger\right) \\
& - 12\lambda_7\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_u\epsilon_d^\dagger\right) + 12\lambda_5\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_u Y_d^\dagger\right) + 6\lambda_6\text{Tr}\left(\epsilon_d Y_d^\dagger\epsilon_d\epsilon_d^\dagger\right) \\
& + \frac{4}{5}g_1^2\text{Tr}\left(\epsilon_d Y_d^\dagger\epsilon_d Y_d^\dagger\right) - 32g_3^2\text{Tr}\left(\epsilon_d Y_d^\dagger\epsilon_d Y_d^\dagger\right) + 12\lambda_1\text{Tr}\left(\epsilon_d Y_d^\dagger\epsilon_d Y_d^\dagger\right) + 12\lambda_2\text{Tr}\left(\epsilon_d Y_d^\dagger\epsilon_d Y_d^\dagger\right)
\end{aligned}$$

$$\begin{aligned}
& -9\lambda_5 \text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger) + 12\lambda_7 \text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger) - 12\lambda_7 \text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) \\
& -24\lambda_4 \text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u Y_d^\dagger) + 3\lambda_5 \text{Tr}(\epsilon_d Y_u^\dagger Y_u \epsilon_d^\dagger) - 12\lambda_7 \text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger) \\
& -\frac{1}{2}\lambda_5 \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger) + 2\lambda_6 \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger) + 4\lambda_6 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) - 6\lambda_5 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger) \\
& + 2\lambda_6 \text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger) - \frac{12}{5}g_1^2 \text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e Y_e^\dagger) + 4\lambda_1 \text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e Y_e^\dagger) + 4\lambda_2 \text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e Y_e^\dagger) \\
& - 3\lambda_5 \text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger) + 4\lambda_7 \text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger) - 12\lambda_6 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_d \epsilon_u^\dagger) \\
& + 12\lambda_5 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_d Y_u^\dagger) - \frac{3}{2}\lambda_5 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger) + 6\lambda_7 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger) + 12\lambda_7 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) \\
& - 18\lambda_5 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger) + 3\lambda_5 \text{Tr}(\epsilon_u Y_d^\dagger Y_d \epsilon_u^\dagger) - 12\lambda_6 \text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger) \\
& + 6\lambda_7 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger) - \frac{8}{5}g_1^2 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u Y_u^\dagger) - 32g_3^2 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u Y_u^\dagger) + 12\lambda_1 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u Y_u^\dagger) \\
& + 12\lambda_2 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u Y_u^\dagger) - 9\lambda_5 \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) + 12\lambda_6 \text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger) + \frac{4}{5}g_1^2 \text{Tr}(Y_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger) \\
& - 32g_3^2 \text{Tr}(Y_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger) + 12\lambda_1 \text{Tr}(Y_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger) + 12\lambda_2 \text{Tr}(Y_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger) + 6\lambda_7 \text{Tr}(Y_d \epsilon_d^\dagger Y_d Y_d^\dagger) \\
& - 24\lambda_4 \text{Tr}(Y_d \epsilon_u^\dagger Y_u \epsilon_d^\dagger) - 12\lambda_6 \text{Tr}(Y_d \epsilon_u^\dagger Y_u Y_d^\dagger) + 6\lambda_7 \text{Tr}(Y_d Y_d^\dagger Y_d \epsilon_d^\dagger) \\
& - \frac{3}{2}\lambda_5 \text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) - 12\lambda_7 \text{Tr}(Y_d Y_u^\dagger Y_u \epsilon_d^\dagger) - 33\lambda_5 \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) \\
& - \frac{12}{5}g_1^2 \text{Tr}(Y_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) + 4\lambda_1 \text{Tr}(Y_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) + 4\lambda_2 \text{Tr}(Y_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) + 2\lambda_7 \text{Tr}(Y_e \epsilon_e^\dagger Y_e Y_e^\dagger) \\
& + 2\lambda_7 \text{Tr}(Y_e Y_e^\dagger Y_e \epsilon_e^\dagger) - \frac{1}{2}\lambda_5 \text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) - \frac{8}{5}g_1^2 \text{Tr}(Y_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) - 32g_3^2 \text{Tr}(Y_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) \\
& + 12\lambda_1 \text{Tr}(Y_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) + 12\lambda_2 \text{Tr}(Y_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) + 6\lambda_6 \text{Tr}(Y_u \epsilon_u^\dagger Y_u Y_u^\dagger) + 6\lambda_6 \text{Tr}(Y_u Y_u^\dagger Y_u \epsilon_u^\dagger) \\
& - \frac{3}{2}\lambda_5 \text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger) + 6\text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d Y_d^\dagger \epsilon_d Y_d^\dagger) + 30\text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger) - 6\text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_d^\dagger \epsilon_d Y_d^\dagger) \\
& - 6\text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u \epsilon_u^\dagger Y_u \epsilon_d^\dagger) + 6\text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger \epsilon_d Y_d^\dagger) + 18\text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger) + 18\text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d Y_d^\dagger Y_d Y_d^\dagger) \\
& + 6\text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) + 12\text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger \epsilon_d Y_d^\dagger) - 6\text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger \epsilon_d Y_d^\dagger) \\
& - 6\text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u Y_u^\dagger \epsilon_u \epsilon_d^\dagger) - 6\text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger \epsilon_d Y_d^\dagger) + 2\text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger \epsilon_e Y_e^\dagger) + 10\text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) \\
& + 2\text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger) + 6\text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger) + 6\text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e Y_e^\dagger Y_e Y_e^\dagger) + 4\text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger \epsilon_e Y_e^\dagger) \\
& - 6\text{Tr}(\epsilon_u \epsilon_u^\dagger Y_d \epsilon_d^\dagger Y_d \epsilon_u^\dagger) + 6\text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger \epsilon_u Y_u^\dagger) + 30\text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) - 6\text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger \epsilon_u Y_u^\dagger) \\
& + 6\text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger) + 18\text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger) + 18\text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u Y_u^\dagger Y_u Y_u^\dagger) + 12\text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger \epsilon_u Y_u^\dagger) \\
& + 18\text{Tr}(Y_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger Y_d Y_d^\dagger) + 6\text{Tr}(Y_d \epsilon_d^\dagger Y_d \epsilon_u^\dagger Y_u Y_d^\dagger) + 6\text{Tr}(Y_d \epsilon_d^\dagger Y_d Y_d^\dagger Y_d \epsilon_d^\dagger) \\
& - 6\text{Tr}(Y_d \epsilon_u^\dagger Y_u \epsilon_u^\dagger Y_u Y_d^\dagger) - 6\text{Tr}(Y_d \epsilon_u^\dagger Y_u Y_d^\dagger Y_d \epsilon_d^\dagger) + 6\text{Tr}(Y_d Y_d^\dagger Y_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger)
\end{aligned}$$

$$\begin{aligned}
& -6\text{Tr}\left(Y_d Y_u^\dagger Y_u \epsilon_d^\dagger Y_d \epsilon_d^\dagger\right) + 6\text{Tr}\left(Y_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger Y_e Y_e^\dagger\right) + 2\text{Tr}\left(Y_e \epsilon_e^\dagger Y_e Y_e^\dagger Y_e \epsilon_e^\dagger\right) + 2\text{Tr}\left(Y_e Y_e^\dagger Y_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger\right) \\
& + 18\text{Tr}\left(Y_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger Y_u Y_u^\dagger\right) + 6\text{Tr}\left(Y_u \epsilon_u^\dagger Y_u Y_u^\dagger Y_u \epsilon_u^\dagger\right) + 6\text{Tr}\left(Y_u Y_u^\dagger Y_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger\right)
\end{aligned} \tag{13}$$

$$\begin{aligned}
\beta_{\lambda_7}^{(1)} = & +6\lambda_3\lambda_6 + 4\lambda_4\lambda_6 + 2\lambda_5\lambda_6 - \frac{9}{5}g_1^2\lambda_7 - 9g_2^2\lambda_7 + 24\lambda_2\lambda_7 + 6\lambda_3\lambda_7 + 8\lambda_4\lambda_7 + 10\lambda_5\lambda_7 + 9\lambda_7\text{Tr}\left(\epsilon_d \epsilon_d^\dagger\right) \\
& + 6\lambda_2\text{Tr}\left(\epsilon_d Y_d^\dagger\right) + 3\lambda_7\text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) + 2\lambda_2\text{Tr}\left(\epsilon_e Y_e^\dagger\right) + 3\lambda_7\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) + 3\lambda_3\text{Tr}\left(\epsilon_u Y_u^\dagger\right) \\
& + 3\lambda_4\text{Tr}\left(\epsilon_u Y_u^\dagger\right) + 3\lambda_5\text{Tr}\left(\epsilon_u Y_u^\dagger\right) + 3\lambda_3\text{Tr}\left(Y_d \epsilon_d^\dagger\right) + 3\lambda_4\text{Tr}\left(Y_d \epsilon_d^\dagger\right) + 3\lambda_5\text{Tr}\left(Y_d \epsilon_d^\dagger\right) \\
& + 3\lambda_7\text{Tr}\left(Y_d Y_d^\dagger\right) + \lambda_3\text{Tr}\left(Y_e \epsilon_e^\dagger\right) + \lambda_4\text{Tr}\left(Y_e \epsilon_e^\dagger\right) + \lambda_5\text{Tr}\left(Y_e \epsilon_e^\dagger\right) + \lambda_7\text{Tr}\left(Y_e Y_e^\dagger\right) \\
& + 6\lambda_2\text{Tr}\left(Y_u \epsilon_u^\dagger\right) + 9\lambda_7\text{Tr}\left(Y_u Y_u^\dagger\right) - 3\text{Tr}\left(\epsilon_d \epsilon_d^\dagger \epsilon_d Y_d^\dagger\right) - 6\text{Tr}\left(\epsilon_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger\right) - 3\text{Tr}\left(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger\right) \\
& - \text{Tr}\left(\epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger\right) - 2\text{Tr}\left(\epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger\right) - \text{Tr}\left(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger\right) - 6\text{Tr}\left(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger\right) \\
& - 3\text{Tr}\left(Y_u \epsilon_u^\dagger Y_u Y_u^\dagger\right) - 3\text{Tr}\left(Y_u Y_u^\dagger Y_u \epsilon_u^\dagger\right)
\end{aligned} \tag{14}$$

$$\begin{aligned}
\beta_{\lambda_7}^{(2)} = & +\frac{27}{20}g_1^4\lambda_6 + \frac{3}{2}g_1^2g_2^2\lambda_6 + \frac{45}{4}g_2^4\lambda_6 + \frac{36}{5}g_1^2\lambda_3\lambda_6 + 36g_2^2\lambda_3\lambda_6 - 36\lambda_1\lambda_3\lambda_6 - 36\lambda_2\lambda_3\lambda_6 \\
& - 36\lambda_3^2\lambda_6 + \frac{24}{5}g_1^2\lambda_4\lambda_6 + 18g_2^2\lambda_4\lambda_6 - 28\lambda_1\lambda_4\lambda_6 - 28\lambda_2\lambda_4\lambda_6 - 56\lambda_3\lambda_4\lambda_6 - 34\lambda_4^2\lambda_6 \\
& - \frac{6}{5}g_1^2\lambda_5\lambda_6 - 20\lambda_1\lambda_5\lambda_6 - 20\lambda_2\lambda_5\lambda_6 - 40\lambda_3\lambda_5\lambda_6 - 44\lambda_4\lambda_5\lambda_6 - 42\lambda_5^2\lambda_6 - 42\lambda_6^3 + \frac{1683}{200}g_1^4\lambda_7 \\
& + \frac{87}{20}g_1^2g_2^2\lambda_7 - \frac{141}{8}g_2^4\lambda_7 + 6\lambda_1^2\lambda_7 + \frac{108}{5}g_1^2\lambda_2\lambda_7 + 108g_2^2\lambda_2\lambda_7 - 318\lambda_2^2\lambda_7 + \frac{18}{5}g_1^2\lambda_3\lambda_7 \\
& + 18g_2^2\lambda_3\lambda_7 - 36\lambda_1\lambda_3\lambda_7 - 132\lambda_2\lambda_3\lambda_7 - 32\lambda_3^2\lambda_7 + 6g_1^2\lambda_4\lambda_7 + 36g_2^2\lambda_4\lambda_7 - 28\lambda_1\lambda_4\lambda_7 \\
& - 140\lambda_2\lambda_4\lambda_7 - 68\lambda_3\lambda_4\lambda_7 - 34\lambda_4^2\lambda_7 + 12g_1^2\lambda_5\lambda_7 + 54g_2^2\lambda_5\lambda_7 - 20\lambda_1\lambda_5\lambda_7 - 148\lambda_2\lambda_5\lambda_7 \\
& - 72\lambda_3\lambda_5\lambda_7 - 76\lambda_4\lambda_5\lambda_7 - 36\lambda_5^2\lambda_7 - 33\lambda_6^2\lambda_7 - 126\lambda_6\lambda_7^2 - 111\lambda_7^3 \\
& + \frac{3}{8}\left(16\left(10g_3^2 - 24\lambda_2 - 3\lambda_3 - 4\lambda_4 - 5\lambda_5\right) + 45g_2^2 + 5g_1^2\right)\lambda_7\text{Tr}\left(\epsilon_d \epsilon_d^\dagger\right) \\
& + \frac{3}{40}\left(3g_1^4 + 18g_1^2g_2^2\right. \\
& \left. - 5\left(3g_2^4 + 8\left(10\lambda_2\lambda_5 + 12\lambda_6\lambda_7 + 12\lambda_7^2 + 3\lambda_3\lambda_4 + 3\lambda_4\lambda_5 + 4\lambda_3\lambda_5 + 6\lambda_2\lambda_3 + 8\lambda_2\lambda_4 + \lambda_3^2 + \lambda_4^2\right)\right)\right)\text{Tr}\left(\epsilon_d Y_d^\dagger\right) \\
& + \frac{45}{8}g_1^2\lambda_7\text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) + \frac{45}{8}g_2^2\lambda_7\text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) - 48\lambda_2\lambda_7\text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) - 6\lambda_3\lambda_7\text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) \\
& - 8\lambda_4\lambda_7\text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) - 10\lambda_5\lambda_7\text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) - \frac{9}{8}g_1^4\text{Tr}\left(\epsilon_e Y_e^\dagger\right) + \frac{33}{20}g_1^2g_2^2\text{Tr}\left(\epsilon_e Y_e^\dagger\right) \\
& - \frac{3}{8}g_2^4\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 6\lambda_2\lambda_3\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - \lambda_3^2\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 8\lambda_2\lambda_4\text{Tr}\left(\epsilon_e Y_e^\dagger\right) \\
& - 3\lambda_3\lambda_4\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - \lambda_4^2\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 10\lambda_2\lambda_5\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 4\lambda_3\lambda_5\text{Tr}\left(\epsilon_e Y_e^\dagger\right) \\
& - 3\lambda_4\lambda_5\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 12\lambda_6\lambda_7\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 12\lambda_7^2\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 36\lambda_3\lambda_6\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) \\
& - 24\lambda_4\lambda_6\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) - 12\lambda_5\lambda_6\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) + \frac{17}{8}g_1^2\lambda_7\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) + \frac{45}{8}g_2^2\lambda_7\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right)
\end{aligned}$$

$$\begin{aligned}
& + 20g_3^2\lambda_7\text{Tr}(\epsilon_u\epsilon_u^\dagger) - 18\lambda_3\lambda_7\text{Tr}(\epsilon_u\epsilon_u^\dagger) - 24\lambda_4\lambda_7\text{Tr}(\epsilon_u\epsilon_u^\dagger) - 30\lambda_5\lambda_7\text{Tr}(\epsilon_u\epsilon_u^\dagger) \\
& - \frac{171}{200}g_1^4\text{Tr}(\epsilon_u Y_u^\dagger) + \frac{63}{20}g_1^2g_2^2\text{Tr}(\epsilon_u Y_u^\dagger) - \frac{9}{8}g_2^4\text{Tr}(\epsilon_u Y_u^\dagger) - 18\lambda_2\lambda_3\text{Tr}(\epsilon_u Y_u^\dagger) \\
& - 3\lambda_3^2\text{Tr}(\epsilon_u Y_u^\dagger) - 24\lambda_2\lambda_4\text{Tr}(\epsilon_u Y_u^\dagger) - 9\lambda_3\lambda_4\text{Tr}(\epsilon_u Y_u^\dagger) - 3\lambda_4^2\text{Tr}(\epsilon_u Y_u^\dagger) \\
& - 30\lambda_2\lambda_5\text{Tr}(\epsilon_u Y_u^\dagger) - 12\lambda_3\lambda_5\text{Tr}(\epsilon_u Y_u^\dagger) - 9\lambda_4\lambda_5\text{Tr}(\epsilon_u Y_u^\dagger) - 36\lambda_6\lambda_7\text{Tr}(\epsilon_u Y_u^\dagger) \\
& - 36\lambda_7^2\text{Tr}(\epsilon_u Y_u^\dagger) + \frac{9}{40}g_1^4\text{Tr}(Y_d\epsilon_d^\dagger) + \frac{27}{20}g_1^2g_2^2\text{Tr}(Y_d\epsilon_d^\dagger) - \frac{9}{8}g_2^4\text{Tr}(Y_d\epsilon_d^\dagger) \\
& - 18\lambda_2\lambda_3\text{Tr}(Y_d\epsilon_d^\dagger) - 3\lambda_3^2\text{Tr}(Y_d\epsilon_d^\dagger) - 24\lambda_2\lambda_4\text{Tr}(Y_d\epsilon_d^\dagger) - 9\lambda_3\lambda_4\text{Tr}(Y_d\epsilon_d^\dagger) \\
& - 3\lambda_4^2\text{Tr}(Y_d\epsilon_d^\dagger) - 30\lambda_2\lambda_5\text{Tr}(Y_d\epsilon_d^\dagger) - 12\lambda_3\lambda_5\text{Tr}(Y_d\epsilon_d^\dagger) - 9\lambda_4\lambda_5\text{Tr}(Y_d\epsilon_d^\dagger) \\
& - 36\lambda_6\lambda_7\text{Tr}(Y_d\epsilon_d^\dagger) - 36\lambda_7^2\text{Tr}(Y_d\epsilon_d^\dagger) - 36\lambda_3\lambda_6\text{Tr}(Y_d Y_d^\dagger) - 24\lambda_4\lambda_6\text{Tr}(Y_d Y_d^\dagger) \\
& - 12\lambda_5\lambda_6\text{Tr}(Y_d Y_d^\dagger) + \frac{5}{8}g_1^2\lambda_7\text{Tr}(Y_d Y_d^\dagger) + \frac{45}{8}g_2^2\lambda_7\text{Tr}(Y_d Y_d^\dagger) + 20g_3^2\lambda_7\text{Tr}(Y_d Y_d^\dagger) \\
& - 18\lambda_3\lambda_7\text{Tr}(Y_d Y_d^\dagger) - 24\lambda_4\lambda_7\text{Tr}(Y_d Y_d^\dagger) - 30\lambda_5\lambda_7\text{Tr}(Y_d Y_d^\dagger) - \frac{9}{8}g_1^4\text{Tr}(Y_e\epsilon_e^\dagger) \\
& + \frac{33}{20}g_1^2g_2^2\text{Tr}(Y_e\epsilon_e^\dagger) - \frac{3}{8}g_2^4\text{Tr}(Y_e\epsilon_e^\dagger) - 6\lambda_2\lambda_3\text{Tr}(Y_e\epsilon_e^\dagger) - \lambda_3^2\text{Tr}(Y_e\epsilon_e^\dagger) \\
& - 8\lambda_2\lambda_4\text{Tr}(Y_e\epsilon_e^\dagger) - 3\lambda_3\lambda_4\text{Tr}(Y_e\epsilon_e^\dagger) - \lambda_4^2\text{Tr}(Y_e\epsilon_e^\dagger) - 10\lambda_2\lambda_5\text{Tr}(Y_e\epsilon_e^\dagger) \\
& - 4\lambda_3\lambda_5\text{Tr}(Y_e\epsilon_e^\dagger) - 3\lambda_4\lambda_5\text{Tr}(Y_e\epsilon_e^\dagger) - 12\lambda_6\lambda_7\text{Tr}(Y_e\epsilon_e^\dagger) - 12\lambda_7^2\text{Tr}(Y_e\epsilon_e^\dagger) \\
& - 12\lambda_3\lambda_6\text{Tr}(Y_e Y_e^\dagger) - 8\lambda_4\lambda_6\text{Tr}(Y_e Y_e^\dagger) - 4\lambda_5\lambda_6\text{Tr}(Y_e Y_e^\dagger) + \frac{15}{8}g_1^2\lambda_7\text{Tr}(Y_e Y_e^\dagger) \\
& + \frac{15}{8}g_2^2\lambda_7\text{Tr}(Y_e Y_e^\dagger) - 6\lambda_3\lambda_7\text{Tr}(Y_e Y_e^\dagger) - 8\lambda_4\lambda_7\text{Tr}(Y_e Y_e^\dagger) - 10\lambda_5\lambda_7\text{Tr}(Y_e Y_e^\dagger) \\
& - \frac{171}{200}g_1^4\text{Tr}(Y_u\epsilon_u^\dagger) + \frac{63}{20}g_1^2g_2^2\text{Tr}(Y_u\epsilon_u^\dagger) - \frac{9}{8}g_2^4\text{Tr}(Y_u\epsilon_u^\dagger) - 18\lambda_2\lambda_3\text{Tr}(Y_u\epsilon_u^\dagger) \\
& - 3\lambda_3^2\text{Tr}(Y_u\epsilon_u^\dagger) - 24\lambda_2\lambda_4\text{Tr}(Y_u\epsilon_u^\dagger) - 9\lambda_3\lambda_4\text{Tr}(Y_u\epsilon_u^\dagger) - 3\lambda_4^2\text{Tr}(Y_u\epsilon_u^\dagger) \\
& - 30\lambda_2\lambda_5\text{Tr}(Y_u\epsilon_u^\dagger) - 12\lambda_3\lambda_5\text{Tr}(Y_u\epsilon_u^\dagger) - 9\lambda_4\lambda_5\text{Tr}(Y_u\epsilon_u^\dagger) - 36\lambda_6\lambda_7\text{Tr}(Y_u\epsilon_u^\dagger) \\
& - 36\lambda_7^2\text{Tr}(Y_u\epsilon_u^\dagger) + \frac{51}{8}g_1^2\lambda_7\text{Tr}(Y_u Y_u^\dagger) + \frac{135}{8}g_2^2\lambda_7\text{Tr}(Y_u Y_u^\dagger) + 60g_3^2\lambda_7\text{Tr}(Y_u Y_u^\dagger) \\
& - 144\lambda_2\lambda_7\text{Tr}(Y_u Y_u^\dagger) - 18\lambda_3\lambda_7\text{Tr}(Y_u Y_u^\dagger) - 24\lambda_4\lambda_7\text{Tr}(Y_u Y_u^\dagger) - 30\lambda_5\lambda_7\text{Tr}(Y_u Y_u^\dagger) \\
& - \frac{33}{4}\lambda_7\text{Tr}(\epsilon_d\epsilon_d^\dagger\epsilon_d\epsilon_d^\dagger) + \frac{2}{5}g_1^2\text{Tr}(\epsilon_d\epsilon_d^\dagger\epsilon_d Y_d^\dagger) - 16g_3^2\text{Tr}(\epsilon_d\epsilon_d^\dagger\epsilon_d Y_d^\dagger) \\
& + 6\lambda_2\text{Tr}(\epsilon_d\epsilon_d^\dagger\epsilon_d Y_d^\dagger) + 3\lambda_3\text{Tr}(\epsilon_d\epsilon_d^\dagger\epsilon_d Y_d^\dagger) + 3\lambda_4\text{Tr}(\epsilon_d\epsilon_d^\dagger\epsilon_d Y_d^\dagger) + \frac{4}{5}g_1^2\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d\epsilon_d^\dagger) \\
& - 32g_3^2\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d\epsilon_d^\dagger) + 12\lambda_2\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d\epsilon_d^\dagger) + 6\lambda_3\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d\epsilon_d^\dagger) + 6\lambda_4\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d\epsilon_d^\dagger) \\
& - 6\lambda_7\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d Y_d^\dagger) - 21\lambda_7\text{Tr}(\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger) - 6\lambda_3\text{Tr}(\epsilon_d\epsilon_u^\dagger\epsilon_u Y_d^\dagger)
\end{aligned}$$

$$\begin{aligned}
& -12\lambda_2 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u \epsilon_d^\dagger) - 6\lambda_4 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u \epsilon_d^\dagger) - 6\lambda_5 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u \epsilon_d^\dagger) \\
& -12\lambda_6 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) + 12\lambda_7 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) + \frac{2}{5} g_1^2 \text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger) \\
& -16g_3^2 \text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger) + 6\lambda_2 \text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger) + 3\lambda_3 \text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger) \\
& + 3\lambda_4 \text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger) + 6\lambda_6 \text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d Y_d^\dagger) + 3\lambda_7 \text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger) + 6\lambda_5 \text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger) \\
& -12\lambda_2 \text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) - 6\lambda_4 \text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) - 6\lambda_5 \text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) \\
& -12\lambda_7 \text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u Y_d^\dagger) - \frac{39}{2} \lambda_7 \text{Tr}(\epsilon_d Y_u^\dagger Y_u \epsilon_d^\dagger) - 12\lambda_2 \text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger) \\
& -6\lambda_4 \text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger) - 6\lambda_5 \text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger) - \frac{11}{4} \lambda_7 \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger) \\
& -\frac{6}{5} g_1^2 \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger) + 2\lambda_2 \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger) + \lambda_3 \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger) \\
& + \lambda_4 \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger) - \frac{12}{5} g_1^2 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) + 4\lambda_2 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) + 2\lambda_3 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) \\
& + 2\lambda_4 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) - 2\lambda_7 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger) - \frac{6}{5} g_1^2 \text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger) \\
& + 2\lambda_2 \text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger) + \lambda_3 \text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger) + \lambda_4 \text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger) + 2\lambda_6 \text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e Y_e^\dagger) \\
& + \lambda_7 \text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger) + 2\lambda_5 \text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger) - 6\lambda_3 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_d \epsilon_u^\dagger) \\
& -12\lambda_6 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_d Y_u^\dagger) + 12\lambda_7 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_d Y_u^\dagger) - \frac{27}{4} \lambda_7 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger) \\
& + 3\lambda_5 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger) + 6\lambda_5 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) - 6\lambda_7 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger) \\
& + \frac{3}{2} \lambda_7 \text{Tr}(\epsilon_u Y_d^\dagger Y_d \epsilon_u^\dagger) - 6\lambda_3 \text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger) + 3\lambda_5 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger) \\
& + 6\lambda_6 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u Y_u^\dagger) + 3\lambda_7 \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) - \frac{8}{5} g_1^2 \text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger) - 32g_3^2 \text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger) \\
& + 12\lambda_2 \text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger) + 6\lambda_3 \text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger) + 6\lambda_4 \text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger) + 6\lambda_6 \text{Tr}(Y_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger) \\
& + 3\lambda_5 \text{Tr}(Y_d \epsilon_d^\dagger Y_d Y_d^\dagger) - 12\lambda_7 \text{Tr}(Y_d \epsilon_u^\dagger Y_u \epsilon_d^\dagger) - 6\lambda_3 \text{Tr}(Y_d \epsilon_u^\dagger Y_u Y_d^\dagger) \\
& + 3\lambda_5 \text{Tr}(Y_d Y_d^\dagger Y_d \epsilon_d^\dagger) - \frac{27}{4} \lambda_7 \text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) - 12\lambda_2 \text{Tr}(Y_d Y_u^\dagger Y_u \epsilon_d^\dagger) \\
& -6\lambda_4 \text{Tr}(Y_d Y_u^\dagger Y_u \epsilon_d^\dagger) - 6\lambda_5 \text{Tr}(Y_d Y_u^\dagger Y_u \epsilon_d^\dagger) - 21\lambda_7 \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) \\
& + 2\lambda_6 \text{Tr}(Y_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) + \lambda_5 \text{Tr}(Y_e \epsilon_e^\dagger Y_e Y_e^\dagger) + \lambda_5 \text{Tr}(Y_e Y_e^\dagger Y_e \epsilon_e^\dagger) - \frac{9}{4} \lambda_7 \text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) \\
& + 6\lambda_6 \text{Tr}(Y_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) - \frac{4}{5} g_1^2 \text{Tr}(Y_u \epsilon_u^\dagger Y_u Y_u^\dagger) - 16g_3^2 \text{Tr}(Y_u \epsilon_u^\dagger Y_u Y_u^\dagger) \\
& + 6\lambda_2 \text{Tr}(Y_u \epsilon_u^\dagger Y_u Y_u^\dagger) + 3\lambda_3 \text{Tr}(Y_u \epsilon_u^\dagger Y_u Y_u^\dagger) + 3\lambda_4 \text{Tr}(Y_u \epsilon_u^\dagger Y_u Y_u^\dagger)
\end{aligned}$$



$$+ 6\text{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger Y_u \epsilon_u^\dagger\right) \quad (15)$$

$$\begin{aligned} \beta_{\lambda_1}^{(1)} = & + \frac{27}{200}g_1^4 + \frac{9}{20}g_1^2g_2^2 + \frac{9}{8}g_2^4 - \frac{9}{5}g_1^2\lambda_1 - 9g_2^2\lambda_1 + 24\lambda_1^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + \lambda_5^2 + 12\lambda_6^2 \\ & + 6\lambda_6\text{Tr}\left(\epsilon_d Y_d^\dagger\right) + 2\lambda_6\text{Tr}\left(\epsilon_e Y_e^\dagger\right) + 12\lambda_1\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) + 12\lambda_1\text{Tr}\left(Y_d Y_d^\dagger\right) + 4\lambda_1\text{Tr}\left(Y_e Y_e^\dagger\right) \\ & + 6\lambda_6\text{Tr}\left(Y_u \epsilon_u^\dagger\right) - 6\text{Tr}\left(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger\right) - 6\text{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger\right) - 2\text{Tr}\left(Y_e Y_e^\dagger Y_e Y_e^\dagger\right) \end{aligned} \quad (16)$$

$$\begin{aligned} \beta_{\lambda_1}^{(2)} = & - \frac{3537}{2000}g_1^6 - \frac{1719}{400}g_1^4g_2^2 - \frac{303}{80}g_1^2g_2^4 + \frac{291}{16}g_2^6 + \frac{1953}{200}g_1^4\lambda_1 + \frac{117}{20}g_1^2g_2^2\lambda_1 - \frac{51}{8}g_2^4\lambda_1 + \frac{108}{5}g_1^2\lambda_1^2 \\ & + 108g_2^2\lambda_1^2 - 312\lambda_1^3 + \frac{9}{10}g_1^4\lambda_3 + \frac{15}{2}g_2^4\lambda_3 + \frac{12}{5}g_1^2\lambda_3^2 + 12g_2^2\lambda_3^2 - 20\lambda_1\lambda_3^2 - 8\lambda_3^3 + \frac{9}{20}g_1^4\lambda_4 \\ & + \frac{3}{2}g_1^2g_2^2\lambda_4 + \frac{15}{4}g_2^4\lambda_4 + \frac{12}{5}g_1^2\lambda_3\lambda_4 + 12g_2^2\lambda_3\lambda_4 - 20\lambda_1\lambda_3\lambda_4 - 12\lambda_3^2\lambda_4 + \frac{6}{5}g_1^2\lambda_4^2 \\ & + 3g_2^2\lambda_4^2 - 12\lambda_1\lambda_4^2 - 16\lambda_3\lambda_4^2 - 6\lambda_4^3 - \frac{3}{5}g_1^2\lambda_5^2 - 14\lambda_1\lambda_5^2 - 20\lambda_3\lambda_5^2 - 22\lambda_4\lambda_5^2 \\ & + \frac{54}{5}g_1^2\lambda_6^2 + 54g_2^2\lambda_6^2 - 318\lambda_1\lambda_6^2 - 66\lambda_3\lambda_6^2 - 70\lambda_4\lambda_6^2 - 74\lambda_5\lambda_6^2 - 36\lambda_3\lambda_6\lambda_7 - 28\lambda_4\lambda_6\lambda_7 \\ & - 20\lambda_5\lambda_6\lambda_7 + 6\lambda_1\lambda_7^2 - 18\lambda_3\lambda_7^2 - 14\lambda_4\lambda_7^2 - 10\lambda_5\lambda_7^2 \\ & - 6\left(2\lambda_3^2 + 2\lambda_3\lambda_4 + 6\lambda_6^2 + \lambda_4^2 + \lambda_5^2\right)\text{Tr}\left(\epsilon_d \epsilon_d^\dagger\right) - 6\left(12\lambda_1 + 2\lambda_4 + 3\lambda_3 + \lambda_5\right)\lambda_6\text{Tr}\left(\epsilon_d Y_d^\dagger\right) \\ & - 4\lambda_3^2\text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) - 4\lambda_3\lambda_4\text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) - 2\lambda_4^2\text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) - 2\lambda_5^2\text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) \\ & - 12\lambda_6^2\text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) - 24\lambda_1\lambda_6\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 6\lambda_3\lambda_6\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 4\lambda_4\lambda_6\text{Tr}\left(\epsilon_e Y_e^\dagger\right) \\ & - 2\lambda_5\lambda_6\text{Tr}\left(\epsilon_e Y_e^\dagger\right) - \frac{171}{100}g_1^4\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) + \frac{63}{10}g_1^2g_2^2\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) - \frac{9}{4}g_2^4\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) \\ & + \frac{17}{2}g_1^2\lambda_1\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) + \frac{45}{2}g_2^2\lambda_1\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) + 80g_3^2\lambda_1\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) - 144\lambda_1^2\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) \\ & - 36\lambda_6^2\text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) - 72\lambda_1\lambda_6\text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 18\lambda_3\lambda_6\text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 12\lambda_4\lambda_6\text{Tr}\left(\epsilon_u Y_u^\dagger\right) \\ & - 6\lambda_5\lambda_6\text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 72\lambda_1\lambda_6\text{Tr}\left(Y_d \epsilon_d^\dagger\right) - 18\lambda_3\lambda_6\text{Tr}\left(Y_d \epsilon_d^\dagger\right) - 12\lambda_4\lambda_6\text{Tr}\left(Y_d \epsilon_d^\dagger\right) \\ & - 6\lambda_5\lambda_6\text{Tr}\left(Y_d \epsilon_d^\dagger\right) + \frac{9}{20}g_1^4\text{Tr}\left(Y_d Y_d^\dagger\right) + \frac{27}{10}g_1^2g_2^2\text{Tr}\left(Y_d Y_d^\dagger\right) - \frac{9}{4}g_2^4\text{Tr}\left(Y_d Y_d^\dagger\right) \\ & + \frac{5}{2}g_1^2\lambda_1\text{Tr}\left(Y_d Y_d^\dagger\right) + \frac{45}{2}g_2^2\lambda_1\text{Tr}\left(Y_d Y_d^\dagger\right) + 80g_3^2\lambda_1\text{Tr}\left(Y_d Y_d^\dagger\right) - 144\lambda_1^2\text{Tr}\left(Y_d Y_d^\dagger\right) \\ & - 36\lambda_6^2\text{Tr}\left(Y_d Y_d^\dagger\right) - 24\lambda_1\lambda_6\text{Tr}\left(Y_e \epsilon_e^\dagger\right) - 6\lambda_3\lambda_6\text{Tr}\left(Y_e \epsilon_e^\dagger\right) - 4\lambda_4\lambda_6\text{Tr}\left(Y_e \epsilon_e^\dagger\right) \\ & - 2\lambda_5\lambda_6\text{Tr}\left(Y_e \epsilon_e^\dagger\right) - \frac{9}{4}g_1^4\text{Tr}\left(Y_e Y_e^\dagger\right) + \frac{33}{10}g_1^2g_2^2\text{Tr}\left(Y_e Y_e^\dagger\right) - \frac{3}{4}g_2^4\text{Tr}\left(Y_e Y_e^\dagger\right) \\ & + \frac{15}{2}g_1^2\lambda_1\text{Tr}\left(Y_e Y_e^\dagger\right) + \frac{15}{2}g_2^2\lambda_1\text{Tr}\left(Y_e Y_e^\dagger\right) - 48\lambda_1^2\text{Tr}\left(Y_e Y_e^\dagger\right) - 12\lambda_6^2\text{Tr}\left(Y_e Y_e^\dagger\right) \\ & - 72\lambda_1\lambda_6\text{Tr}\left(Y_u \epsilon_u^\dagger\right) - 18\lambda_3\lambda_6\text{Tr}\left(Y_u \epsilon_u^\dagger\right) - 12\lambda_4\lambda_6\text{Tr}\left(Y_u \epsilon_u^\dagger\right) - 6\lambda_5\lambda_6\text{Tr}\left(Y_u \epsilon_u^\dagger\right) \\ & - 12\lambda_3^2\text{Tr}\left(Y_u Y_u^\dagger\right) - 12\lambda_3\lambda_4\text{Tr}\left(Y_u Y_u^\dagger\right) - 6\lambda_4^2\text{Tr}\left(Y_u Y_u^\dagger\right) - 6\lambda_5^2\text{Tr}\left(Y_u Y_u^\dagger\right) \end{aligned}$$



$$\begin{aligned}
& -36\lambda_6^2 \text{Tr}\left(Y_u Y_u^\dagger\right) - 18\lambda_1 \text{Tr}\left(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger\right) - 9\lambda_1 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger\right) \\
& - 12\lambda_6 \text{Tr}\left(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_d^\dagger\right) + 12\lambda_1 \text{Tr}\left(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger\right) - 12\lambda_3 \text{Tr}\left(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger\right) \\
& + 6\lambda_5 \text{Tr}\left(\epsilon_d Y_d^\dagger \epsilon_d Y_d^\dagger\right) - 9\lambda_1 \text{Tr}\left(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger\right) + 12\lambda_6 \text{Tr}\left(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger\right) - 6\lambda_1 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger\right) \\
& + 2\lambda_5 \text{Tr}\left(\epsilon_e Y_e^\dagger \epsilon_e Y_e^\dagger\right) - 3\lambda_1 \text{Tr}\left(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger\right) + 4\lambda_6 \text{Tr}\left(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger\right) - 12\lambda_6 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger Y_d \epsilon_u^\dagger\right) \\
& + 12\lambda_1 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger Y_d Y_u^\dagger\right) - 12\lambda_3 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger Y_d Y_u^\dagger\right) - \frac{8}{5} g_1^2 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger\right) \\
& - 32g_3^2 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger\right) - 3\lambda_1 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger\right) + 6\lambda_6 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger\right) + 12\lambda_6 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger\right) \\
& - 18\lambda_1 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger\right) - 42\lambda_1 \text{Tr}\left(\epsilon_u Y_d^\dagger Y_d \epsilon_u^\dagger\right) - 12\lambda_6 \text{Tr}\left(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger\right) \\
& + 6\lambda_6 \text{Tr}\left(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger\right) + 6\lambda_5 \text{Tr}\left(\epsilon_u Y_u^\dagger \epsilon_u Y_u^\dagger\right) - 9\lambda_1 \text{Tr}\left(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger\right) + 6\lambda_5 \text{Tr}\left(Y_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger\right) \\
& + 6\lambda_6 \text{Tr}\left(Y_d \epsilon_d^\dagger Y_d Y_d^\dagger\right) - 12\lambda_6 \text{Tr}\left(Y_d \epsilon_u^\dagger Y_u Y_d^\dagger\right) + 6\lambda_6 \text{Tr}\left(Y_d Y_d^\dagger Y_d \epsilon_d^\dagger\right) \\
& + \frac{4}{5} g_1^2 \text{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger\right) - 32g_3^2 \text{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger\right) - 3\lambda_1 \text{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger\right) - 9\lambda_1 \text{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger\right) \\
& + 2\lambda_5 \text{Tr}\left(Y_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger\right) + 2\lambda_6 \text{Tr}\left(Y_e \epsilon_e^\dagger Y_e Y_e^\dagger\right) + 2\lambda_6 \text{Tr}\left(Y_e Y_e^\dagger Y_e \epsilon_e^\dagger\right) - \frac{12}{5} g_1^2 \text{Tr}\left(Y_e Y_e^\dagger Y_e Y_e^\dagger\right) \\
& - \lambda_1 \text{Tr}\left(Y_e Y_e^\dagger Y_e Y_e^\dagger\right) + 6\lambda_5 \text{Tr}\left(Y_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger\right) + 12 \text{Tr}\left(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger Y_d Y_d^\dagger\right) \\
& + 12 \text{Tr}\left(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger Y_d Y_d^\dagger\right) + 6 \text{Tr}\left(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger\right) - 12 \text{Tr}\left(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger Y_u Y_d^\dagger\right) \\
& - 12 \text{Tr}\left(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger Y_d Y_d^\dagger\right) + 12 \text{Tr}\left(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger Y_d Y_d^\dagger\right) + 6 \text{Tr}\left(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger Y_d \epsilon_d^\dagger\right) \\
& + 4 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger Y_e Y_e^\dagger\right) + 4 \text{Tr}\left(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger Y_e Y_e^\dagger\right) + 2 \text{Tr}\left(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger Y_e \epsilon_e^\dagger\right) \\
& - 12 \text{Tr}\left(\epsilon_u \epsilon_d^\dagger Y_d Y_d^\dagger Y_d Y_u^\dagger\right) - 12 \text{Tr}\left(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger \epsilon_u \epsilon_u^\dagger\right) + 30 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger\right) \\
& + 12 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger\right) + 6 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger \epsilon_u Y_d^\dagger Y_d \epsilon_u^\dagger\right) + 6 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger\right) \\
& + 12 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger Y_u Y_d^\dagger Y_d Y_u^\dagger\right) - 12 \text{Tr}\left(\epsilon_u Y_d^\dagger Y_d \epsilon_d^\dagger \epsilon_u \epsilon_u^\dagger\right) - 6 \text{Tr}\left(\epsilon_u Y_d^\dagger Y_d Y_d^\dagger Y_d \epsilon_u^\dagger\right) \\
& + 6 \text{Tr}\left(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger\right) + 6 \text{Tr}\left(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger\right) + 30 \text{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger Y_d Y_d^\dagger\right) + 6 \text{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger Y_d Y_d^\dagger\right) \\
& + 10 \text{Tr}\left(Y_e Y_e^\dagger Y_e Y_e^\dagger Y_e Y_e^\dagger\right)
\end{aligned} \tag{17}$$

$$\begin{aligned}
\beta_{\lambda_4}^{(1)} &= \frac{9}{5} g_1^2 g_2^2 - \frac{9}{5} g_1^2 \lambda_4 - 9g_2^2 \lambda_4 + 4\lambda_1 \lambda_4 + 4\lambda_2 \lambda_4 + 8\lambda_3 \lambda_4 + 4\lambda_4^2 + 8\lambda_5^2 + 10\lambda_6^2 + 4\lambda_6 \lambda_7 + 10\lambda_7^2 \\
& + 6\lambda_4 \text{Tr}\left(\epsilon_d \epsilon_d^\dagger\right) + 6\lambda_7 \text{Tr}\left(\epsilon_d Y_d^\dagger\right) + 2\lambda_4 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) + 2\lambda_7 \text{Tr}\left(\epsilon_e Y_e^\dagger\right) + 6\lambda_4 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) \\
& + 6\lambda_6 \text{Tr}\left(\epsilon_u Y_u^\dagger\right) + 6\lambda_6 \text{Tr}\left(Y_d \epsilon_d^\dagger\right) + 6\lambda_4 \text{Tr}\left(Y_d Y_d^\dagger\right) + 2\lambda_6 \text{Tr}\left(Y_e \epsilon_e^\dagger\right) + 2\lambda_4 \text{Tr}\left(Y_e Y_e^\dagger\right) \\
& + 6\lambda_7 \text{Tr}\left(Y_u \epsilon_u^\dagger\right) + 6\lambda_4 \text{Tr}\left(Y_u Y_u^\dagger\right) + 12 \text{Tr}\left(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger\right) - 12 \text{Tr}\left(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger\right) \\
& - 12 \text{Tr}\left(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger\right) - 4 \text{Tr}\left(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger\right) - 12 \text{Tr}\left(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger\right) - 12 \text{Tr}\left(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger\right)
\end{aligned}$$

(18)

$$\begin{aligned}
& + 12\text{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger\right) \\
\beta_{\lambda_4}^{(2)} = & -\frac{657}{50}g_1^4 g_2^2 - \frac{42}{5}g_1^2 g_2^4 + 6g_1^2 g_2^2 \lambda_1 + 6g_1^2 g_2^2 \lambda_2 + \frac{6}{5}g_1^2 g_2^2 \lambda_3 + \frac{1413}{200}g_1^4 \lambda_4 + \frac{153}{20}g_1^2 g_2^2 \lambda_4 \\
& - \frac{231}{8}g_2^4 \lambda_4 + \frac{24}{5}g_1^2 \lambda_1 \lambda_4 - 28\lambda_1^2 \lambda_4 + \frac{24}{5}g_1^2 \lambda_2 \lambda_4 - 28\lambda_2^2 \lambda_4 + \frac{12}{5}g_1^2 \lambda_3 \lambda_4 + 36g_2^2 \lambda_3 \lambda_4 \\
& - 80\lambda_1 \lambda_3 \lambda_4 - 80\lambda_2 \lambda_3 \lambda_4 - 28\lambda_3^2 \lambda_4 + \frac{24}{5}g_1^2 \lambda_4^2 + 18g_2^2 \lambda_4^2 - 40\lambda_1 \lambda_4^2 - 40\lambda_2 \lambda_4^2 - 28\lambda_3 \lambda_4^2 \\
& + \frac{48}{5}g_1^2 \lambda_5^2 + 54g_2^2 \lambda_5^2 - 48\lambda_1 \lambda_5^2 - 48\lambda_2 \lambda_5^2 - 48\lambda_3 \lambda_5^2 - 26\lambda_4 \lambda_5^2 + \frac{42}{5}g_1^2 \lambda_6^2 + 54g_2^2 \lambda_6^2 \\
& - 148\lambda_1 \lambda_6^2 - 20\lambda_2 \lambda_6^2 - 72\lambda_3 \lambda_6^2 - 68\lambda_4 \lambda_6^2 - 80\lambda_5 \lambda_6^2 + \frac{24}{5}g_1^2 \lambda_6 \lambda_7 - 40\lambda_1 \lambda_6 \lambda_7 - 40\lambda_2 \lambda_6 \lambda_7 \\
& - 80\lambda_3 \lambda_6 \lambda_7 - 160\lambda_4 \lambda_6 \lambda_7 - 96\lambda_5 \lambda_6 \lambda_7 + \frac{42}{5}g_1^2 \lambda_7^2 + 54g_2^2 \lambda_7^2 - 20\lambda_1 \lambda_7^2 - 148\lambda_2 \lambda_7^2 \\
& - 72\lambda_3 \lambda_7^2 - 68\lambda_4 \lambda_7^2 - 80\lambda_5 \lambda_7^2 \\
& + \frac{1}{20}\left(5\left(16\left(10g_3^2 \lambda_4 - 3\left(2\lambda_2 \lambda_4 + 2\lambda_3 \lambda_4 + 2\lambda_5^2 + 5\lambda_7^2 + \lambda_6 \lambda_7 + \lambda_4^2\right)\right) + 45g_2^2 \lambda_4\right) + g_1^2\left(108g_2^2 + 25\lambda_4\right)\right)\text{Tr}\left(\epsilon_d \epsilon_d^\dagger\right) \\
& - 6\left(2\lambda_1 \lambda_7 + 2\lambda_2 \lambda_6 + 2\lambda_3\left(\lambda_6 + \lambda_7\right) + 4\lambda_5 \lambda_6 + 4\lambda_5 \lambda_7 + 5\lambda_4 \lambda_6 + 5\lambda_4 \lambda_7\right)\text{Tr}\left(\epsilon_d Y_d^\dagger\right) \\
& + \frac{33}{5}g_1^2 g_2^2 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) + \frac{15}{4}g_1^2 \lambda_4 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) + \frac{15}{4}g_2^2 \lambda_4 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) - 8\lambda_2 \lambda_4 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) \\
& - 8\lambda_3 \lambda_4 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) - 4\lambda_4^2 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) - 8\lambda_5^2 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) - 4\lambda_6 \lambda_7 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) \\
& - 20\lambda_7^2 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger\right) - 4\lambda_2 \lambda_6 \text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 4\lambda_3 \lambda_6 \text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 10\lambda_4 \lambda_6 \text{Tr}\left(\epsilon_e Y_e^\dagger\right) \\
& - 8\lambda_5 \lambda_6 \text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 4\lambda_1 \lambda_7 \text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 4\lambda_3 \lambda_7 \text{Tr}\left(\epsilon_e Y_e^\dagger\right) - 10\lambda_4 \lambda_7 \text{Tr}\left(\epsilon_e Y_e^\dagger\right) \\
& - 8\lambda_5 \lambda_7 \text{Tr}\left(\epsilon_e Y_e^\dagger\right) + \frac{63}{5}g_1^2 g_2^2 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) + \frac{17}{4}g_1^2 \lambda_4 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) + \frac{45}{4}g_2^2 \lambda_4 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) \\
& + 40g_3^2 \lambda_4 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) - 24\lambda_1 \lambda_4 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) - 24\lambda_3 \lambda_4 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) - 12\lambda_4^2 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) \\
& - 24\lambda_5^2 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) - 60\lambda_6^2 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) - 12\lambda_6 \lambda_7 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) - 12\lambda_2 \lambda_6 \text{Tr}\left(\epsilon_u Y_u^\dagger\right) \\
& - 12\lambda_3 \lambda_6 \text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 30\lambda_4 \lambda_6 \text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 24\lambda_5 \lambda_6 \text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 12\lambda_1 \lambda_7 \text{Tr}\left(\epsilon_u Y_u^\dagger\right) \\
& - 12\lambda_3 \lambda_7 \text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 30\lambda_4 \lambda_7 \text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 24\lambda_5 \lambda_7 \text{Tr}\left(\epsilon_u Y_u^\dagger\right) - 12\lambda_2 \lambda_6 \text{Tr}\left(Y_d \epsilon_d^\dagger\right) \\
& - 12\lambda_3 \lambda_6 \text{Tr}\left(Y_d \epsilon_d^\dagger\right) - 30\lambda_4 \lambda_6 \text{Tr}\left(Y_d \epsilon_d^\dagger\right) - 24\lambda_5 \lambda_6 \text{Tr}\left(Y_d \epsilon_d^\dagger\right) - 12\lambda_1 \lambda_7 \text{Tr}\left(Y_d \epsilon_d^\dagger\right) \\
& - 12\lambda_3 \lambda_7 \text{Tr}\left(Y_d \epsilon_d^\dagger\right) - 30\lambda_4 \lambda_7 \text{Tr}\left(Y_d \epsilon_d^\dagger\right) - 24\lambda_5 \lambda_7 \text{Tr}\left(Y_d \epsilon_d^\dagger\right) + \frac{27}{5}g_1^2 g_2^2 \text{Tr}\left(Y_d Y_d^\dagger\right) \\
& + \frac{5}{4}g_1^2 \lambda_4 \text{Tr}\left(Y_d Y_d^\dagger\right) + \frac{45}{4}g_2^2 \lambda_4 \text{Tr}\left(Y_d Y_d^\dagger\right) + 40g_3^2 \lambda_4 \text{Tr}\left(Y_d Y_d^\dagger\right) - 24\lambda_1 \lambda_4 \text{Tr}\left(Y_d Y_d^\dagger\right) \\
& - 24\lambda_3 \lambda_4 \text{Tr}\left(Y_d Y_d^\dagger\right) - 12\lambda_4^2 \text{Tr}\left(Y_d Y_d^\dagger\right) - 24\lambda_5^2 \text{Tr}\left(Y_d Y_d^\dagger\right) - 60\lambda_6^2 \text{Tr}\left(Y_d Y_d^\dagger\right) \\
& - 12\lambda_6 \lambda_7 \text{Tr}\left(Y_d Y_d^\dagger\right) - 4\lambda_2 \lambda_6 \text{Tr}\left(Y_e \epsilon_e^\dagger\right) - 4\lambda_3 \lambda_6 \text{Tr}\left(Y_e \epsilon_e^\dagger\right) - 10\lambda_4 \lambda_6 \text{Tr}\left(Y_e \epsilon_e^\dagger\right)
\end{aligned}$$

$$\begin{aligned}
& - 8\lambda_5\lambda_6\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - 4\lambda_1\lambda_7\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - 4\lambda_3\lambda_7\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - 10\lambda_4\lambda_7\text{Tr}\left(Y_e\epsilon_e^\dagger\right) \\
& - 8\lambda_5\lambda_7\text{Tr}\left(Y_e\epsilon_e^\dagger\right) + \frac{33}{5}g_1^2g_2^2\text{Tr}\left(Y_eY_e^\dagger\right) + \frac{15}{4}g_1^2\lambda_4\text{Tr}\left(Y_eY_e^\dagger\right) + \frac{15}{4}g_2^2\lambda_4\text{Tr}\left(Y_eY_e^\dagger\right) \\
& - 8\lambda_1\lambda_4\text{Tr}\left(Y_eY_e^\dagger\right) - 8\lambda_3\lambda_4\text{Tr}\left(Y_eY_e^\dagger\right) - 4\lambda_4^2\text{Tr}\left(Y_eY_e^\dagger\right) - 8\lambda_5^2\text{Tr}\left(Y_eY_e^\dagger\right) \\
& - 20\lambda_6^2\text{Tr}\left(Y_eY_e^\dagger\right) - 4\lambda_6\lambda_7\text{Tr}\left(Y_eY_e^\dagger\right) - 12\lambda_2\lambda_6\text{Tr}\left(Y_u\epsilon_u^\dagger\right) - 12\lambda_3\lambda_6\text{Tr}\left(Y_u\epsilon_u^\dagger\right) \\
& - 30\lambda_4\lambda_6\text{Tr}\left(Y_u\epsilon_u^\dagger\right) - 24\lambda_5\lambda_6\text{Tr}\left(Y_u\epsilon_u^\dagger\right) - 12\lambda_1\lambda_7\text{Tr}\left(Y_u\epsilon_u^\dagger\right) - 12\lambda_3\lambda_7\text{Tr}\left(Y_u\epsilon_u^\dagger\right) \\
& - 30\lambda_4\lambda_7\text{Tr}\left(Y_u\epsilon_u^\dagger\right) - 24\lambda_5\lambda_7\text{Tr}\left(Y_u\epsilon_u^\dagger\right) + \frac{63}{5}g_1^2g_2^2\text{Tr}\left(Y_uY_u^\dagger\right) + \frac{17}{4}g_1^2\lambda_4\text{Tr}\left(Y_uY_u^\dagger\right) \\
& + \frac{45}{4}g_2^2\lambda_4\text{Tr}\left(Y_uY_u^\dagger\right) + 40g_3^2\lambda_4\text{Tr}\left(Y_uY_u^\dagger\right) - 24\lambda_2\lambda_4\text{Tr}\left(Y_uY_u^\dagger\right) - 24\lambda_3\lambda_4\text{Tr}\left(Y_uY_u^\dagger\right) \\
& - 12\lambda_4^2\text{Tr}\left(Y_uY_u^\dagger\right) - 24\lambda_5^2\text{Tr}\left(Y_uY_u^\dagger\right) - 12\lambda_6\lambda_7\text{Tr}\left(Y_uY_u^\dagger\right) - 60\lambda_7^2\text{Tr}\left(Y_uY_u^\dagger\right) \\
& - \frac{27}{2}\lambda_4\text{Tr}\left(\epsilon_d\epsilon_d^\dagger\epsilon_d\epsilon_d^\dagger\right) + 6\lambda_7\text{Tr}\left(\epsilon_d\epsilon_d^\dagger\epsilon_dY_d^\dagger\right) + 12\lambda_7\text{Tr}\left(\epsilon_d\epsilon_d^\dagger Y_d\epsilon_d^\dagger\right) + 6\lambda_4\text{Tr}\left(\epsilon_d\epsilon_d^\dagger Y_dY_d^\dagger\right) \\
& + \frac{4}{5}g_1^2\text{Tr}\left(\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger\right) + 64g_3^2\text{Tr}\left(\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger\right) - 24\lambda_3\text{Tr}\left(\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger\right) \\
& - 33\lambda_4\text{Tr}\left(\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger\right) - 24\lambda_6\text{Tr}\left(\epsilon_d\epsilon_u^\dagger\epsilon_uY_d^\dagger\right) + 12\lambda_7\text{Tr}\left(\epsilon_d\epsilon_u^\dagger\epsilon_uY_d^\dagger\right) \\
& + 12\lambda_6\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_u\epsilon_d^\dagger\right) - 24\lambda_7\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_u\epsilon_d^\dagger\right) - \frac{4}{5}g_1^2\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_uY_d^\dagger\right) \\
& - 64g_3^2\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_uY_d^\dagger\right) + 24\lambda_1\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_uY_d^\dagger\right) + 24\lambda_2\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_uY_d^\dagger\right) \\
& + 12\lambda_4\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_uY_d^\dagger\right) + 6\lambda_7\text{Tr}\left(\epsilon_dY_d^\dagger\epsilon_d\epsilon_d^\dagger\right) + \frac{8}{5}g_1^2\text{Tr}\left(\epsilon_dY_d^\dagger Y_d\epsilon_d^\dagger\right) \\
& - 64g_3^2\text{Tr}\left(\epsilon_dY_d^\dagger Y_d\epsilon_d^\dagger\right) + 24\lambda_3\text{Tr}\left(\epsilon_dY_d^\dagger Y_d\epsilon_d^\dagger\right) - 9\lambda_4\text{Tr}\left(\epsilon_dY_d^\dagger Y_d\epsilon_d^\dagger\right) \\
& + 12\lambda_6\text{Tr}\left(\epsilon_dY_d^\dagger Y_dY_d^\dagger\right) + 12\lambda_6\text{Tr}\left(\epsilon_dY_u^\dagger\epsilon_u\epsilon_d^\dagger\right) - 24\lambda_7\text{Tr}\left(\epsilon_dY_u^\dagger\epsilon_u\epsilon_d^\dagger\right) \\
& - 48\lambda_5\text{Tr}\left(\epsilon_dY_u^\dagger\epsilon_uY_d^\dagger\right) + 27\lambda_4\text{Tr}\left(\epsilon_dY_u^\dagger Y_u\epsilon_d^\dagger\right) + 12\lambda_6\text{Tr}\left(\epsilon_dY_u^\dagger Y_uY_d^\dagger\right) \\
& - 24\lambda_7\text{Tr}\left(\epsilon_dY_u^\dagger Y_uY_d^\dagger\right) - \frac{9}{2}\lambda_4\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\epsilon_e\epsilon_e^\dagger\right) + 2\lambda_7\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\epsilon_eY_e^\dagger\right) + 4\lambda_7\text{Tr}\left(\epsilon_e\epsilon_e^\dagger Y_e\epsilon_e^\dagger\right) \\
& + 2\lambda_4\text{Tr}\left(\epsilon_e\epsilon_e^\dagger Y_eY_e^\dagger\right) + 2\lambda_7\text{Tr}\left(\epsilon_eY_e^\dagger\epsilon_e\epsilon_e^\dagger\right) - \frac{24}{5}g_1^2\text{Tr}\left(\epsilon_eY_e^\dagger Y_e\epsilon_e^\dagger\right) \\
& + 8\lambda_3\text{Tr}\left(\epsilon_eY_e^\dagger Y_e\epsilon_e^\dagger\right) - 3\lambda_4\text{Tr}\left(\epsilon_eY_e^\dagger Y_e\epsilon_e^\dagger\right) + 4\lambda_6\text{Tr}\left(\epsilon_eY_e^\dagger Y_eY_e^\dagger\right) \\
& - 24\lambda_6\text{Tr}\left(\epsilon_u\epsilon_d^\dagger Y_d\epsilon_u^\dagger\right) + 12\lambda_7\text{Tr}\left(\epsilon_u\epsilon_d^\dagger Y_d\epsilon_u^\dagger\right) - \frac{4}{5}g_1^2\text{Tr}\left(\epsilon_u\epsilon_d^\dagger Y_dY_u^\dagger\right) \\
& - 64g_3^2\text{Tr}\left(\epsilon_u\epsilon_d^\dagger Y_dY_u^\dagger\right) + 24\lambda_1\text{Tr}\left(\epsilon_u\epsilon_d^\dagger Y_dY_u^\dagger\right) + 24\lambda_2\text{Tr}\left(\epsilon_u\epsilon_d^\dagger Y_dY_u^\dagger\right) \\
& + 12\lambda_4\text{Tr}\left(\epsilon_u\epsilon_d^\dagger Y_dY_u^\dagger\right) - \frac{27}{2}\lambda_4\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\epsilon_u\epsilon_u^\dagger\right) + 6\lambda_6\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\epsilon_uY_u^\dagger\right) + 12\lambda_6\text{Tr}\left(\epsilon_u\epsilon_u^\dagger Y_u\epsilon_u^\dagger\right) \\
& + 6\lambda_4\text{Tr}\left(\epsilon_u\epsilon_u^\dagger Y_uY_u^\dagger\right) + 27\lambda_4\text{Tr}\left(\epsilon_uY_d^\dagger Y_d\epsilon_u^\dagger\right) - 24\lambda_6\text{Tr}\left(\epsilon_uY_d^\dagger Y_dY_u^\dagger\right)
\end{aligned}$$

$$\begin{aligned}
& + 12\lambda_7 \text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger) + 6\lambda_6 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger) - \frac{16}{5} g_1^2 \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) \\
& - 64g_3^2 \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) + 24\lambda_3 \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) - 9\lambda_4 \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) \\
& + 12\lambda_7 \text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger) + 6\lambda_6 \text{Tr}(Y_d \epsilon_d^\dagger Y_d Y_d^\dagger) - 48\lambda_5 \text{Tr}(Y_d \epsilon_u^\dagger Y_u \epsilon_d^\dagger) \\
& - 24\lambda_6 \text{Tr}(Y_d \epsilon_u^\dagger Y_u Y_d^\dagger) + 12\lambda_7 \text{Tr}(Y_d \epsilon_u^\dagger Y_u Y_d^\dagger) + 6\lambda_6 \text{Tr}(Y_d Y_d^\dagger Y_d \epsilon_d^\dagger) \\
& - \frac{27}{2} \lambda_4 \text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) + 12\lambda_6 \text{Tr}(Y_d Y_u^\dagger Y_u \epsilon_d^\dagger) - 24\lambda_7 \text{Tr}(Y_d Y_u^\dagger Y_u \epsilon_d^\dagger) \\
& + \frac{4}{5} g_1^2 \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) + 64g_3^2 \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) - 24\lambda_3 \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) \\
& - 33\lambda_4 \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) + 2\lambda_6 \text{Tr}(Y_e \epsilon_e^\dagger Y_e Y_e^\dagger) + 2\lambda_6 \text{Tr}(Y_e Y_e^\dagger Y_e \epsilon_e^\dagger) - \frac{9}{2} \lambda_4 \text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) \\
& + 6\lambda_7 \text{Tr}(Y_u \epsilon_u^\dagger Y_u Y_u^\dagger) + 6\lambda_7 \text{Tr}(Y_u Y_u^\dagger Y_u \epsilon_u^\dagger) - \frac{27}{2} \lambda_4 \text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger) - 12 \text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) \\
& + 18 \text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) + 12 \text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger) + 6 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger \epsilon_d Y_d^\dagger) \\
& + 30 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) - 36 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_u^\dagger Y_u Y_d^\dagger) - 12 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger) \\
& - 36 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger Y_d Y_d^\dagger) - 24 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger \epsilon_u \epsilon_d^\dagger) + 30 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger Y_u Y_d^\dagger) \\
& - 6 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u \epsilon_d^\dagger \epsilon_d Y_d^\dagger) - 6 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u \epsilon_u^\dagger \epsilon_u Y_d^\dagger) + 12 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger \epsilon_d \epsilon_d^\dagger) \\
& + 30 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger Y_d Y_d^\dagger) - 36 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_u^\dagger \epsilon_u \epsilon_d^\dagger) + 30 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_u^\dagger Y_u Y_d^\dagger) \\
& + 30 \text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger) + 12 \text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger) + 36 \text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger Y_d Y_d^\dagger) \\
& - 6 \text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger Y_u Y_d^\dagger) + 24 \text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger Y_d \epsilon_d^\dagger) - 6 \text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger Y_d \epsilon_d^\dagger) \\
& - 6 \text{Tr}(\epsilon_d Y_u^\dagger Y_u \epsilon_u^\dagger Y_u Y_d^\dagger) + 4 \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger) + 2 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger) \\
& + 10 \text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) + 4 \text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger) + 12 \text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger Y_e Y_e^\dagger) \\
& + 8 \text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger Y_e \epsilon_e^\dagger) - 6 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d \epsilon_u^\dagger \epsilon_u Y_u^\dagger) + 30 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_d^\dagger Y_d Y_u^\dagger) \\
& + 12 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger \epsilon_u \epsilon_u^\dagger) + 30 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger Y_u Y_u^\dagger) + 18 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) \\
& + 12 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) + 6 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger) - 36 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u Y_d^\dagger Y_d Y_u^\dagger) \\
& - 6 \text{Tr}(\epsilon_u Y_d^\dagger Y_d \epsilon_d^\dagger Y_d Y_u^\dagger) + 30 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) - 6 \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_d^\dagger Y_d Y_u^\dagger) \\
& + 12 \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger) + 36 \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger Y_u Y_u^\dagger) + 24 \text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger Y_u \epsilon_u^\dagger) \\
& - 12 \text{Tr}(Y_d Y_d^\dagger Y_d Y_u^\dagger Y_u Y_d^\dagger) - 12 \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger Y_d Y_d^\dagger) - 24 \text{Tr}(Y_d Y_u^\dagger Y_u Y_u^\dagger Y_u Y_d^\dagger)
\end{aligned} \tag{19}$$

$$\beta_{\lambda_3}^{(1)} = + \frac{27}{100} g_1^4 - \frac{9}{10} g_1^2 g_2^2 + \frac{9}{4} g_2^4 - \frac{9}{5} g_1^2 \lambda_3 - 9 g_2^2 \lambda_3 + 12 \lambda_1 \lambda_3 + 12 \lambda_2 \lambda_3 + 4 \lambda_3^2 + 4 \lambda_1 \lambda_4 + 4 \lambda_2 \lambda_4 + 2 \lambda_4^2$$

$$\begin{aligned}
& + 2\lambda_5^2 + 4\lambda_6^2 + 16\lambda_6\lambda_7 + 4\lambda_7^2 + 6\lambda_3\text{Tr}(\epsilon_d\epsilon_d^\dagger) + 6\lambda_7\text{Tr}(\epsilon_d Y_d^\dagger) + 2\lambda_3\text{Tr}(\epsilon_e\epsilon_e^\dagger) + 2\lambda_7\text{Tr}(\epsilon_e Y_e^\dagger) \\
& + 6\lambda_3\text{Tr}(\epsilon_u\epsilon_u^\dagger) + 6\lambda_6\text{Tr}(\epsilon_u Y_u^\dagger) + 6\lambda_6\text{Tr}(Y_d\epsilon_d^\dagger) + 6\lambda_3\text{Tr}(Y_d Y_d^\dagger) + 2\lambda_6\text{Tr}(Y_e\epsilon_e^\dagger) \\
& + 2\lambda_3\text{Tr}(Y_e Y_e^\dagger) + 6\lambda_7\text{Tr}(Y_u\epsilon_u^\dagger) + 6\lambda_3\text{Tr}(Y_u Y_u^\dagger) - 12\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d Y_d^\dagger) \\
& - 12\text{Tr}(\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger) + 12\text{Tr}(\epsilon_d\epsilon_u^\dagger Y_u Y_d^\dagger) - 4\text{Tr}(\epsilon_e\epsilon_e^\dagger Y_e Y_e^\dagger) + 12\text{Tr}(\epsilon_u\epsilon_d^\dagger Y_d Y_u^\dagger) \\
& - 12\text{Tr}(\epsilon_u\epsilon_u^\dagger Y_u Y_u^\dagger) - 12\text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) \tag{20} \\
\beta_{\lambda_3}^{(2)} = & -\frac{3537}{1000}g_1^6 + \frac{909}{200}g_1^4 g_2^2 + \frac{33}{40}g_1^2 g_2^4 + \frac{291}{8}g_2^6 + \frac{27}{10}g_1^4 \lambda_1 - 3g_1^2 g_2^2 \lambda_1 + \frac{45}{2}g_2^4 \lambda_1 + \frac{27}{10}g_1^4 \lambda_2 \\
& - 3g_1^2 g_2^2 \lambda_2 + \frac{45}{2}g_2^4 \lambda_2 + \frac{1773}{200}g_1^4 \lambda_3 + \frac{33}{20}g_1^2 g_2^2 \lambda_3 - \frac{111}{8}g_2^4 \lambda_3 + \frac{72}{5}g_1^2 \lambda_1 \lambda_3 + 72g_2^2 \lambda_1 \lambda_3 \\
& - 60\lambda_1^2 \lambda_3 + \frac{72}{5}g_1^2 \lambda_2 \lambda_3 + 72g_2^2 \lambda_2 \lambda_3 - 60\lambda_2^2 \lambda_3 + \frac{6}{5}g_1^2 \lambda_3^2 + 6g_2^2 \lambda_3^2 - 72\lambda_1 \lambda_3^2 - 72\lambda_2 \lambda_3^2 \\
& - 12\lambda_3^3 + \frac{9}{10}g_1^4 \lambda_4 - \frac{9}{5}g_1^2 g_2^2 \lambda_4 + \frac{15}{2}g_2^4 \lambda_4 + \frac{24}{5}g_1^2 \lambda_1 \lambda_4 + 36g_2^2 \lambda_1 \lambda_4 - 16\lambda_1^2 \lambda_4 + \frac{24}{5}g_1^2 \lambda_2 \lambda_4 \\
& + 36g_2^2 \lambda_2 \lambda_4 - 16\lambda_2^2 \lambda_4 - 12g_2^2 \lambda_3 \lambda_4 - 32\lambda_1 \lambda_3 \lambda_4 - 32\lambda_2 \lambda_3 \lambda_4 - 4\lambda_3^2 \lambda_4 - \frac{6}{5}g_1^2 \lambda_4^2 \\
& + 6g_2^2 \lambda_4^2 - 28\lambda_1 \lambda_4^2 - 28\lambda_2 \lambda_4^2 - 16\lambda_3 \lambda_4^2 - 12\lambda_4^3 + \frac{12}{5}g_1^2 \lambda_5^2 - 36\lambda_1 \lambda_5^2 - 36\lambda_2 \lambda_5^2 \\
& - 18\lambda_3 \lambda_5^2 - 44\lambda_4 \lambda_5^2 + \frac{6}{5}g_1^2 \lambda_6^2 - 124\lambda_1 \lambda_6^2 - 44\lambda_2 \lambda_6^2 - 60\lambda_3 \lambda_6^2 - 68\lambda_4 \lambda_6^2 - 68\lambda_5 \lambda_6^2 \\
& + \frac{96}{5}g_1^2 \lambda_6 \lambda_7 + 108g_2^2 \lambda_6 \lambda_7 - 88\lambda_1 \lambda_6 \lambda_7 - 88\lambda_2 \lambda_6 \lambda_7 - 176\lambda_3 \lambda_6 \lambda_7 - 88\lambda_4 \lambda_6 \lambda_7 - 72\lambda_5 \lambda_6 \lambda_7 \\
& + \frac{6}{5}g_1^2 \lambda_7^2 - 44\lambda_1 \lambda_7^2 - 124\lambda_2 \lambda_7^2 - 60\lambda_3 \lambda_7^2 - 68\lambda_4 \lambda_7^2 - 68\lambda_5 \lambda_7^2 \\
& + \frac{1}{20}(9g_1^4 + g_1^2(25\lambda_3 - 54g_2^2) \\
& - 5(-45g_2^2 \lambda_3 + 8(-20g_3^2 \lambda_3 + 3(2\lambda_3^2 + 4\lambda_2(3\lambda_3 + \lambda_4) + 4\lambda_7^2 + 8\lambda_6 \lambda_7 + \lambda_4^2 + \lambda_5^2)) + 9g_2^4))\text{Tr}(\epsilon_d\epsilon_d^\dagger) \\
& - 6(2\lambda_4 \lambda_6 + 2\lambda_4 \lambda_7 + 5\lambda_3(\lambda_6 + \lambda_7) + 8\lambda_1 \lambda_7 + 8\lambda_2 \lambda_6 + \lambda_5 \lambda_6 + \lambda_5 \lambda_7)\text{Tr}(\epsilon_d Y_d^\dagger) - \frac{9}{4}g_1^4 \text{Tr}(\epsilon_e\epsilon_e^\dagger) \\
& - \frac{33}{10}g_1^2 g_2^2 \text{Tr}(\epsilon_e\epsilon_e^\dagger) - \frac{3}{4}g_2^4 \text{Tr}(\epsilon_e\epsilon_e^\dagger) + \frac{15}{4}g_1^2 \lambda_3 \text{Tr}(\epsilon_e\epsilon_e^\dagger) + \frac{15}{4}g_2^2 \lambda_3 \text{Tr}(\epsilon_e\epsilon_e^\dagger) \\
& - 24\lambda_2 \lambda_3 \text{Tr}(\epsilon_e\epsilon_e^\dagger) - 4\lambda_3^2 \text{Tr}(\epsilon_e\epsilon_e^\dagger) - 8\lambda_2 \lambda_4 \text{Tr}(\epsilon_e\epsilon_e^\dagger) - 2\lambda_4^2 \text{Tr}(\epsilon_e\epsilon_e^\dagger) \\
& - 2\lambda_5^2 \text{Tr}(\epsilon_e\epsilon_e^\dagger) - 16\lambda_6 \lambda_7 \text{Tr}(\epsilon_e\epsilon_e^\dagger) - 8\lambda_7^2 \text{Tr}(\epsilon_e\epsilon_e^\dagger) - 16\lambda_2 \lambda_6 \text{Tr}(\epsilon_e Y_e^\dagger) \\
& - 10\lambda_3 \lambda_6 \text{Tr}(\epsilon_e Y_e^\dagger) - 4\lambda_4 \lambda_6 \text{Tr}(\epsilon_e Y_e^\dagger) - 2\lambda_5 \lambda_6 \text{Tr}(\epsilon_e Y_e^\dagger) - 16\lambda_1 \lambda_7 \text{Tr}(\epsilon_e Y_e^\dagger) \\
& - 10\lambda_3 \lambda_7 \text{Tr}(\epsilon_e Y_e^\dagger) - 4\lambda_4 \lambda_7 \text{Tr}(\epsilon_e Y_e^\dagger) - 2\lambda_5 \lambda_7 \text{Tr}(\epsilon_e Y_e^\dagger) - \frac{171}{100}g_1^4 \text{Tr}(\epsilon_u\epsilon_u^\dagger) \\
& - \frac{63}{10}g_1^2 g_2^2 \text{Tr}(\epsilon_u\epsilon_u^\dagger) - \frac{9}{4}g_2^4 \text{Tr}(\epsilon_u\epsilon_u^\dagger) + \frac{17}{4}g_1^2 \lambda_3 \text{Tr}(\epsilon_u\epsilon_u^\dagger) + \frac{45}{4}g_2^2 \lambda_3 \text{Tr}(\epsilon_u\epsilon_u^\dagger) \\
& + 40g_3^2 \lambda_3 \text{Tr}(\epsilon_u\epsilon_u^\dagger) - 72\lambda_1 \lambda_3 \text{Tr}(\epsilon_u\epsilon_u^\dagger) - 12\lambda_3^2 \text{Tr}(\epsilon_u\epsilon_u^\dagger) - 24\lambda_1 \lambda_4 \text{Tr}(\epsilon_u\epsilon_u^\dagger)
\end{aligned}$$

$$\begin{aligned}
& -6\lambda_4^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger) - 6\lambda_5^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger) - 24\lambda_6^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger) - 48\lambda_6 \lambda_7 \text{Tr}(\epsilon_u \epsilon_u^\dagger) \\
& -48\lambda_2 \lambda_6 \text{Tr}(\epsilon_u Y_u^\dagger) - 30\lambda_3 \lambda_6 \text{Tr}(\epsilon_u Y_u^\dagger) - 12\lambda_4 \lambda_6 \text{Tr}(\epsilon_u Y_u^\dagger) - 6\lambda_5 \lambda_6 \text{Tr}(\epsilon_u Y_u^\dagger) \\
& -48\lambda_1 \lambda_7 \text{Tr}(\epsilon_u Y_u^\dagger) - 30\lambda_3 \lambda_7 \text{Tr}(\epsilon_u Y_u^\dagger) - 12\lambda_4 \lambda_7 \text{Tr}(\epsilon_u Y_u^\dagger) - 6\lambda_5 \lambda_7 \text{Tr}(\epsilon_u Y_u^\dagger) \\
& -48\lambda_2 \lambda_6 \text{Tr}(Y_d \epsilon_d^\dagger) - 30\lambda_3 \lambda_6 \text{Tr}(Y_d \epsilon_d^\dagger) - 12\lambda_4 \lambda_6 \text{Tr}(Y_d \epsilon_d^\dagger) - 6\lambda_5 \lambda_6 \text{Tr}(Y_d \epsilon_d^\dagger) \\
& -48\lambda_1 \lambda_7 \text{Tr}(Y_d \epsilon_d^\dagger) - 30\lambda_3 \lambda_7 \text{Tr}(Y_d \epsilon_d^\dagger) - 12\lambda_4 \lambda_7 \text{Tr}(Y_d \epsilon_d^\dagger) - 6\lambda_5 \lambda_7 \text{Tr}(Y_d \epsilon_d^\dagger) \\
& + \frac{9}{20} g_1^4 \text{Tr}(Y_d Y_d^\dagger) - \frac{27}{10} g_1^2 g_2^2 \text{Tr}(Y_d Y_d^\dagger) - \frac{9}{4} g_2^4 \text{Tr}(Y_d Y_d^\dagger) + \frac{5}{4} g_1^2 \lambda_3 \text{Tr}(Y_d Y_d^\dagger) \\
& + \frac{45}{4} g_2^2 \lambda_3 \text{Tr}(Y_d Y_d^\dagger) + 40g_3^2 \lambda_3 \text{Tr}(Y_d Y_d^\dagger) - 72\lambda_1 \lambda_3 \text{Tr}(Y_d Y_d^\dagger) - 12\lambda_3^2 \text{Tr}(Y_d Y_d^\dagger) \\
& -24\lambda_1 \lambda_4 \text{Tr}(Y_d Y_d^\dagger) - 6\lambda_4^2 \text{Tr}(Y_d Y_d^\dagger) - 6\lambda_5^2 \text{Tr}(Y_d Y_d^\dagger) - 24\lambda_6^2 \text{Tr}(Y_d Y_d^\dagger) \\
& -48\lambda_6 \lambda_7 \text{Tr}(Y_d Y_d^\dagger) - 16\lambda_2 \lambda_6 \text{Tr}(Y_e \epsilon_e^\dagger) - 10\lambda_3 \lambda_6 \text{Tr}(Y_e \epsilon_e^\dagger) - 4\lambda_4 \lambda_6 \text{Tr}(Y_e \epsilon_e^\dagger) \\
& -2\lambda_5 \lambda_6 \text{Tr}(Y_e \epsilon_e^\dagger) - 16\lambda_1 \lambda_7 \text{Tr}(Y_e \epsilon_e^\dagger) - 10\lambda_3 \lambda_7 \text{Tr}(Y_e \epsilon_e^\dagger) - 4\lambda_4 \lambda_7 \text{Tr}(Y_e \epsilon_e^\dagger) \\
& -2\lambda_5 \lambda_7 \text{Tr}(Y_e \epsilon_e^\dagger) - \frac{9}{4} g_1^4 \text{Tr}(Y_e Y_e^\dagger) - \frac{33}{10} g_1^2 g_2^2 \text{Tr}(Y_e Y_e^\dagger) - \frac{3}{4} g_2^4 \text{Tr}(Y_e Y_e^\dagger) \\
& + \frac{15}{4} g_1^2 \lambda_3 \text{Tr}(Y_e Y_e^\dagger) + \frac{15}{4} g_2^2 \lambda_3 \text{Tr}(Y_e Y_e^\dagger) - 24\lambda_1 \lambda_3 \text{Tr}(Y_e Y_e^\dagger) - 4\lambda_3^2 \text{Tr}(Y_e Y_e^\dagger) \\
& -8\lambda_1 \lambda_4 \text{Tr}(Y_e Y_e^\dagger) - 2\lambda_4^2 \text{Tr}(Y_e Y_e^\dagger) - 2\lambda_5^2 \text{Tr}(Y_e Y_e^\dagger) - 8\lambda_6^2 \text{Tr}(Y_e Y_e^\dagger) \\
& -16\lambda_6 \lambda_7 \text{Tr}(Y_e Y_e^\dagger) - 48\lambda_2 \lambda_6 \text{Tr}(Y_u \epsilon_u^\dagger) - 30\lambda_3 \lambda_6 \text{Tr}(Y_u \epsilon_u^\dagger) - 12\lambda_4 \lambda_6 \text{Tr}(Y_u \epsilon_u^\dagger) \\
& -6\lambda_5 \lambda_6 \text{Tr}(Y_u \epsilon_u^\dagger) - 48\lambda_1 \lambda_7 \text{Tr}(Y_u \epsilon_u^\dagger) - 30\lambda_3 \lambda_7 \text{Tr}(Y_u \epsilon_u^\dagger) - 12\lambda_4 \lambda_7 \text{Tr}(Y_u \epsilon_u^\dagger) \\
& -6\lambda_5 \lambda_7 \text{Tr}(Y_u \epsilon_u^\dagger) - \frac{171}{100} g_1^4 \text{Tr}(Y_u Y_u^\dagger) - \frac{63}{10} g_1^2 g_2^2 \text{Tr}(Y_u Y_u^\dagger) - \frac{9}{4} g_2^4 \text{Tr}(Y_u Y_u^\dagger) \\
& + \frac{17}{4} g_1^2 \lambda_3 \text{Tr}(Y_u Y_u^\dagger) + \frac{45}{4} g_2^2 \lambda_3 \text{Tr}(Y_u Y_u^\dagger) + 40g_3^2 \lambda_3 \text{Tr}(Y_u Y_u^\dagger) - 72\lambda_2 \lambda_3 \text{Tr}(Y_u Y_u^\dagger) \\
& -12\lambda_3^2 \text{Tr}(Y_u Y_u^\dagger) - 24\lambda_2 \lambda_4 \text{Tr}(Y_u Y_u^\dagger) - 6\lambda_4^2 \text{Tr}(Y_u Y_u^\dagger) - 6\lambda_5^2 \text{Tr}(Y_u Y_u^\dagger) \\
& -48\lambda_6 \lambda_7 \text{Tr}(Y_u Y_u^\dagger) - 24\lambda_7^2 \text{Tr}(Y_u Y_u^\dagger) - \frac{27}{2} \lambda_3 \text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger) + 6\lambda_7 \text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d Y_d^\dagger) \\
& + 12\lambda_7 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger) + \frac{8}{5} g_1^2 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger) - 64g_3^2 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger) \\
& + 6\lambda_3 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger) - \frac{4}{5} g_1^2 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) - 64g_3^2 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) \\
& + 15\lambda_3 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) + 12\lambda_6 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_d^\dagger) - 24\lambda_7 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_d^\dagger) \\
& -24\lambda_6 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u \epsilon_d^\dagger) + 12\lambda_7 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u \epsilon_d^\dagger) + \frac{4}{5} g_1^2 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) \\
& + 64g_3^2 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) - 48\lambda_1 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) - 48\lambda_2 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger)
\end{aligned}$$

$$\begin{aligned}
& + 12\lambda_3 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) + 6\lambda_7 \text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger) - 9\lambda_3 \text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger) \\
& + 24\lambda_4 \text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger) + 12\lambda_6 \text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger) - 24\lambda_6 \text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) \\
& + 12\lambda_7 \text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) + 24\lambda_5 \text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u Y_d^\dagger) - 21\lambda_3 \text{Tr}(\epsilon_d Y_u^\dagger Y_u \epsilon_d^\dagger) \\
& - 24\lambda_4 \text{Tr}(\epsilon_d Y_u^\dagger Y_u \epsilon_d^\dagger) - 24\lambda_6 \text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger) + 12\lambda_7 \text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger) \\
& - \frac{9}{2} \lambda_3 \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger) + 2\lambda_7 \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger) + 4\lambda_7 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) - \frac{24}{5} g_1^2 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger) \\
& + 2\lambda_3 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger) + 2\lambda_7 \text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger) - 3\lambda_3 \text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger) \\
& + 8\lambda_4 \text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger) + 4\lambda_6 \text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger) + 12\lambda_6 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d \epsilon_u^\dagger) \\
& - 24\lambda_7 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d \epsilon_u^\dagger) + \frac{4}{5} g_1^2 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) + 64g_3^2 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) \\
& - 48\lambda_1 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) - 48\lambda_2 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) + 12\lambda_3 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) \\
& - \frac{27}{2} \lambda_3 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger) + 6\lambda_6 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger) + 12\lambda_6 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) - \frac{16}{5} g_1^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger) \\
& - 64g_3^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger) + 6\lambda_3 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger) - 21\lambda_3 \text{Tr}(\epsilon_u Y_d^\dagger Y_d \epsilon_u^\dagger) \\
& - 24\lambda_4 \text{Tr}(\epsilon_u Y_d^\dagger Y_d \epsilon_u^\dagger) + 12\lambda_6 \text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger) - 24\lambda_7 \text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger) \\
& + 6\lambda_6 \text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger) - 9\lambda_3 \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) + 24\lambda_4 \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) \\
& + 12\lambda_7 \text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger) + 6\lambda_6 \text{Tr}(Y_d \epsilon_d^\dagger Y_d Y_d^\dagger) + 24\lambda_5 \text{Tr}(Y_d \epsilon_u^\dagger Y_u \epsilon_d^\dagger) \\
& + 12\lambda_6 \text{Tr}(Y_d \epsilon_u^\dagger Y_u Y_d^\dagger) - 24\lambda_7 \text{Tr}(Y_d \epsilon_u^\dagger Y_u Y_d^\dagger) + 6\lambda_6 \text{Tr}(Y_d Y_d^\dagger Y_d \epsilon_d^\dagger) \\
& - \frac{27}{2} \lambda_3 \text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) - 24\lambda_6 \text{Tr}(Y_d Y_u^\dagger Y_u \epsilon_d^\dagger) + 12\lambda_7 \text{Tr}(Y_d Y_u^\dagger Y_u \epsilon_d^\dagger) \\
& - \frac{4}{5} g_1^2 \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) - 64g_3^2 \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) + 15\lambda_3 \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) \\
& + 2\lambda_6 \text{Tr}(Y_e \epsilon_e^\dagger Y_e Y_e^\dagger) + 2\lambda_6 \text{Tr}(Y_e Y_e^\dagger Y_e \epsilon_e^\dagger) - \frac{9}{2} \lambda_3 \text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) + 6\lambda_7 \text{Tr}(Y_u \epsilon_u^\dagger Y_u Y_u^\dagger) \\
& + 6\lambda_7 \text{Tr}(Y_u Y_u^\dagger Y_u \epsilon_u^\dagger) - \frac{27}{2} \lambda_3 \text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger) + 30 \text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger) \\
& + 18 \text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) - 18 \text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) + 6 \text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger) \\
& - 6 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d \epsilon_u^\dagger \epsilon_u Y_d^\dagger) + 12 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger) + 42 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger Y_d Y_d^\dagger) \\
& - 42 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) + 42 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_u^\dagger Y_u Y_d^\dagger) + 18 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger) \\
& + 42 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger Y_d Y_d^\dagger) + 36 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) - 42 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger Y_u Y_d^\dagger) \\
& - 6 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_d^\dagger Y_d \epsilon_d^\dagger) - 6 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_u^\dagger Y_u \epsilon_d^\dagger) + 6 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u \epsilon_u^\dagger \epsilon_u Y_d^\dagger)
\end{aligned}$$

$$\begin{aligned}
& -24\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_u Y_d^\dagger \epsilon_d \epsilon_d^\dagger\right) - 42\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_u Y_d^\dagger Y_d Y_d^\dagger\right) + 42\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_u Y_u^\dagger \epsilon_u \epsilon_d^\dagger\right) \\
& - 42\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_u Y_u^\dagger Y_u Y_d^\dagger\right) + 6\text{Tr}\left(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger\right) + 6\text{Tr}\left(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_u^\dagger Y_u \epsilon_d^\dagger\right) \\
& + 6\text{Tr}\left(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger\right) + 6\text{Tr}\left(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger Y_d Y_d^\dagger\right) - 6\text{Tr}\left(\epsilon_d Y_d^\dagger Y_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger\right) \\
& + 6\text{Tr}\left(\epsilon_d Y_d^\dagger Y_d \epsilon_u^\dagger Y_u Y_d^\dagger\right) + 12\text{Tr}\left(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger Y_d \epsilon_d^\dagger\right) - 6\text{Tr}\left(\epsilon_d Y_d^\dagger Y_d Y_u^\dagger Y_u \epsilon_d^\dagger\right) \\
& + 6\text{Tr}\left(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger Y_d \epsilon_d^\dagger\right) - 6\text{Tr}\left(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_u^\dagger Y_u \epsilon_d^\dagger\right) - 6\text{Tr}\left(\epsilon_d Y_u^\dagger Y_u \epsilon_d^\dagger Y_d Y_d^\dagger\right) \\
& - 6\text{Tr}\left(\epsilon_d Y_u^\dagger Y_u \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger\right) + 6\text{Tr}\left(\epsilon_d Y_u^\dagger Y_u \epsilon_u^\dagger Y_u Y_d^\dagger\right) - 6\text{Tr}\left(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger Y_d \epsilon_d^\dagger\right) \\
& + 10\text{Tr}\left(\epsilon_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger\right) + 2\text{Tr}\left(\epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger\right) + 4\text{Tr}\left(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger\right) \\
& + 14\text{Tr}\left(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger Y_e Y_e^\dagger\right) + 2\text{Tr}\left(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger\right) + 2\text{Tr}\left(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger\right) \\
& + 2\text{Tr}\left(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger Y_e Y_e^\dagger\right) + 4\text{Tr}\left(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger Y_e \epsilon_e^\dagger\right) - 42\text{Tr}\left(\epsilon_u \epsilon_d^\dagger Y_d Y_d^\dagger Y_d Y_u^\dagger\right) \\
& - 24\text{Tr}\left(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger \epsilon_u \epsilon_u^\dagger\right) - 42\text{Tr}\left(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger Y_u Y_u^\dagger\right) - 18\text{Tr}\left(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger\right) \\
& + 30\text{Tr}\left(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger\right) + 6\text{Tr}\left(\epsilon_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger\right) + 42\text{Tr}\left(\epsilon_u \epsilon_u^\dagger Y_u Y_d^\dagger Y_d Y_u^\dagger\right) \\
& + 12\text{Tr}\left(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger\right) + 42\text{Tr}\left(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger Y_u Y_u^\dagger\right) + 6\text{Tr}\left(\epsilon_u Y_d^\dagger Y_d \epsilon_d^\dagger Y_d Y_u^\dagger\right) \\
& - 6\text{Tr}\left(\epsilon_u Y_d^\dagger Y_d \epsilon_u^\dagger Y_u Y_u^\dagger\right) - 6\text{Tr}\left(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger Y_u \epsilon_u^\dagger\right) + 6\text{Tr}\left(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_d^\dagger Y_d \epsilon_u^\dagger\right) \\
& + 6\text{Tr}\left(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger\right) + 6\text{Tr}\left(\epsilon_u Y_u^\dagger Y_u \epsilon_d^\dagger Y_d Y_u^\dagger\right) + 6\text{Tr}\left(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger\right) \\
& + 6\text{Tr}\left(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger Y_u Y_u^\dagger\right) - 6\text{Tr}\left(\epsilon_u Y_u^\dagger Y_u Y_d^\dagger Y_d \epsilon_u^\dagger\right) + 12\text{Tr}\left(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger Y_u \epsilon_u^\dagger\right) \\
& + 18\text{Tr}\left(Y_d Y_d^\dagger Y_d Y_u^\dagger Y_u Y_d^\dagger\right) + 18\text{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger Y_d Y_d^\dagger\right) + 36\text{Tr}\left(Y_d Y_u^\dagger Y_u Y_u^\dagger Y_u Y_d^\dagger\right)
\end{aligned} \tag{21}$$

$$\begin{aligned}
\beta_{\lambda_2}^{(1)} &= +\frac{27}{200}g_1^4 + \frac{9}{20}g_1^2g_2^2 + \frac{9}{8}g_2^4 - \frac{9}{5}g_1^2\lambda_2 - 9g_2^2\lambda_2 + 24\lambda_2^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + \lambda_5^2 + 12\lambda_7^2 \\
& + 12\lambda_2\text{Tr}\left(\epsilon_d\epsilon_d^\dagger\right) + 4\lambda_2\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) + 6\lambda_7\text{Tr}\left(\epsilon_u Y_u^\dagger\right) + 6\lambda_7\text{Tr}\left(Y_d\epsilon_d^\dagger\right) + 2\lambda_7\text{Tr}\left(Y_e\epsilon_e^\dagger\right) \\
& + 12\lambda_2\text{Tr}\left(Y_u Y_u^\dagger\right) - 6\text{Tr}\left(\epsilon_d\epsilon_d^\dagger\epsilon_d\epsilon_d^\dagger\right) - 2\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\epsilon_e\epsilon_e^\dagger\right) - 6\text{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger\right)
\end{aligned} \tag{22}$$

$$\begin{aligned}
\beta_{\lambda_2}^{(2)} &= -\frac{3537}{2000}g_1^6 - \frac{1719}{400}g_1^4g_2^2 - \frac{303}{80}g_1^2g_2^4 + \frac{291}{16}g_2^6 + \frac{1953}{200}g_1^4\lambda_2 + \frac{117}{20}g_1^2g_2^2\lambda_2 - \frac{51}{8}g_2^4\lambda_2 + \frac{108}{5}g_1^2\lambda_2^2 \\
& + 108g_2^2\lambda_2^2 - 312\lambda_2^3 + \frac{9}{10}g_1^4\lambda_3 + \frac{15}{2}g_2^4\lambda_3 + \frac{12}{5}g_1^2\lambda_3^2 + 12g_2^2\lambda_3^2 - 20\lambda_2\lambda_3^2 - 8\lambda_3^3 + \frac{9}{20}g_1^4\lambda_4 \\
& + \frac{3}{2}g_1^2g_2^2\lambda_4 + \frac{15}{4}g_2^4\lambda_4 + \frac{12}{5}g_1^2\lambda_3\lambda_4 + 12g_2^2\lambda_3\lambda_4 - 20\lambda_2\lambda_3\lambda_4 - 12\lambda_3^2\lambda_4 + \frac{6}{5}g_1^2\lambda_4^2 \\
& + 3g_2^2\lambda_4^2 - 12\lambda_2\lambda_4^2 - 16\lambda_3\lambda_4^2 - 6\lambda_4^3 - \frac{3}{5}g_1^2\lambda_5^2 - 14\lambda_2\lambda_5^2 - 20\lambda_3\lambda_5^2 - 22\lambda_4\lambda_5^2 + 6\lambda_2\lambda_6^2 \\
& - 18\lambda_3\lambda_6^2 - 14\lambda_4\lambda_6^2 - 10\lambda_5\lambda_6^2 - 36\lambda_3\lambda_6\lambda_7 - 28\lambda_4\lambda_6\lambda_7 - 20\lambda_5\lambda_6\lambda_7 + \frac{54}{5}g_1^2\lambda_7^2 + 54g_2^2\lambda_7^2 \\
& - 318\lambda_2\lambda_7^2 - 66\lambda_3\lambda_7^2 - 70\lambda_4\lambda_7^2 - 74\lambda_5\lambda_7^2
\end{aligned}$$



$$\begin{aligned}
& + \frac{1}{20} \left( -5 \left( 144\lambda_7^2 - 320g_3^2\lambda_2 + 576\lambda_2^2 - 90g_2^2\lambda_2 + 9g_2^4 \right) + 9g_1^4 + g_1^2 \left( 50\lambda_2 + 54g_2^2 \right) \right) \text{Tr} \left( \epsilon_d \epsilon_d^\dagger \right) \\
& - 6 \left( 12\lambda_2 + 2\lambda_4 + 3\lambda_3 + \lambda_5 \right) \lambda_7 \text{Tr} \left( \epsilon_d Y_d^\dagger \right) - \frac{9}{4} g_1^4 \text{Tr} \left( \epsilon_e \epsilon_e^\dagger \right) + \frac{33}{10} g_1^2 g_2^2 \text{Tr} \left( \epsilon_e \epsilon_e^\dagger \right) \\
& - \frac{3}{4} g_2^4 \text{Tr} \left( \epsilon_e \epsilon_e^\dagger \right) + \frac{15}{2} g_1^2 \lambda_2 \text{Tr} \left( \epsilon_e \epsilon_e^\dagger \right) + \frac{15}{2} g_2^2 \lambda_2 \text{Tr} \left( \epsilon_e \epsilon_e^\dagger \right) - 48\lambda_2^2 \text{Tr} \left( \epsilon_e \epsilon_e^\dagger \right) \\
& - 12\lambda_7^2 \text{Tr} \left( \epsilon_e \epsilon_e^\dagger \right) - 24\lambda_2 \lambda_7 \text{Tr} \left( \epsilon_e Y_e^\dagger \right) - 6\lambda_3 \lambda_7 \text{Tr} \left( \epsilon_e Y_e^\dagger \right) - 4\lambda_4 \lambda_7 \text{Tr} \left( \epsilon_e Y_e^\dagger \right) \\
& - 2\lambda_5 \lambda_7 \text{Tr} \left( \epsilon_e Y_e^\dagger \right) - 12\lambda_3^2 \text{Tr} \left( \epsilon_u \epsilon_u^\dagger \right) - 12\lambda_3 \lambda_4 \text{Tr} \left( \epsilon_u \epsilon_u^\dagger \right) - 6\lambda_4^2 \text{Tr} \left( \epsilon_u \epsilon_u^\dagger \right) \\
& - 6\lambda_5^2 \text{Tr} \left( \epsilon_u \epsilon_u^\dagger \right) - 36\lambda_7^2 \text{Tr} \left( \epsilon_u \epsilon_u^\dagger \right) - 72\lambda_2 \lambda_7 \text{Tr} \left( \epsilon_u Y_u^\dagger \right) - 18\lambda_3 \lambda_7 \text{Tr} \left( \epsilon_u Y_u^\dagger \right) \\
& - 12\lambda_4 \lambda_7 \text{Tr} \left( \epsilon_u Y_u^\dagger \right) - 6\lambda_5 \lambda_7 \text{Tr} \left( \epsilon_u Y_u^\dagger \right) - 72\lambda_2 \lambda_7 \text{Tr} \left( Y_d \epsilon_d^\dagger \right) - 18\lambda_3 \lambda_7 \text{Tr} \left( Y_d \epsilon_d^\dagger \right) \\
& - 12\lambda_4 \lambda_7 \text{Tr} \left( Y_d \epsilon_d^\dagger \right) - 6\lambda_5 \lambda_7 \text{Tr} \left( Y_d \epsilon_d^\dagger \right) - 12\lambda_3^2 \text{Tr} \left( Y_d Y_d^\dagger \right) - 12\lambda_3 \lambda_4 \text{Tr} \left( Y_d Y_d^\dagger \right) \\
& - 6\lambda_4^2 \text{Tr} \left( Y_d Y_d^\dagger \right) - 6\lambda_5^2 \text{Tr} \left( Y_d Y_d^\dagger \right) - 36\lambda_7^2 \text{Tr} \left( Y_d Y_d^\dagger \right) - 24\lambda_2 \lambda_7 \text{Tr} \left( Y_e \epsilon_e^\dagger \right) \\
& - 6\lambda_3 \lambda_7 \text{Tr} \left( Y_e \epsilon_e^\dagger \right) - 4\lambda_4 \lambda_7 \text{Tr} \left( Y_e \epsilon_e^\dagger \right) - 2\lambda_5 \lambda_7 \text{Tr} \left( Y_e \epsilon_e^\dagger \right) - 4\lambda_3^2 \text{Tr} \left( Y_e Y_e^\dagger \right) \\
& - 4\lambda_3 \lambda_4 \text{Tr} \left( Y_e Y_e^\dagger \right) - 2\lambda_4^2 \text{Tr} \left( Y_e Y_e^\dagger \right) - 2\lambda_5^2 \text{Tr} \left( Y_e Y_e^\dagger \right) - 12\lambda_7^2 \text{Tr} \left( Y_e Y_e^\dagger \right) \\
& - 72\lambda_2 \lambda_7 \text{Tr} \left( Y_u \epsilon_u^\dagger \right) - 18\lambda_3 \lambda_7 \text{Tr} \left( Y_u \epsilon_u^\dagger \right) - 12\lambda_4 \lambda_7 \text{Tr} \left( Y_u \epsilon_u^\dagger \right) - 6\lambda_5 \lambda_7 \text{Tr} \left( Y_u \epsilon_u^\dagger \right) \\
& - \frac{171}{100} g_1^4 \text{Tr} \left( Y_u Y_u^\dagger \right) + \frac{63}{10} g_1^2 g_2^2 \text{Tr} \left( Y_u Y_u^\dagger \right) - \frac{9}{4} g_2^4 \text{Tr} \left( Y_u Y_u^\dagger \right) + \frac{17}{2} g_1^2 \lambda_2 \text{Tr} \left( Y_u Y_u^\dagger \right) \\
& + \frac{45}{2} g_2^2 \lambda_2 \text{Tr} \left( Y_u Y_u^\dagger \right) + 80g_3^2 \lambda_2 \text{Tr} \left( Y_u Y_u^\dagger \right) - 144\lambda_2^2 \text{Tr} \left( Y_u Y_u^\dagger \right) - 36\lambda_7^2 \text{Tr} \left( Y_u Y_u^\dagger \right) \\
& + \frac{4}{5} g_1^2 \text{Tr} \left( \epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger \right) - 32g_3^2 \text{Tr} \left( \epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger \right) - 3\lambda_2 \text{Tr} \left( \epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger \right) + 6\lambda_7 \text{Tr} \left( \epsilon_d \epsilon_d^\dagger \epsilon_d Y_d^\dagger \right) \\
& + 12\lambda_7 \text{Tr} \left( \epsilon_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger \right) - 18\lambda_2 \text{Tr} \left( \epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger \right) - 9\lambda_2 \text{Tr} \left( \epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger \right) \\
& - 12\lambda_7 \text{Tr} \left( \epsilon_d \epsilon_u^\dagger Y_u \epsilon_d^\dagger \right) + 12\lambda_2 \text{Tr} \left( \epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger \right) - 12\lambda_3 \text{Tr} \left( \epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger \right) \\
& + 6\lambda_7 \text{Tr} \left( \epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger \right) + 6\lambda_5 \text{Tr} \left( \epsilon_d Y_d^\dagger \epsilon_d Y_d^\dagger \right) - 9\lambda_2 \text{Tr} \left( \epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger \right) \\
& - 12\lambda_7 \text{Tr} \left( \epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger \right) - 42\lambda_2 \text{Tr} \left( \epsilon_d Y_u^\dagger Y_u \epsilon_d^\dagger \right) - 12\lambda_7 \text{Tr} \left( \epsilon_d Y_u^\dagger Y_u Y_d^\dagger \right) \\
& - \frac{12}{5} g_1^2 \text{Tr} \left( \epsilon_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger \right) - \lambda_2 \text{Tr} \left( \epsilon_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger \right) + 2\lambda_7 \text{Tr} \left( \epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger \right) + 4\lambda_7 \text{Tr} \left( \epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger \right) \\
& - 6\lambda_2 \text{Tr} \left( \epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger \right) + 2\lambda_7 \text{Tr} \left( \epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger \right) + 2\lambda_5 \text{Tr} \left( \epsilon_e Y_e^\dagger \epsilon_e Y_e^\dagger \right) \\
& - 3\lambda_2 \text{Tr} \left( \epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger \right) + 12\lambda_2 \text{Tr} \left( \epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger \right) - 12\lambda_3 \text{Tr} \left( \epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger \right) \\
& - 18\lambda_2 \text{Tr} \left( \epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger \right) + 6\lambda_5 \text{Tr} \left( \epsilon_u Y_u^\dagger \epsilon_u Y_u^\dagger \right) - 9\lambda_2 \text{Tr} \left( \epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger \right) + 12\lambda_7 \text{Tr} \left( \epsilon_u Y_u^\dagger Y_u Y_u^\dagger \right) \\
& + 6\lambda_5 \text{Tr} \left( Y_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger \right) - 12\lambda_7 \text{Tr} \left( Y_d Y_u^\dagger Y_u \epsilon_d^\dagger \right) - 9\lambda_2 \text{Tr} \left( Y_d Y_u^\dagger Y_u Y_d^\dagger \right) + 2\lambda_5 \text{Tr} \left( Y_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger \right)
\end{aligned}$$

$$\begin{aligned}
& + 6\lambda_5 \text{Tr}\left(Y_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger\right) + 6\lambda_7 \text{Tr}\left(Y_u \epsilon_u^\dagger Y_u Y_u^\dagger\right) + 6\lambda_7 \text{Tr}\left(Y_u Y_u^\dagger Y_u \epsilon_u^\dagger\right) - \frac{8}{5} g_1^2 \text{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger\right) \\
& - 32g_3^2 \text{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger\right) - 3\lambda_2 \text{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger\right) + 30 \text{Tr}\left(\epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger\right) + 12 \text{Tr}\left(\epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger\right) \\
& + 6 \text{Tr}\left(\epsilon_d \epsilon_d^\dagger \epsilon_d Y_u^\dagger Y_u \epsilon_u^\dagger\right) + 6 \text{Tr}\left(\epsilon_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger \epsilon_d Y_d^\dagger\right) - 12 \text{Tr}\left(\epsilon_d \epsilon_d^\dagger Y_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger\right) \\
& + 12 \text{Tr}\left(\epsilon_d \epsilon_d^\dagger Y_d Y_u^\dagger Y_u Y_d^\dagger\right) + 6 \text{Tr}\left(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger\right) - 12 \text{Tr}\left(\epsilon_d \epsilon_u^\dagger Y_u Y_u^\dagger \epsilon_d \epsilon_d^\dagger\right) \\
& + 12 \text{Tr}\left(\epsilon_d \epsilon_u^\dagger Y_u Y_u^\dagger \epsilon_u \epsilon_d^\dagger\right) - 12 \text{Tr}\left(\epsilon_d \epsilon_u^\dagger Y_u Y_u^\dagger Y_u Y_d^\dagger\right) + 6 \text{Tr}\left(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger\right) \\
& + 6 \text{Tr}\left(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger\right) - 12 \text{Tr}\left(\epsilon_d Y_u^\dagger Y_u \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger\right) - 6 \text{Tr}\left(\epsilon_d Y_u^\dagger Y_u Y_u^\dagger Y_u \epsilon_d^\dagger\right) \\
& + 10 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger\right) + 4 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger\right) + 2 \text{Tr}\left(\epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger\right) + 2 \text{Tr}\left(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger\right) \\
& + 2 \text{Tr}\left(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger\right) - 12 \text{Tr}\left(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger Y_u Y_u^\dagger\right) + 12 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger Y_u Y_u^\dagger\right) \\
& + 12 \text{Tr}\left(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger Y_u Y_u^\dagger\right) + 6 \text{Tr}\left(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger Y_u \epsilon_u^\dagger\right) + 6 \text{Tr}\left(Y_d Y_u^\dagger Y_u Y_u^\dagger Y_u Y_d^\dagger\right) + 30 \text{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger Y_u Y_u^\dagger\right) \quad (23)
\end{aligned}$$

### 3.3 Yukawa Couplings

$$\begin{aligned}
\beta_{\epsilon_u}^{(1)} &= +\frac{1}{2} \left( \epsilon_u \epsilon_d^\dagger \epsilon_d + 3\epsilon_u \epsilon_u^\dagger \epsilon_u - 3\epsilon_u Y_d^\dagger Y_d + \epsilon_u Y_u^\dagger Y_u - 4Y_u Y_d^\dagger \epsilon_d + 2Y_u Y_u^\dagger \epsilon_u \right. \\
&\quad \left. + 6Y_u \text{Tr}\left(\epsilon_d Y_d^\dagger\right) + 2Y_u \text{Tr}\left(\epsilon_e Y_e^\dagger\right) + 6Y_u \text{Tr}\left(\epsilon_u Y_u^\dagger\right) \right) \\
&\quad + \epsilon_u \left( 3 \text{Tr}\left(\epsilon_u \epsilon_u^\dagger\right) + 3 \text{Tr}\left(Y_d Y_d^\dagger\right) - 8g_3^2 - \frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 + \text{Tr}\left(Y_e Y_e^\dagger\right) \right) \quad (24) \\
\beta_{\epsilon_u}^{(2)} &= -\frac{41}{240}g_1^2 \epsilon_u \epsilon_d^\dagger \epsilon_d + \frac{33}{16}g_2^2 \epsilon_u \epsilon_d^\dagger \epsilon_d + \frac{16}{3}g_3^2 \epsilon_u \epsilon_d^\dagger \epsilon_d - 2\lambda_3 \epsilon_u \epsilon_d^\dagger \epsilon_d \\
&\quad + 2\lambda_4 \epsilon_u \epsilon_d^\dagger \epsilon_d + \frac{223}{80}g_1^2 \epsilon_u \epsilon_u^\dagger \epsilon_u + \frac{135}{16}g_2^2 \epsilon_u \epsilon_u^\dagger \epsilon_u + 16g_3^2 \epsilon_u \epsilon_u^\dagger \epsilon_u \\
&\quad - 12\lambda_1 \epsilon_u \epsilon_u^\dagger \epsilon_u - 6\lambda_6 \epsilon_u \epsilon_u^\dagger Y_u - \frac{43}{80}g_1^2 \epsilon_u Y_d^\dagger Y_d + \frac{9}{16}g_2^2 \epsilon_u Y_d^\dagger Y_d \\
&\quad - 16g_3^2 \epsilon_u Y_d^\dagger Y_d - 6\lambda_6 \epsilon_u Y_u^\dagger \epsilon_u + \frac{523}{240}g_1^2 \epsilon_u Y_u^\dagger Y_u + \frac{33}{16}g_2^2 \epsilon_u Y_u^\dagger Y_u \\
&\quad + \frac{16}{3}g_3^2 \epsilon_u Y_u^\dagger Y_u - 2\lambda_3 \epsilon_u Y_u^\dagger Y_u - 4\lambda_4 \epsilon_u Y_u^\dagger Y_u - 6\lambda_6 Y_u \epsilon_u^\dagger \epsilon_u \\
&\quad - 6\lambda_5 Y_u \epsilon_u^\dagger Y_u - \frac{11}{30}g_1^2 Y_u Y_d^\dagger \epsilon_d - \frac{3}{2}g_2^2 Y_u Y_d^\dagger \epsilon_d - \frac{64}{3}g_3^2 Y_u Y_d^\dagger \epsilon_d \\
&\quad + 2\lambda_3 Y_u Y_d^\dagger \epsilon_d - 2\lambda_4 Y_u Y_d^\dagger \epsilon_d + \frac{73}{120}g_1^2 Y_u Y_u^\dagger \epsilon_u + \frac{51}{8}g_2^2 Y_u Y_u^\dagger \epsilon_u \\
&\quad + \frac{32}{3}g_3^2 Y_u Y_u^\dagger \epsilon_u - 4\lambda_3 Y_u Y_u^\dagger \epsilon_u - 2\lambda_4 Y_u Y_u^\dagger \epsilon_u - 6\lambda_7 Y_u Y_u^\dagger Y_u \\
&\quad - \frac{1}{4} \epsilon_u \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger \epsilon_d - \frac{1}{4} \epsilon_u \epsilon_d^\dagger \epsilon_d \epsilon_u^\dagger \epsilon_u + \epsilon_u \epsilon_d^\dagger \epsilon_d Y_d^\dagger Y_d - \frac{1}{4} \epsilon_u \epsilon_d^\dagger Y_d Y_d^\dagger \epsilon_d
\end{aligned}$$

$$\begin{aligned}
& + \frac{3}{2}\epsilon_u\epsilon_d^\dagger\epsilon_u\epsilon_u^\dagger\epsilon_u - \epsilon_u\epsilon_u^\dagger\epsilon_u Y_d^\dagger Y_d - \frac{1}{4}\epsilon_u\epsilon_u^\dagger Y_u Y_u^\dagger \epsilon_u + \frac{7}{4}\epsilon_u Y_d^\dagger \epsilon_d \epsilon_d^\dagger Y_d \\
& - \frac{1}{4}\epsilon_u Y_d^\dagger Y_d \epsilon_u^\dagger \epsilon_u + \frac{11}{4}\epsilon_u Y_d^\dagger Y_d Y_d^\dagger Y_d - 2\epsilon_u Y_u^\dagger \epsilon_u \epsilon_d^\dagger Y_d + \frac{7}{4}\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger Y_u \\
& - \frac{1}{4}\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger \epsilon_u + \epsilon_u Y_u^\dagger Y_u Y_d^\dagger Y_d - \frac{1}{4}\epsilon_u Y_u^\dagger Y_u Y_u^\dagger Y_u + Y_u \epsilon_d^\dagger \epsilon_d Y_d^\dagger \epsilon_d \\
& - \frac{1}{4}Y_u \epsilon_d^\dagger \epsilon_d Y_u^\dagger \epsilon_u - Y_u \epsilon_u^\dagger \epsilon_u Y_d^\dagger \epsilon_d + \frac{7}{4}Y_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger \epsilon_u \\
& + 2Y_u Y_d^\dagger \epsilon_d \epsilon_d^\dagger \epsilon_d + 3Y_u Y_d^\dagger Y_d Y_d^\dagger \epsilon_d - \frac{1}{4}Y_u Y_d^\dagger Y_d Y_u^\dagger \epsilon_u - 2Y_u Y_u^\dagger \epsilon_u \epsilon_d^\dagger \epsilon_d \\
& + 2Y_u Y_u^\dagger \epsilon_u Y_u^\dagger Y_u + Y_u Y_u^\dagger Y_u Y_d^\dagger \epsilon_d - \frac{1}{4}Y_u Y_u^\dagger Y_u Y_u^\dagger \epsilon_u - \frac{9}{4}\epsilon_u \epsilon_d^\dagger \epsilon_d \text{Tr}(\epsilon_d \epsilon_d^\dagger) \\
& - \frac{9}{4}\epsilon_u Y_u^\dagger Y_u \text{Tr}(\epsilon_d \epsilon_d^\dagger) - \frac{9}{2}Y_u Y_u^\dagger \epsilon_u \text{Tr}(\epsilon_d \epsilon_d^\dagger) + \frac{15}{4}\epsilon_u \epsilon_d^\dagger Y_d \text{Tr}(\epsilon_d Y_d^\dagger) \\
& - \frac{9}{4}\epsilon_u \epsilon_u^\dagger Y_u \text{Tr}(\epsilon_d Y_d^\dagger) + 6Y_u \epsilon_d^\dagger \epsilon_d \text{Tr}(\epsilon_d Y_d^\dagger) - \frac{9}{2}Y_u \epsilon_u^\dagger \epsilon_u \text{Tr}(\epsilon_d Y_d^\dagger) \\
& - \frac{3}{4}\epsilon_u \epsilon_d^\dagger \epsilon_d \text{Tr}(\epsilon_e \epsilon_e^\dagger) - \frac{3}{4}\epsilon_u Y_u^\dagger Y_u \text{Tr}(\epsilon_e \epsilon_e^\dagger) - \frac{3}{2}Y_u Y_u^\dagger \epsilon_u \text{Tr}(\epsilon_e \epsilon_e^\dagger) \\
& + \frac{5}{4}\epsilon_u \epsilon_d^\dagger Y_d \text{Tr}(\epsilon_e Y_e^\dagger) - \frac{3}{4}\epsilon_u \epsilon_u^\dagger Y_u \text{Tr}(\epsilon_e Y_e^\dagger) + 2Y_u \epsilon_d^\dagger \epsilon_d \text{Tr}(\epsilon_e Y_e^\dagger) \\
& - \frac{3}{2}Y_u \epsilon_u^\dagger \epsilon_u \text{Tr}(\epsilon_e Y_e^\dagger) - \frac{27}{4}\epsilon_u \epsilon_u^\dagger \epsilon_u \text{Tr}(\epsilon_u \epsilon_u^\dagger) + \frac{15}{4}\epsilon_u Y_d^\dagger Y_d \text{Tr}(\epsilon_u \epsilon_u^\dagger) \\
& + 6Y_u Y_d^\dagger \epsilon_d \text{Tr}(\epsilon_u \epsilon_u^\dagger) + \frac{15}{4}\epsilon_u \epsilon_d^\dagger Y_d \text{Tr}(\epsilon_u Y_u^\dagger) - \frac{9}{4}\epsilon_u \epsilon_u^\dagger Y_u \text{Tr}(\epsilon_u Y_u^\dagger) \\
& + 6Y_u \epsilon_d^\dagger \epsilon_d \text{Tr}(\epsilon_u Y_u^\dagger) - \frac{9}{2}Y_u \epsilon_u^\dagger \epsilon_u \text{Tr}(\epsilon_u Y_u^\dagger) - \frac{9}{4}\epsilon_u Y_d^\dagger \epsilon_d \text{Tr}(Y_d \epsilon_d^\dagger) \\
& - \frac{27}{4}\epsilon_u Y_u^\dagger \epsilon_u \text{Tr}(Y_d \epsilon_d^\dagger) - \frac{27}{4}\epsilon_u \epsilon_u^\dagger \epsilon_u \text{Tr}(Y_d Y_d^\dagger) + \frac{15}{4}\epsilon_u Y_d^\dagger Y_d \text{Tr}(Y_d Y_d^\dagger) \\
& + 6Y_u Y_d^\dagger \epsilon_d \text{Tr}(Y_d Y_d^\dagger) - \frac{3}{4}\epsilon_u Y_d^\dagger \epsilon_d \text{Tr}(Y_e \epsilon_e^\dagger) - \frac{9}{4}\epsilon_u Y_u^\dagger \epsilon_u \text{Tr}(Y_e \epsilon_e^\dagger) \\
& - \frac{9}{4}\epsilon_u \epsilon_u^\dagger \epsilon_u \text{Tr}(Y_e Y_e^\dagger) + \frac{5}{4}\epsilon_u Y_d^\dagger Y_d \text{Tr}(Y_e Y_e^\dagger) + 2Y_u Y_d^\dagger \epsilon_d \text{Tr}(Y_e Y_e^\dagger) \\
& - \frac{9}{4}\epsilon_u Y_d^\dagger \epsilon_d \text{Tr}(Y_u \epsilon_u^\dagger) - \frac{27}{4}\epsilon_u Y_u^\dagger \epsilon_u \text{Tr}(Y_u \epsilon_u^\dagger) - \frac{9}{4}\epsilon_u \epsilon_d^\dagger \epsilon_d \text{Tr}(Y_u Y_u^\dagger) \\
& - \frac{9}{4}\epsilon_u Y_u^\dagger Y_u \text{Tr}(Y_u Y_u^\dagger) - \frac{9}{2}Y_u Y_u^\dagger \epsilon_u \text{Tr}(Y_u Y_u^\dagger) \\
& + \frac{1}{8}Y_u \left( 24\lambda_1\lambda_6 + 12\lambda_3\lambda_6 + 12\lambda_4\lambda_6 + 12\lambda_5\lambda_6 + 24\lambda_2\lambda_7 + 12\lambda_3\lambda_7 + 12\lambda_4\lambda_7 + 12\lambda_5\lambda_7 \right. \\
& + 5(32g_3^2 + 9g_2^2 + g_1^2) \text{Tr}(\epsilon_d Y_d^\dagger) + 15(g_1^2 + g_2^2) \text{Tr}(\epsilon_e Y_e^\dagger) + 17g_1^2 \text{Tr}(\epsilon_u Y_u^\dagger) + 45g_2^2 \text{Tr}(\epsilon_u Y_u^\dagger) \\
& + 160g_3^2 \text{Tr}(\epsilon_u Y_u^\dagger) + 6\text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_d^\dagger) - 54\text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger) - 54\text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger) \\
& + 6\text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) + 6\text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger) - 18\text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger) - 18\text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger) \\
& \left. - 18\text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger) + 6\text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger) - 36\text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger) - 54\text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger) \right)
\end{aligned}$$

$$\begin{aligned}
& + \epsilon_u \left( \frac{1267}{600} g_1^4 - \frac{9}{20} g_1^2 g_2^2 - \frac{21}{4} g_2^4 + \frac{19}{15} g_1^2 g_3^2 + 9 g_2^2 g_3^2 - 108 g_3^4 + 6 \lambda_1^2 + \lambda_3^2 + \lambda_3 \lambda_4 + \lambda_4^2 + \frac{3}{2} \lambda_5^2 + \frac{9}{2} \lambda_6^2 \right. \\
& + \frac{3}{2} \lambda_7^2 + \frac{1}{8} (160 g_3^2 + 17 g_1^2 + 45 g_2^2) \text{Tr}(\epsilon_u \epsilon_u^\dagger) + \frac{5}{8} (32 g_3^2 + 9 g_2^2 + g_1^2) \text{Tr}(Y_d Y_d^\dagger) + \frac{15}{8} g_1^2 \text{Tr}(Y_e Y_e^\dagger) \\
& + \frac{15}{8} g_2^2 \text{Tr}(Y_e Y_e^\dagger) - \frac{9}{2} \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger) - \frac{9}{4} \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) + 3 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_u^\dagger) \\
& - \frac{9}{4} \text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger) - \frac{3}{2} \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger) - \frac{3}{4} \text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger) + 3 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_d Y_u^\dagger) \\
& - \frac{27}{4} \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger) - \frac{9}{2} \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger) + \frac{3}{2} \text{Tr}(\epsilon_u Y_d^\dagger Y_d \epsilon_u^\dagger) - \frac{9}{4} \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) \\
& \left. - \frac{27}{4} \text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) - \frac{9}{4} \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) - \frac{9}{4} \text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) \right) \tag{25}
\end{aligned}$$

$$\begin{aligned}
\beta_{Y_u}^{(1)} & = + \frac{1}{2} (2 \epsilon_u \epsilon_u^\dagger Y_u - 3 Y_u \epsilon_d^\dagger \epsilon_d + 3 Y_u Y_u^\dagger Y_u - 4 \epsilon_u \epsilon_d^\dagger Y_d + Y_u \epsilon_u^\dagger \epsilon_u + Y_u Y_d^\dagger Y_d) \\
& + \epsilon_u (3 \text{Tr}(Y_d \epsilon_d^\dagger) + 3 \text{Tr}(Y_u \epsilon_u^\dagger) + \text{Tr}(Y_e \epsilon_e^\dagger)) \\
& + Y_u (3 \text{Tr}(\epsilon_d \epsilon_d^\dagger) + 3 \text{Tr}(Y_u Y_u^\dagger) - 8 g_3^2 - \frac{17}{20} g_1^2 - \frac{9}{4} g_2^2 + \text{Tr}(\epsilon_e \epsilon_e^\dagger)) \tag{26}
\end{aligned}$$

$$\begin{aligned}
\beta_{Y_u}^{(2)} & = - \frac{11}{30} g_1^2 \epsilon_u \epsilon_d^\dagger Y_d - \frac{3}{2} g_2^2 \epsilon_u \epsilon_d^\dagger Y_d - \frac{64}{3} g_3^2 \epsilon_u \epsilon_d^\dagger Y_d + 2 \lambda_3 \epsilon_u \epsilon_d^\dagger Y_d \\
& - 2 \lambda_4 \epsilon_u \epsilon_d^\dagger Y_d - 6 \lambda_6 \epsilon_u \epsilon_u^\dagger \epsilon_u + \frac{73}{120} g_1^2 \epsilon_u \epsilon_u^\dagger Y_u + \frac{51}{8} g_2^2 \epsilon_u \epsilon_u^\dagger Y_u \\
& + \frac{32}{3} g_3^2 \epsilon_u \epsilon_u^\dagger Y_u - 4 \lambda_3 \epsilon_u \epsilon_u^\dagger Y_u - 2 \lambda_4 \epsilon_u \epsilon_u^\dagger Y_u - 6 \lambda_5 \epsilon_u Y_u^\dagger \epsilon_u \\
& - 6 \lambda_7 \epsilon_u Y_u^\dagger Y_u - \frac{43}{80} g_1^2 Y_u \epsilon_d^\dagger \epsilon_d + \frac{9}{16} g_2^2 Y_u \epsilon_d^\dagger \epsilon_d - 16 g_3^2 Y_u \epsilon_d^\dagger \epsilon_d \\
& + \frac{523}{240} g_1^2 Y_u \epsilon_u^\dagger \epsilon_u + \frac{33}{16} g_2^2 Y_u \epsilon_u^\dagger \epsilon_u + \frac{16}{3} g_3^2 Y_u \epsilon_u^\dagger \epsilon_u - 2 \lambda_3 Y_u \epsilon_u^\dagger \epsilon_u \\
& - 4 \lambda_4 Y_u \epsilon_u^\dagger \epsilon_u - 6 \lambda_7 Y_u \epsilon_u^\dagger Y_u - \frac{41}{240} g_1^2 Y_u Y_d^\dagger Y_d + \frac{33}{16} g_2^2 Y_u Y_d^\dagger Y_d \\
& + \frac{16}{3} g_3^2 Y_u Y_d^\dagger Y_d - 2 \lambda_3 Y_u Y_d^\dagger Y_d + 2 \lambda_4 Y_u Y_d^\dagger Y_d - 6 \lambda_7 Y_u Y_u^\dagger \epsilon_u \\
& + \frac{223}{80} g_1^2 Y_u Y_u^\dagger Y_u + \frac{135}{16} g_2^2 Y_u Y_u^\dagger Y_u + 16 g_3^2 Y_u Y_u^\dagger Y_u - 12 \lambda_2 Y_u Y_u^\dagger Y_u \\
& + 3 \epsilon_u \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger Y_d - \frac{1}{4} \epsilon_u \epsilon_d^\dagger \epsilon_d \epsilon_u^\dagger Y_u + 2 \epsilon_u \epsilon_d^\dagger Y_d Y_d^\dagger Y_d + \epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger Y_d \\
& - \frac{1}{4} \epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger Y_u + 2 \epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger \epsilon_u - 2 \epsilon_u \epsilon_u^\dagger Y_u Y_d^\dagger Y_d + \epsilon_u Y_d^\dagger Y_d \epsilon_d^\dagger Y_d \\
& - \frac{1}{4} \epsilon_u Y_d^\dagger Y_d \epsilon_u^\dagger Y_u - \epsilon_u Y_u^\dagger Y_u \epsilon_d^\dagger Y_d + \frac{7}{4} \epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger Y_u + \frac{11}{4} Y_u \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger \epsilon_d \\
& - \frac{1}{4} Y_u \epsilon_d^\dagger \epsilon_d Y_u^\dagger Y_u + \frac{7}{4} Y_u \epsilon_d^\dagger Y_d Y_d^\dagger \epsilon_d + Y_u \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger \epsilon_d - \frac{1}{4} Y_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger \epsilon_u \\
& - \frac{1}{4} Y_u \epsilon_u^\dagger \epsilon_u Y_u^\dagger Y_u - 2 Y_u \epsilon_u^\dagger Y_u Y_d^\dagger \epsilon_d + \frac{7}{4} Y_u \epsilon_u^\dagger Y_u Y_u^\dagger \epsilon_u \\
& - \frac{1}{4} Y_u Y_d^\dagger \epsilon_d \epsilon_d^\dagger Y_d + Y_u Y_d^\dagger Y_d \epsilon_d^\dagger \epsilon_d - \frac{1}{4} Y_u Y_d^\dagger Y_d Y_d^\dagger Y_d - \frac{1}{4} Y_u Y_d^\dagger Y_d Y_u^\dagger Y_u
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{4}Y_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger Y_u - Y_u Y_u^\dagger Y_u \epsilon_d^\dagger \epsilon_d + \frac{3}{2}Y_u Y_u^\dagger Y_u Y_u^\dagger Y_u + 6\epsilon_u \epsilon_d^\dagger Y_d \text{Tr}(\epsilon_d \epsilon_d^\dagger) \\
& + \frac{15}{4}Y_u \epsilon_d^\dagger \epsilon_d \text{Tr}(\epsilon_d \epsilon_d^\dagger) - \frac{27}{4}Y_u Y_u^\dagger Y_u \text{Tr}(\epsilon_d \epsilon_d^\dagger) - \frac{9}{4}Y_u \epsilon_d^\dagger Y_d \text{Tr}(\epsilon_d Y_d^\dagger) \\
& - \frac{27}{4}Y_u \epsilon_u^\dagger Y_u \text{Tr}(\epsilon_d Y_d^\dagger) + 2\epsilon_u \epsilon_d^\dagger Y_d \text{Tr}(\epsilon_e \epsilon_e^\dagger) + \frac{5}{4}Y_u \epsilon_d^\dagger \epsilon_d \text{Tr}(\epsilon_e \epsilon_e^\dagger) \\
& - \frac{9}{4}Y_u Y_u^\dagger Y_u \text{Tr}(\epsilon_e \epsilon_e^\dagger) - \frac{3}{4}Y_u \epsilon_d^\dagger Y_d \text{Tr}(\epsilon_e Y_e^\dagger) - \frac{9}{4}Y_u \epsilon_u^\dagger Y_u \text{Tr}(\epsilon_e Y_e^\dagger) \\
& - \frac{9}{2}\epsilon_u \epsilon_u^\dagger Y_u \text{Tr}(\epsilon_u \epsilon_u^\dagger) - \frac{9}{4}Y_u \epsilon_u^\dagger \epsilon_u \text{Tr}(\epsilon_u \epsilon_u^\dagger) - \frac{9}{4}Y_u Y_d^\dagger Y_d \text{Tr}(\epsilon_u \epsilon_u^\dagger) \\
& - \frac{9}{4}Y_u \epsilon_d^\dagger Y_d \text{Tr}(\epsilon_u Y_u^\dagger) - \frac{27}{4}Y_u \epsilon_u^\dagger Y_u \text{Tr}(\epsilon_u Y_u^\dagger) + 6\epsilon_u Y_d^\dagger Y_d \text{Tr}(Y_d \epsilon_d^\dagger) \\
& - \frac{9}{2}\epsilon_u Y_u^\dagger Y_u \text{Tr}(Y_d \epsilon_d^\dagger) + \frac{15}{4}Y_u Y_d^\dagger \epsilon_d \text{Tr}(Y_d \epsilon_d^\dagger) - \frac{9}{4}Y_u Y_u^\dagger \epsilon_u \text{Tr}(Y_d \epsilon_d^\dagger) \\
& - \frac{9}{2}\epsilon_u \epsilon_u^\dagger Y_u \text{Tr}(Y_d Y_d^\dagger) - \frac{9}{4}Y_u \epsilon_u^\dagger \epsilon_u \text{Tr}(Y_d Y_d^\dagger) - \frac{9}{4}Y_u Y_d^\dagger Y_d \text{Tr}(Y_d Y_d^\dagger) \\
& + 2\epsilon_u Y_d^\dagger Y_d \text{Tr}(Y_e \epsilon_e^\dagger) - \frac{3}{2}\epsilon_u Y_u^\dagger Y_u \text{Tr}(Y_e \epsilon_e^\dagger) + \frac{5}{4}Y_u Y_d^\dagger \epsilon_d \text{Tr}(Y_e \epsilon_e^\dagger) \\
& - \frac{3}{4}Y_u Y_u^\dagger \epsilon_u \text{Tr}(Y_e \epsilon_e^\dagger) - \frac{3}{2}\epsilon_u \epsilon_u^\dagger Y_u \text{Tr}(Y_e Y_e^\dagger) - \frac{3}{4}Y_u \epsilon_u^\dagger \epsilon_u \text{Tr}(Y_e Y_e^\dagger) \\
& - \frac{3}{4}Y_u Y_d^\dagger Y_d \text{Tr}(Y_e Y_e^\dagger) + 6\epsilon_u Y_d^\dagger Y_d \text{Tr}(Y_u \epsilon_u^\dagger) - \frac{9}{2}\epsilon_u Y_u^\dagger Y_u \text{Tr}(Y_u \epsilon_u^\dagger) \\
& + \frac{15}{4}Y_u Y_d^\dagger \epsilon_d \text{Tr}(Y_u \epsilon_u^\dagger) - \frac{9}{4}Y_u Y_u^\dagger \epsilon_u \text{Tr}(Y_u \epsilon_u^\dagger) + 6\epsilon_u \epsilon_d^\dagger Y_d \text{Tr}(Y_u Y_u^\dagger) \\
& + \frac{15}{4}Y_u \epsilon_d^\dagger \epsilon_d \text{Tr}(Y_u Y_u^\dagger) - \frac{27}{4}Y_u Y_u^\dagger Y_u \text{Tr}(Y_u Y_u^\dagger) \\
& + \frac{1}{8}\epsilon_u (24\lambda_1\lambda_6 + 12\lambda_3\lambda_6 + 12\lambda_4\lambda_6 + 12\lambda_5\lambda_6 + 24\lambda_2\lambda_7 + 12\lambda_3\lambda_7 + 12\lambda_4\lambda_7 + 12\lambda_5\lambda_7 \\
& + 5(32g_3^2 + 9g_2^2 + g_1^2)\text{Tr}(Y_d \epsilon_d^\dagger) + 15(g_1^2 + g_2^2)\text{Tr}(Y_e \epsilon_e^\dagger) + 17g_1^2\text{Tr}(Y_u \epsilon_u^\dagger) + 45g_2^2\text{Tr}(Y_u \epsilon_u^\dagger) \\
& + 160g_3^2\text{Tr}(Y_u \epsilon_u^\dagger) - 54\text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d \epsilon_d^\dagger) + 6\text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u \epsilon_d^\dagger) - 18\text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e \epsilon_e^\dagger) \\
& + 6\text{Tr}(\epsilon_u \epsilon_u^\dagger Y_d \epsilon_u^\dagger) - 54\text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u \epsilon_u^\dagger) - 54\text{Tr}(Y_d \epsilon_d^\dagger Y_d Y_d^\dagger) + 6\text{Tr}(Y_d \epsilon_u^\dagger Y_u Y_d^\dagger) \\
& + 6\text{Tr}(Y_d Y_u^\dagger Y_u \epsilon_d^\dagger) - 18\text{Tr}(Y_e \epsilon_e^\dagger Y_e Y_e^\dagger) - 36\text{Tr}(Y_u \epsilon_u^\dagger Y_u Y_u^\dagger) - 18\text{Tr}(Y_u Y_u^\dagger Y_u \epsilon_u^\dagger)) \\
& + Y_u \left( \frac{1267}{600}g_1^4 - \frac{9}{20}g_1^2 g_2^2 - \frac{21}{4}g_2^4 + \frac{19}{15}g_1^2 g_3^2 + 9g_2^2 g_3^2 - 108g_3^4 + 6\lambda_2^2 + \lambda_3^2 + \lambda_3\lambda_4 + \lambda_4^2 + \frac{3}{2}\lambda_5^2 + \frac{3}{2}\lambda_6^2 \right. \\
& + \frac{9}{2}\lambda_7^2 + \frac{5}{8}(32g_3^2 + 9g_2^2 + g_1^2)\text{Tr}(\epsilon_d \epsilon_d^\dagger) + \frac{15}{8}(g_1^2 + g_2^2)\text{Tr}(\epsilon_e \epsilon_e^\dagger) + \frac{17}{8}g_1^2\text{Tr}(Y_u Y_u^\dagger) \\
& + \frac{45}{8}g_2^2\text{Tr}(Y_u Y_u^\dagger) + 20g_3^2\text{Tr}(Y_u Y_u^\dagger) - \frac{27}{4}\text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger) - \frac{9}{2}\text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger) \\
& - \frac{9}{4}\text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) + 3\text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) - \frac{9}{4}\text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger) + \frac{3}{2}\text{Tr}(\epsilon_d Y_u^\dagger Y_u \epsilon_d^\dagger) \\
& \left. - \frac{9}{4}\text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger) - \frac{3}{2}\text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger) - \frac{3}{4}\text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger) + 3\text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) \right)
\end{aligned}$$

$$-\frac{9}{2}\text{Tr}\left(\epsilon_u\epsilon_u^\dagger Y_u Y_u^\dagger\right) - \frac{9}{4}\text{Tr}\left(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger\right) - \frac{9}{4}\text{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger\right) - \frac{27}{4}\text{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger\right) \quad (27)$$

$$\begin{aligned} \beta_{Y_d}^{(1)} = & +\frac{1}{4}\left(2\left(2\epsilon_d\epsilon_d^\dagger Y_d - 3Y_d\epsilon_u^\dagger\epsilon_u + 3Y_d Y_d^\dagger Y_d - 4\epsilon_d\epsilon_u^\dagger Y_u + Y_d\epsilon_d^\dagger\epsilon_d + Y_d Y_u^\dagger Y_u\right)\right. \\ & \left.- Y_d\left(-12\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) - 12\text{Tr}\left(Y_d Y_d^\dagger\right) + 32g_3^2 - 4\text{Tr}\left(Y_e Y_e^\dagger\right) + 9g_2^2 + g_1^2\right)\right) \\ & + \epsilon_d\left(3\text{Tr}\left(Y_d\epsilon_d^\dagger\right) + 3\text{Tr}\left(Y_u\epsilon_u^\dagger\right) + \text{Tr}\left(Y_e\epsilon_e^\dagger\right)\right) \end{aligned} \quad (28)$$

$$\begin{aligned} \beta_{Y_d}^{(2)} = & -6\lambda_7\epsilon_d\epsilon_d^\dagger\epsilon_d + \frac{157}{120}g_1^2\epsilon_d\epsilon_d^\dagger Y_d + \frac{51}{8}g_2^2\epsilon_d\epsilon_d^\dagger Y_d + \frac{32}{3}g_3^2\epsilon_d\epsilon_d^\dagger Y_d \\ & - 4\lambda_3\epsilon_d\epsilon_d^\dagger Y_d - 2\lambda_4\epsilon_d\epsilon_d^\dagger Y_d - \frac{23}{30}g_1^2\epsilon_d\epsilon_u^\dagger Y_u - \frac{3}{2}g_2^2\epsilon_d\epsilon_u^\dagger Y_u \\ & - \frac{64}{3}g_3^2\epsilon_d\epsilon_u^\dagger Y_u + 2\lambda_3\epsilon_d\epsilon_u^\dagger Y_u - 2\lambda_4\epsilon_d\epsilon_u^\dagger Y_u - 6\lambda_5\epsilon_d Y_d^\dagger \epsilon_d \\ & - 6\lambda_6\epsilon_d Y_d^\dagger Y_d + \frac{247}{240}g_1^2 Y_d\epsilon_d^\dagger\epsilon_d + \frac{33}{16}g_2^2 Y_d\epsilon_d^\dagger\epsilon_d + \frac{16}{3}g_3^2 Y_d\epsilon_d^\dagger\epsilon_d \\ & - 2\lambda_3 Y_d\epsilon_d^\dagger\epsilon_d - 4\lambda_4 Y_d\epsilon_d^\dagger\epsilon_d - 6\lambda_6 Y_d\epsilon_d^\dagger Y_d - \frac{79}{80}g_1^2 Y_d\epsilon_u^\dagger\epsilon_u \\ & + \frac{9}{16}g_2^2 Y_d\epsilon_u^\dagger\epsilon_u - 16g_3^2 Y_d\epsilon_u^\dagger\epsilon_u - 6\lambda_6 Y_d Y_d^\dagger \epsilon_d + \frac{187}{80}g_1^2 Y_d Y_d^\dagger Y_d \\ & + \frac{135}{16}g_2^2 Y_d Y_d^\dagger Y_d + 16g_3^2 Y_d Y_d^\dagger Y_d - 12\lambda_1 Y_d Y_d^\dagger Y_d - \frac{53}{240}g_1^2 Y_d Y_u^\dagger Y_u \\ & + \frac{33}{16}g_2^2 Y_d Y_u^\dagger Y_u + \frac{16}{3}g_3^2 Y_d Y_u^\dagger Y_u - 2\lambda_3 Y_d Y_u^\dagger Y_u + 2\lambda_4 Y_d Y_u^\dagger Y_u \\ & - \frac{1}{4}\epsilon_d\epsilon_d^\dagger\epsilon_d\epsilon_d^\dagger Y_d + \epsilon_d\epsilon_d^\dagger\epsilon_d\epsilon_u^\dagger Y_u + 2\epsilon_d\epsilon_d^\dagger Y_d\epsilon_d^\dagger\epsilon_d - 2\epsilon_d\epsilon_d^\dagger Y_d Y_u^\dagger Y_u \\ & - \frac{1}{4}\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger Y_d + 3\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_u^\dagger Y_u + 2\epsilon_d\epsilon_u^\dagger Y_u Y_u^\dagger Y_u + \frac{7}{4}\epsilon_d Y_d^\dagger Y_d\epsilon_d^\dagger Y_d \\ & - \epsilon_d Y_d^\dagger Y_d\epsilon_u^\dagger Y_u - \frac{1}{4}\epsilon_d Y_u^\dagger Y_u\epsilon_d^\dagger Y_d + \epsilon_d Y_u^\dagger Y_u\epsilon_u^\dagger Y_u - \frac{1}{4}Y_d\epsilon_d^\dagger\epsilon_d\epsilon_d^\dagger\epsilon_d \\ & + Y_d\epsilon_d^\dagger\epsilon_d\epsilon_u^\dagger\epsilon_u - \frac{1}{4}Y_d\epsilon_d^\dagger\epsilon_d Y_d^\dagger Y_d + \frac{7}{4}Y_d\epsilon_d^\dagger Y_d Y_d^\dagger \epsilon_d - 2Y_d\epsilon_d^\dagger Y_d Y_u^\dagger \epsilon_u \\ & + \frac{11}{4}Y_d\epsilon_u^\dagger\epsilon_u\epsilon_u^\dagger\epsilon_u - \frac{1}{4}Y_d\epsilon_u^\dagger\epsilon_u Y_d^\dagger Y_d + \frac{7}{4}Y_d\epsilon_u^\dagger Y_u Y_u^\dagger \epsilon_u - \frac{1}{4}Y_d Y_d^\dagger \epsilon_d\epsilon_d^\dagger Y_d \\ & - Y_d Y_d^\dagger Y_d\epsilon_u^\dagger\epsilon_u + \frac{3}{2}Y_d Y_d^\dagger Y_d Y_d^\dagger Y_d - \frac{1}{4}Y_d Y_u^\dagger \epsilon_u\epsilon_u^\dagger Y_u + Y_d Y_u^\dagger Y_u\epsilon_u^\dagger\epsilon_u \\ & - \frac{1}{4}Y_d Y_u^\dagger Y_u Y_d^\dagger Y_d - \frac{1}{4}Y_d Y_u^\dagger Y_u Y_u^\dagger Y_u - \frac{9}{2}\epsilon_d\epsilon_d^\dagger Y_d \text{Tr}\left(\epsilon_d\epsilon_d^\dagger\right) \\ & - \frac{9}{4}Y_d\epsilon_d^\dagger\epsilon_d \text{Tr}\left(\epsilon_d\epsilon_d^\dagger\right) - \frac{9}{4}Y_d Y_u^\dagger Y_u \text{Tr}\left(\epsilon_d\epsilon_d^\dagger\right) - \frac{27}{4}Y_d\epsilon_d^\dagger Y_d \text{Tr}\left(\epsilon_d Y_d^\dagger\right) \\ & - \frac{9}{4}Y_d\epsilon_u^\dagger Y_u \text{Tr}\left(\epsilon_d Y_d^\dagger\right) - \frac{3}{2}\epsilon_d\epsilon_d^\dagger Y_d \text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - \frac{3}{4}Y_d\epsilon_d^\dagger\epsilon_d \text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) \\ & - \frac{3}{4}Y_d Y_u^\dagger Y_u \text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - \frac{9}{4}Y_d\epsilon_d^\dagger Y_d \text{Tr}\left(\epsilon_e Y_e^\dagger\right) - \frac{3}{4}Y_d\epsilon_u^\dagger Y_u \text{Tr}\left(\epsilon_e Y_e^\dagger\right) \\ & + 6\epsilon_d\epsilon_u^\dagger Y_u \text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) + \frac{15}{4}Y_d\epsilon_u^\dagger\epsilon_u \text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) - \frac{27}{4}Y_d Y_d^\dagger Y_d \text{Tr}\left(\epsilon_u\epsilon_u^\dagger\right) \end{aligned}$$

$$\begin{aligned}
& -\frac{27}{4}Y_d\epsilon_d^\dagger Y_d\text{Tr}(\epsilon_u Y_u^\dagger) - \frac{9}{4}Y_d\epsilon_u^\dagger Y_u\text{Tr}(\epsilon_u Y_u^\dagger) - \frac{9}{2}\epsilon_d Y_d^\dagger Y_d\text{Tr}(Y_d\epsilon_d^\dagger) \\
& + 6\epsilon_d Y_u^\dagger Y_u\text{Tr}(Y_d\epsilon_d^\dagger) - \frac{9}{4}Y_d Y_d^\dagger \epsilon_d\text{Tr}(Y_d\epsilon_d^\dagger) + \frac{15}{4}Y_d Y_u^\dagger \epsilon_u\text{Tr}(Y_d\epsilon_d^\dagger) \\
& + 6\epsilon_d\epsilon_u^\dagger Y_u\text{Tr}(Y_d Y_d^\dagger) + \frac{15}{4}Y_d\epsilon_u^\dagger \epsilon_u\text{Tr}(Y_d Y_d^\dagger) - \frac{27}{4}Y_d Y_d^\dagger Y_d\text{Tr}(Y_d Y_d^\dagger) \\
& - \frac{3}{2}\epsilon_d Y_d^\dagger Y_d\text{Tr}(Y_e\epsilon_e^\dagger) + 2\epsilon_d Y_u^\dagger Y_u\text{Tr}(Y_e\epsilon_e^\dagger) - \frac{3}{4}Y_d Y_d^\dagger \epsilon_d\text{Tr}(Y_e\epsilon_e^\dagger) \\
& + \frac{5}{4}Y_d Y_u^\dagger \epsilon_u\text{Tr}(Y_e\epsilon_e^\dagger) + 2\epsilon_d\epsilon_u^\dagger Y_u\text{Tr}(Y_e Y_e^\dagger) + \frac{5}{4}Y_d\epsilon_u^\dagger \epsilon_u\text{Tr}(Y_e Y_e^\dagger) \\
& - \frac{9}{4}Y_d Y_d^\dagger Y_d\text{Tr}(Y_e Y_e^\dagger) - \frac{9}{2}\epsilon_d Y_d^\dagger Y_d\text{Tr}(Y_u\epsilon_u^\dagger) + 6\epsilon_d Y_u^\dagger Y_u\text{Tr}(Y_u\epsilon_u^\dagger) \\
& - \frac{9}{4}Y_d Y_d^\dagger \epsilon_d\text{Tr}(Y_u\epsilon_u^\dagger) + \frac{15}{4}Y_d Y_u^\dagger \epsilon_u\text{Tr}(Y_u\epsilon_u^\dagger) - \frac{9}{2}\epsilon_d\epsilon_u^\dagger Y_d\text{Tr}(Y_u Y_u^\dagger) \\
& - \frac{9}{4}Y_d\epsilon_d^\dagger \epsilon_d\text{Tr}(Y_u Y_u^\dagger) - \frac{9}{4}Y_d Y_u^\dagger Y_u\text{Tr}(Y_u Y_u^\dagger) \\
& + Y_d\left(-\frac{113}{600}g_1^4 - \frac{27}{20}g_1^2 g_2^2 - \frac{21}{4}g_2^4 + \frac{31}{15}g_1^2 g_3^2 + 9g_2^2 g_3^2 - 108g_3^4 + 6\lambda_1^2 + \lambda_3^2 + \lambda_3\lambda_4 + \lambda_4^2 + \frac{3}{2}\lambda_5^2 + \frac{9}{2}\lambda_6^2\right. \\
& \left. + \frac{3}{2}\lambda_7^2 + \frac{1}{8}(160g_3^2 + 17g_1^2 + 45g_2^2)\text{Tr}(\epsilon_u\epsilon_u^\dagger) + \frac{5}{8}(32g_3^2 + 9g_2^2 + g_1^2)\text{Tr}(Y_d Y_d^\dagger) + \frac{15}{8}g_1^2\text{Tr}(Y_e Y_e^\dagger)\right) \\
& + \frac{15}{8}g_2^2\text{Tr}(Y_e Y_e^\dagger) - \frac{9}{2}\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d Y_d^\dagger) - \frac{9}{4}\text{Tr}(\epsilon_d\epsilon_u^\dagger \epsilon_u\epsilon_d^\dagger) + 3\text{Tr}(\epsilon_d\epsilon_u^\dagger Y_u Y_d^\dagger) \\
& - \frac{9}{4}\text{Tr}(\epsilon_d Y_d^\dagger Y_d\epsilon_d^\dagger) - \frac{3}{2}\text{Tr}(\epsilon_e\epsilon_e^\dagger Y_e Y_e^\dagger) - \frac{3}{4}\text{Tr}(\epsilon_e Y_e^\dagger Y_e\epsilon_e^\dagger) + 3\text{Tr}(\epsilon_u\epsilon_d^\dagger Y_d Y_u^\dagger) \\
& - \frac{27}{4}\text{Tr}(\epsilon_u\epsilon_u^\dagger \epsilon_u\epsilon_u^\dagger) - \frac{9}{2}\text{Tr}(\epsilon_u\epsilon_u^\dagger Y_u Y_u^\dagger) + \frac{3}{2}\text{Tr}(\epsilon_u Y_d^\dagger Y_d\epsilon_u^\dagger) - \frac{9}{4}\text{Tr}(\epsilon_u Y_u^\dagger Y_u\epsilon_u^\dagger) \\
& - \frac{27}{4}\text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) - \frac{9}{4}\text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) - \frac{9}{4}\text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) \\
& + \frac{1}{8}\epsilon_d(24\lambda_1\lambda_6 + 12\lambda_3\lambda_6 + 12\lambda_4\lambda_6 + 12\lambda_5\lambda_6 + 24\lambda_2\lambda_7 + 12\lambda_3\lambda_7 + 12\lambda_4\lambda_7 + 12\lambda_5\lambda_7 \\
& + 5(32g_3^2 + 9g_2^2 + g_1^2)\text{Tr}(Y_d\epsilon_d^\dagger) + 15(g_1^2 + g_2^2)\text{Tr}(Y_e\epsilon_e^\dagger) + 17g_1^2\text{Tr}(Y_u\epsilon_u^\dagger) + 45g_2^2\text{Tr}(Y_u\epsilon_u^\dagger) \\
& + 160g_3^2\text{Tr}(Y_u\epsilon_u^\dagger) - 54\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d\epsilon_d^\dagger) + 6\text{Tr}(\epsilon_d\epsilon_u^\dagger Y_u\epsilon_d^\dagger) - 18\text{Tr}(\epsilon_e\epsilon_e^\dagger Y_e\epsilon_e^\dagger) \\
& + 6\text{Tr}(\epsilon_u\epsilon_d^\dagger Y_d\epsilon_u^\dagger) - 54\text{Tr}(\epsilon_u\epsilon_u^\dagger Y_u\epsilon_u^\dagger) - 36\text{Tr}(Y_d\epsilon_d^\dagger Y_d Y_d^\dagger) + 6\text{Tr}(Y_d\epsilon_u^\dagger Y_u Y_d^\dagger) \\
& - 18\text{Tr}(Y_d Y_d^\dagger Y_d\epsilon_d^\dagger) + 6\text{Tr}(Y_d Y_u^\dagger Y_u\epsilon_d^\dagger) - 12\text{Tr}(Y_e\epsilon_e^\dagger Y_e Y_e^\dagger) - 6\text{Tr}(Y_e Y_e^\dagger Y_e\epsilon_e^\dagger) \\
& - 54\text{Tr}(Y_u\epsilon_u^\dagger Y_u Y_u^\dagger)) \tag{29}
\end{aligned}$$

$$\begin{aligned}
\beta_{Y_e}^{(1)} & = +\frac{1}{2}(2\epsilon_e\epsilon_e^\dagger Y_e + 3Y_e Y_e^\dagger Y_e + Y_e\epsilon_e^\dagger \epsilon_e) \\
& + Y_e\left(3\text{Tr}(\epsilon_u\epsilon_u^\dagger) + 3\text{Tr}(Y_d Y_d^\dagger) - \frac{9}{4}g_1^2 - \frac{9}{4}g_2^2 + \text{Tr}(Y_e Y_e^\dagger)\right) \\
& + \epsilon_e\left(3\text{Tr}(Y_d\epsilon_d^\dagger) + 3\text{Tr}(Y_u\epsilon_u^\dagger) + \text{Tr}(Y_e\epsilon_e^\dagger)\right) \tag{30}
\end{aligned}$$

$$\begin{aligned}
\beta_{Y_e}^{(2)} = & -6\lambda_7\epsilon_e\epsilon_e^\dagger\epsilon_e + \frac{39}{40}g_1^2\epsilon_e\epsilon_e^\dagger Y_e + \frac{51}{8}g_2^2\epsilon_e\epsilon_e^\dagger Y_e - 4\lambda_3\epsilon_e\epsilon_e^\dagger Y_e \\
& - 2\lambda_4\epsilon_e\epsilon_e^\dagger Y_e - 6\lambda_5\epsilon_e Y_e^\dagger\epsilon_e - 6\lambda_6\epsilon_e Y_e^\dagger Y_e + \frac{309}{80}g_1^2 Y_e\epsilon_e^\dagger\epsilon_e \\
& + \frac{33}{16}g_2^2 Y_e\epsilon_e^\dagger\epsilon_e - 2\lambda_3 Y_e\epsilon_e^\dagger\epsilon_e - 4\lambda_4 Y_e\epsilon_e^\dagger\epsilon_e - 6\lambda_6 Y_e\epsilon_e^\dagger Y_e \\
& - 6\lambda_6 Y_e Y_e^\dagger\epsilon_e + \frac{387}{80}g_1^2 Y_e Y_e^\dagger Y_e + \frac{135}{16}g_2^2 Y_e Y_e^\dagger Y_e - 12\lambda_1 Y_e Y_e^\dagger Y_e \\
& - \frac{1}{4}\epsilon_e\epsilon_e^\dagger\epsilon_e\epsilon_e^\dagger Y_e + 2\epsilon_e\epsilon_e^\dagger Y_e\epsilon_e^\dagger\epsilon_e + \frac{7}{4}\epsilon_e Y_e^\dagger Y_e\epsilon_e^\dagger Y_e - \frac{1}{4}Y_e\epsilon_e^\dagger\epsilon_e\epsilon_e^\dagger\epsilon_e \\
& - \frac{1}{4}Y_e\epsilon_e^\dagger\epsilon_e Y_e^\dagger Y_e + \frac{7}{4}Y_e\epsilon_e^\dagger Y_e Y_e^\dagger\epsilon_e - \frac{1}{4}Y_e Y_e^\dagger\epsilon_e\epsilon_e^\dagger Y_e + \frac{3}{2}Y_e Y_e^\dagger Y_e Y_e^\dagger Y_e \\
& - \frac{9}{2}\epsilon_e\epsilon_e^\dagger Y_e \text{Tr}(\epsilon_d\epsilon_d^\dagger) - \frac{9}{4}Y_e\epsilon_e^\dagger\epsilon_e \text{Tr}(\epsilon_d\epsilon_d^\dagger) - \frac{27}{4}Y_e\epsilon_e^\dagger Y_e \text{Tr}(\epsilon_d Y_d^\dagger) \\
& - \frac{3}{2}\epsilon_e\epsilon_e^\dagger Y_e \text{Tr}(\epsilon_e\epsilon_e^\dagger) - \frac{3}{4}Y_e\epsilon_e^\dagger\epsilon_e \text{Tr}(\epsilon_e\epsilon_e^\dagger) - \frac{9}{4}Y_e\epsilon_e^\dagger Y_e \text{Tr}(\epsilon_e Y_e^\dagger) \\
& - \frac{27}{4}Y_e Y_e^\dagger Y_e \text{Tr}(\epsilon_u\epsilon_u^\dagger) - \frac{27}{4}Y_e\epsilon_e^\dagger Y_e \text{Tr}(\epsilon_u Y_u^\dagger) - \frac{9}{2}\epsilon_e Y_e^\dagger Y_e \text{Tr}(Y_d\epsilon_d^\dagger) \\
& - \frac{9}{4}Y_e Y_e^\dagger\epsilon_e \text{Tr}(Y_d\epsilon_d^\dagger) - \frac{27}{4}Y_e Y_e^\dagger Y_e \text{Tr}(Y_d Y_d^\dagger) - \frac{3}{2}\epsilon_e Y_e^\dagger Y_e \text{Tr}(Y_e\epsilon_e^\dagger) \\
& - \frac{3}{4}Y_e Y_e^\dagger\epsilon_e \text{Tr}(Y_e\epsilon_e^\dagger) - \frac{9}{4}Y_e Y_e^\dagger Y_e \text{Tr}(Y_e Y_e^\dagger) - \frac{9}{2}\epsilon_e Y_e^\dagger Y_e \text{Tr}(Y_u\epsilon_u^\dagger) \\
& - \frac{9}{4}Y_e Y_e^\dagger\epsilon_e \text{Tr}(Y_u\epsilon_u^\dagger) - \frac{9}{2}\epsilon_e\epsilon_e^\dagger Y_e \text{Tr}(Y_u Y_u^\dagger) - \frac{9}{4}Y_e\epsilon_e^\dagger\epsilon_e \text{Tr}(Y_u Y_u^\dagger) \\
& + Y_e \left( \frac{1449}{200}g_1^4 + \frac{27}{20}g_1^2 g_2^2 - \frac{21}{4}g_2^4 + 6\lambda_1^2 + \lambda_3^2 + \lambda_3\lambda_4 + \lambda_4^2 + \frac{3}{2}\lambda_5^2 + \frac{9}{2}\lambda_6^2 + \frac{3}{2}\lambda_7^2 \right) \\
& + \frac{1}{8} \left( 160g_3^2 + 17g_1^2 + 45g_2^2 \right) \text{Tr}(\epsilon_u\epsilon_u^\dagger) + \frac{5}{8} \left( 32g_3^2 + 9g_2^2 + g_1^2 \right) \text{Tr}(Y_d Y_d^\dagger) + \frac{15}{8}g_1^2 \text{Tr}(Y_e Y_e^\dagger) \\
& + \frac{15}{8}g_2^2 \text{Tr}(Y_e Y_e^\dagger) - \frac{9}{2}\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d Y_d^\dagger) - \frac{9}{4}\text{Tr}(\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger) + 3\text{Tr}(\epsilon_d\epsilon_u^\dagger Y_u Y_d^\dagger) \\
& - \frac{9}{4}\text{Tr}(\epsilon_d Y_d^\dagger Y_d\epsilon_d^\dagger) - \frac{3}{2}\text{Tr}(\epsilon_e\epsilon_e^\dagger Y_e Y_e^\dagger) - \frac{3}{4}\text{Tr}(\epsilon_e Y_e^\dagger Y_e\epsilon_e^\dagger) + 3\text{Tr}(\epsilon_u\epsilon_d^\dagger Y_d Y_u^\dagger) \\
& - \frac{27}{4}\text{Tr}(\epsilon_u\epsilon_u^\dagger\epsilon_u\epsilon_u^\dagger) - \frac{9}{2}\text{Tr}(\epsilon_u\epsilon_u^\dagger Y_u Y_u^\dagger) + \frac{3}{2}\text{Tr}(\epsilon_u Y_d^\dagger Y_d\epsilon_u^\dagger) - \frac{9}{4}\text{Tr}(\epsilon_u Y_u^\dagger Y_u\epsilon_u^\dagger) \\
& - \frac{27}{4}\text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) - \frac{9}{4}\text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) - \frac{9}{4}\text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) \\
& + \frac{1}{8}\epsilon_e \left( 24\lambda_1\lambda_6 + 12\lambda_3\lambda_6 + 12\lambda_4\lambda_6 + 12\lambda_5\lambda_6 + 24\lambda_2\lambda_7 + 12\lambda_3\lambda_7 + 12\lambda_4\lambda_7 + 12\lambda_5\lambda_7 \right) \\
& + 5 \left( 32g_3^2 + 9g_2^2 + g_1^2 \right) \text{Tr}(Y_d\epsilon_d^\dagger) + 15 \left( g_1^2 + g_2^2 \right) \text{Tr}(Y_e\epsilon_e^\dagger) + 17g_1^2 \text{Tr}(Y_u\epsilon_u^\dagger) + 45g_2^2 \text{Tr}(Y_u\epsilon_u^\dagger) \\
& + 160g_3^2 \text{Tr}(Y_u\epsilon_u^\dagger) - 54\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d\epsilon_d^\dagger) + 6\text{Tr}(\epsilon_d\epsilon_u^\dagger Y_u\epsilon_d^\dagger) - 18\text{Tr}(\epsilon_e\epsilon_e^\dagger Y_e\epsilon_e^\dagger) \\
& + 6\text{Tr}(\epsilon_u\epsilon_d^\dagger Y_d\epsilon_u^\dagger) - 54\text{Tr}(\epsilon_u\epsilon_u^\dagger Y_u\epsilon_u^\dagger) - 36\text{Tr}(Y_d\epsilon_d^\dagger Y_d Y_d^\dagger) + 6\text{Tr}(Y_d\epsilon_u^\dagger Y_u Y_d^\dagger) \\
& - 18\text{Tr}(Y_d Y_d^\dagger Y_d\epsilon_d^\dagger) + 6\text{Tr}(Y_d Y_u^\dagger Y_u\epsilon_d^\dagger) - 12\text{Tr}(Y_e\epsilon_e^\dagger Y_e Y_e^\dagger) - 6\text{Tr}(Y_e Y_e^\dagger Y_e\epsilon_e^\dagger)
\end{aligned}$$



$$- 54\text{Tr}\left(Y_u\epsilon_u^\dagger Y_u Y_u^\dagger\right) \quad (31)$$

$$\begin{aligned} \beta_{\epsilon_d}^{(1)} = & \frac{1}{2}\left(3\epsilon_d\epsilon_d^\dagger\epsilon_d + \epsilon_d\epsilon_u^\dagger\epsilon_u + \epsilon_d Y_d^\dagger Y_d - 3\epsilon_d Y_u^\dagger Y_u + 2Y_d Y_d^\dagger \epsilon_d - 4Y_d Y_u^\dagger \epsilon_u\right. \\ & + 6Y_d \text{Tr}\left(\epsilon_d Y_d^\dagger\right) + 2Y_d \text{Tr}\left(\epsilon_e Y_e^\dagger\right) + 6Y_d \text{Tr}\left(\epsilon_u Y_u^\dagger\right) \\ & \left. + \epsilon_d\left(3\text{Tr}\left(\epsilon_d\epsilon_d^\dagger\right) + 3\text{Tr}\left(Y_u Y_u^\dagger\right) - 8g_3^2 - \frac{1}{4}g_1^2 - \frac{9}{4}g_2^2 + \text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right)\right) \end{aligned} \quad (32)$$

$$\begin{aligned} \beta_{\epsilon_d}^{(2)} = & \frac{187}{80}g_1^2\epsilon_d\epsilon_d^\dagger\epsilon_d + \frac{135}{16}g_2^2\epsilon_d\epsilon_d^\dagger\epsilon_d + 16g_3^2\epsilon_d\epsilon_d^\dagger\epsilon_d - 12\lambda_2\epsilon_d\epsilon_d^\dagger\epsilon_d \\ & - 6\lambda_7\epsilon_d\epsilon_d^\dagger Y_d - \frac{53}{240}g_1^2\epsilon_d\epsilon_u^\dagger\epsilon_u + \frac{33}{16}g_2^2\epsilon_d\epsilon_u^\dagger\epsilon_u + \frac{16}{3}g_3^2\epsilon_d\epsilon_u^\dagger\epsilon_u \\ & - 2\lambda_3\epsilon_d\epsilon_u^\dagger\epsilon_u + 2\lambda_4\epsilon_d\epsilon_u^\dagger\epsilon_u - 6\lambda_7\epsilon_d Y_d^\dagger \epsilon_d + \frac{247}{240}g_1^2\epsilon_d Y_d^\dagger Y_d \\ & + \frac{33}{16}g_2^2\epsilon_d Y_d^\dagger Y_d + \frac{16}{3}g_3^2\epsilon_d Y_d^\dagger Y_d - 2\lambda_3\epsilon_d Y_d^\dagger Y_d - 4\lambda_4\epsilon_d Y_d^\dagger Y_d \\ & - \frac{79}{80}g_1^2\epsilon_d Y_u^\dagger Y_u + \frac{9}{16}g_2^2\epsilon_d Y_u^\dagger Y_u - 16g_3^2\epsilon_d Y_u^\dagger Y_u - 6\lambda_7 Y_d \epsilon_d^\dagger \epsilon_d \\ & - 6\lambda_5 Y_d \epsilon_d^\dagger Y_d + \frac{157}{120}g_1^2 Y_d Y_d^\dagger \epsilon_d + \frac{51}{8}g_2^2 Y_d Y_d^\dagger \epsilon_d + \frac{32}{3}g_3^2 Y_d Y_d^\dagger \epsilon_d \\ & - 4\lambda_3 Y_d Y_d^\dagger \epsilon_d - 2\lambda_4 Y_d Y_d^\dagger \epsilon_d - 6\lambda_6 Y_d Y_d^\dagger Y_d - \frac{23}{30}g_1^2 Y_d Y_u^\dagger \epsilon_u \\ & - \frac{3}{2}g_2^2 Y_d Y_u^\dagger \epsilon_u - \frac{64}{3}g_3^2 Y_d Y_u^\dagger \epsilon_u + 2\lambda_3 Y_d Y_u^\dagger \epsilon_u - 2\lambda_4 Y_d Y_u^\dagger \epsilon_u \\ & + \frac{3}{2}\epsilon_d\epsilon_d^\dagger\epsilon_d\epsilon_d^\dagger\epsilon_d - \epsilon_d\epsilon_d^\dagger\epsilon_d Y_u^\dagger Y_u - \frac{1}{4}\epsilon_d\epsilon_d^\dagger Y_d Y_d^\dagger \epsilon_d - \frac{1}{4}\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger\epsilon_d \\ & - \frac{1}{4}\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_u^\dagger\epsilon_u + \epsilon_d\epsilon_u^\dagger\epsilon_u Y_u^\dagger Y_u - \frac{1}{4}\epsilon_d\epsilon_u^\dagger Y_u Y_u^\dagger \epsilon_u + \frac{7}{4}\epsilon_d Y_d^\dagger \epsilon_d\epsilon_d^\dagger Y_d \\ & - 2\epsilon_d Y_d^\dagger \epsilon_d\epsilon_d^\dagger Y_u - \frac{1}{4}\epsilon_d Y_d^\dagger Y_d\epsilon_d^\dagger\epsilon_d - \frac{1}{4}\epsilon_d Y_d^\dagger Y_d Y_d^\dagger Y_d + \epsilon_d Y_d^\dagger Y_d Y_u^\dagger Y_u \\ & + \frac{7}{4}\epsilon_d Y_u^\dagger \epsilon_u\epsilon_u^\dagger Y_u - \frac{1}{4}\epsilon_d Y_u^\dagger Y_u\epsilon_d^\dagger\epsilon_d + \frac{11}{4}\epsilon_d Y_u^\dagger Y_u Y_u^\dagger Y_u + \frac{7}{4}Y_d\epsilon_d^\dagger\epsilon_d Y_d^\dagger \epsilon_d \\ & - Y_d\epsilon_d^\dagger\epsilon_d Y_u^\dagger \epsilon_u - \frac{1}{4}Y_d\epsilon_u^\dagger\epsilon_u Y_d^\dagger \epsilon_d + Y_d\epsilon_u^\dagger\epsilon_u Y_u^\dagger \epsilon_u \\ & - 2Y_d Y_d^\dagger \epsilon_d\epsilon_u^\dagger\epsilon_u + 2Y_d Y_d^\dagger \epsilon_d Y_d^\dagger Y_d - \frac{1}{4}Y_d Y_d^\dagger Y_d Y_d^\dagger \epsilon_d + Y_d Y_d^\dagger Y_d Y_u^\dagger \epsilon_u \\ & + 2Y_d Y_u^\dagger \epsilon_u\epsilon_u^\dagger\epsilon_u - \frac{1}{4}Y_d Y_u^\dagger Y_u Y_d^\dagger \epsilon_d + 3Y_d Y_u^\dagger Y_u Y_u^\dagger \epsilon_u \\ & - \frac{27}{4}\epsilon_d\epsilon_d^\dagger\epsilon_d \text{Tr}\left(\epsilon_d\epsilon_d^\dagger\right) + \frac{15}{4}\epsilon_d Y_u^\dagger Y_u \text{Tr}\left(\epsilon_d\epsilon_d^\dagger\right) + 6Y_d Y_u^\dagger \epsilon_u \text{Tr}\left(\epsilon_d\epsilon_d^\dagger\right) \\ & - \frac{9}{4}\epsilon_d\epsilon_d^\dagger Y_d \text{Tr}\left(\epsilon_d Y_d^\dagger\right) + \frac{15}{4}\epsilon_d\epsilon_u^\dagger Y_u \text{Tr}\left(\epsilon_d Y_d^\dagger\right) - \frac{9}{2}Y_d\epsilon_d^\dagger\epsilon_d \text{Tr}\left(\epsilon_d Y_d^\dagger\right) \\ & + 6Y_d\epsilon_u^\dagger\epsilon_u \text{Tr}\left(\epsilon_d Y_d^\dagger\right) - \frac{9}{4}\epsilon_d\epsilon_d^\dagger\epsilon_d \text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) + \frac{5}{4}\epsilon_d Y_u^\dagger Y_u \text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) \\ & + 2Y_d Y_u^\dagger \epsilon_u \text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - \frac{3}{4}\epsilon_d\epsilon_d^\dagger Y_d \text{Tr}\left(\epsilon_e Y_e^\dagger\right) + \frac{5}{4}\epsilon_d\epsilon_u^\dagger Y_u \text{Tr}\left(\epsilon_e Y_e^\dagger\right) \end{aligned}$$

$$\begin{aligned}
& -\frac{3}{2}Y_d\epsilon_d^\dagger\epsilon_d\text{Tr}(\epsilon_e Y_e^\dagger) + 2Y_d\epsilon_u^\dagger\epsilon_u\text{Tr}(\epsilon_e Y_e^\dagger) - \frac{9}{4}\epsilon_d\epsilon_d^\dagger\epsilon_u\text{Tr}(\epsilon_u\epsilon_u^\dagger) \\
& -\frac{9}{4}\epsilon_d Y_d^\dagger Y_d\text{Tr}(\epsilon_u\epsilon_u^\dagger) - \frac{9}{2}Y_d Y_d^\dagger \epsilon_d\text{Tr}(\epsilon_u\epsilon_u^\dagger) - \frac{9}{4}\epsilon_d\epsilon_d^\dagger Y_d\text{Tr}(\epsilon_u Y_u^\dagger) \\
& +\frac{15}{4}\epsilon_d\epsilon_u^\dagger Y_u\text{Tr}(\epsilon_u Y_u^\dagger) - \frac{9}{2}Y_d\epsilon_d^\dagger\epsilon_d\text{Tr}(\epsilon_u Y_u^\dagger) + 6Y_d\epsilon_u^\dagger\epsilon_u\text{Tr}(\epsilon_u Y_u^\dagger) \\
& -\frac{27}{4}\epsilon_d Y_d^\dagger \epsilon_d\text{Tr}(Y_d\epsilon_d^\dagger) - \frac{9}{4}\epsilon_d Y_u^\dagger \epsilon_u\text{Tr}(Y_d\epsilon_d^\dagger) - \frac{9}{4}\epsilon_d\epsilon_u^\dagger\epsilon_u\text{Tr}(Y_d Y_d^\dagger) \\
& -\frac{9}{4}\epsilon_d Y_d^\dagger Y_d\text{Tr}(Y_d Y_d^\dagger) - \frac{9}{2}Y_d Y_d^\dagger \epsilon_d\text{Tr}(Y_d Y_d^\dagger) - \frac{9}{4}\epsilon_d Y_d^\dagger \epsilon_d\text{Tr}(Y_e\epsilon_e^\dagger) \\
& -\frac{3}{4}\epsilon_d Y_u^\dagger \epsilon_u\text{Tr}(Y_e\epsilon_e^\dagger) - \frac{3}{4}\epsilon_d\epsilon_u^\dagger\epsilon_u\text{Tr}(Y_e Y_e^\dagger) - \frac{3}{4}\epsilon_d Y_d^\dagger Y_d\text{Tr}(Y_e Y_e^\dagger) \\
& -\frac{3}{2}Y_d Y_d^\dagger \epsilon_d\text{Tr}(Y_e Y_e^\dagger) - \frac{27}{4}\epsilon_d Y_d^\dagger \epsilon_d\text{Tr}(Y_u\epsilon_u^\dagger) - \frac{9}{4}\epsilon_d Y_u^\dagger \epsilon_u\text{Tr}(Y_u\epsilon_u^\dagger) \\
& -\frac{27}{4}\epsilon_d\epsilon_d^\dagger\epsilon_d\text{Tr}(Y_u Y_u^\dagger) + \frac{15}{4}\epsilon_d Y_u^\dagger Y_u\text{Tr}(Y_u Y_u^\dagger) + 6Y_d Y_u^\dagger \epsilon_u\text{Tr}(Y_u Y_u^\dagger) \\
& +\frac{1}{8}Y_d(24\lambda_1\lambda_6 + 12\lambda_3\lambda_6 + 12\lambda_4\lambda_6 + 12\lambda_5\lambda_6 + 24\lambda_2\lambda_7 + 12\lambda_3\lambda_7 + 12\lambda_4\lambda_7 + 12\lambda_5\lambda_7 \\
& + 5(32g_3^2 + 9g_2^2 + g_1^2)\text{Tr}(\epsilon_d Y_d^\dagger) + 15(g_1^2 + g_2^2)\text{Tr}(\epsilon_e Y_e^\dagger) + 17g_1^2\text{Tr}(\epsilon_u Y_u^\dagger) + 45g_2^2\text{Tr}(\epsilon_u Y_u^\dagger) \\
& + 160g_3^2\text{Tr}(\epsilon_u Y_u^\dagger) - 18\text{Tr}(\epsilon_d\epsilon_d^\dagger\epsilon_d Y_d^\dagger) + 6\text{Tr}(\epsilon_d\epsilon_u^\dagger\epsilon_u Y_d^\dagger) - 36\text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d\epsilon_d^\dagger) \\
& - 54\text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger) + 6\text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u\epsilon_d^\dagger) + 6\text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger) - 6\text{Tr}(\epsilon_e\epsilon_e^\dagger\epsilon_e Y_e^\dagger) \\
& - 12\text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e\epsilon_e^\dagger) - 18\text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger) + 6\text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger) - 54\text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u\epsilon_u^\dagger) \\
& - 54\text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger)) \\
& +\epsilon_d\left(-\frac{113}{600}g_1^4 - \frac{27}{20}g_1^2 g_2^2 - \frac{21}{4}g_2^4 + \frac{31}{15}g_1^2 g_3^2 + 9g_2^2 g_3^2 - 108g_3^4 + 6\lambda_2^2 + \lambda_3^2 + \lambda_3\lambda_4 + \lambda_4^2 + \frac{3}{2}\lambda_5^2 + \frac{3}{2}\lambda_6^2\right. \\
& +\frac{9}{2}\lambda_7^2 + \frac{5}{8}(32g_3^2 + 9g_2^2 + g_1^2)\text{Tr}(\epsilon_d\epsilon_d^\dagger) + \frac{15}{8}(g_1^2 + g_2^2)\text{Tr}(\epsilon_e\epsilon_e^\dagger) + \frac{17}{8}g_1^2\text{Tr}(Y_u Y_u^\dagger) \\
& +\frac{45}{8}g_2^2\text{Tr}(Y_u Y_u^\dagger) + 20g_3^2\text{Tr}(Y_u Y_u^\dagger) - \frac{27}{4}\text{Tr}(\epsilon_d\epsilon_d^\dagger\epsilon_d\epsilon_d^\dagger) - \frac{9}{2}\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d Y_d^\dagger) \\
& -\frac{9}{4}\text{Tr}(\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger) + 3\text{Tr}(\epsilon_d\epsilon_u^\dagger Y_u Y_d^\dagger) - \frac{9}{4}\text{Tr}(\epsilon_d Y_d^\dagger Y_d\epsilon_d^\dagger) + \frac{3}{2}\text{Tr}(\epsilon_d Y_u^\dagger Y_u\epsilon_d^\dagger) \\
& -\frac{9}{4}\text{Tr}(\epsilon_e\epsilon_e^\dagger\epsilon_e\epsilon_e^\dagger) - \frac{3}{2}\text{Tr}(\epsilon_e\epsilon_e^\dagger Y_e Y_e^\dagger) - \frac{3}{4}\text{Tr}(\epsilon_e Y_e^\dagger Y_e\epsilon_e^\dagger) + 3\text{Tr}(\epsilon_u\epsilon_d^\dagger Y_d Y_u^\dagger) \\
& \left.-\frac{9}{2}\text{Tr}(\epsilon_u\epsilon_u^\dagger Y_u Y_u^\dagger) - \frac{9}{4}\text{Tr}(\epsilon_u Y_u^\dagger Y_u\epsilon_u^\dagger) - \frac{9}{4}\text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) - \frac{27}{4}\text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger)\right) \tag{33}
\end{aligned}$$

$$\beta_{\epsilon_e}^{(1)} = +\frac{3}{2}\epsilon_e\epsilon_e^\dagger\epsilon_e + \frac{1}{2}\epsilon_e Y_e^\dagger Y_e + Y_e Y_e^\dagger \epsilon_e + 3Y_e\text{Tr}(\epsilon_d Y_d^\dagger) + Y_e\text{Tr}(\epsilon_e Y_e^\dagger) + 3Y_e\text{Tr}(\epsilon_u Y_u^\dagger)$$

$$+ \epsilon_e\left(3\text{Tr}(\epsilon_d\epsilon_d^\dagger) + 3\text{Tr}(Y_u Y_u^\dagger) - \frac{9}{4}g_1^2 - \frac{9}{4}g_2^2 + \text{Tr}(\epsilon_e\epsilon_e^\dagger)\right) \tag{34}$$

$$\beta_{\epsilon_e}^{(2)} = +\frac{1}{80}\left(-480\lambda_7\epsilon_e\epsilon_e^\dagger Y_e - 480\lambda_7\epsilon_e Y_e^\dagger \epsilon_e + 309g_1^2\epsilon_e Y_e^\dagger Y_e + 165g_2^2\epsilon_e Y_e^\dagger Y_e\right)$$

$$\begin{aligned}
& -160\lambda_3\epsilon_e Y_e^\dagger Y_e - 320\lambda_4\epsilon_e Y_e^\dagger Y_e - 480\lambda_7 Y_e \epsilon_e^\dagger \epsilon_e - 480\lambda_5 Y_e \epsilon_e^\dagger Y_e \\
& + 78g_1^2 Y_e Y_e^\dagger \epsilon_e + 510g_2^2 Y_e Y_e^\dagger \epsilon_e - 320\lambda_3 Y_e Y_e^\dagger \epsilon_e - 160\lambda_4 Y_e Y_e^\dagger \epsilon_e \\
& - 480\lambda_6 Y_e Y_e^\dagger Y_e + 120\epsilon_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger \epsilon_e - 20\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger \epsilon_e + 140\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger Y_e \\
& - 20\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger \epsilon_e - 20\epsilon_e Y_e^\dagger Y_e Y_e^\dagger Y_e + 140Y_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger \epsilon_e + 160Y_e Y_e^\dagger \epsilon_e Y_e^\dagger Y_e \\
& - 20Y_e Y_e^\dagger Y_e Y_e^\dagger \epsilon_e - 180\epsilon_e \epsilon_e^\dagger Y_e \text{Tr}(\epsilon_d Y_d^\dagger) - 360Y_e \epsilon_e^\dagger \epsilon_e \text{Tr}(\epsilon_d Y_d^\dagger) \\
& - 60\epsilon_e \epsilon_e^\dagger Y_e \text{Tr}(\epsilon_e Y_e^\dagger) - 120Y_e \epsilon_e^\dagger \epsilon_e \text{Tr}(\epsilon_e Y_e^\dagger) - 180\epsilon_e Y_e^\dagger Y_e \text{Tr}(\epsilon_u \epsilon_u^\dagger) \\
& - 360Y_e Y_e^\dagger \epsilon_e \text{Tr}(\epsilon_u \epsilon_u^\dagger) - 180\epsilon_e \epsilon_e^\dagger Y_e \text{Tr}(\epsilon_u Y_u^\dagger) - 360Y_e \epsilon_e^\dagger \epsilon_e \text{Tr}(\epsilon_u Y_u^\dagger) \\
& - 540\epsilon_e Y_e^\dagger \epsilon_e \text{Tr}(Y_d \epsilon_d^\dagger) - 180\epsilon_e Y_e^\dagger Y_e \text{Tr}(Y_d Y_d^\dagger) - 360Y_e Y_e^\dagger \epsilon_e \text{Tr}(Y_d Y_d^\dagger) \\
& - 180\epsilon_e Y_e^\dagger \epsilon_e \text{Tr}(Y_e \epsilon_e^\dagger) - 60\epsilon_e Y_e^\dagger Y_e \text{Tr}(Y_e Y_e^\dagger) - 120Y_e Y_e^\dagger \epsilon_e \text{Tr}(Y_e Y_e^\dagger) \\
& - 540\epsilon_e Y_e^\dagger \epsilon_e \text{Tr}(Y_u \epsilon_u^\dagger) \\
& + 3\epsilon_e \epsilon_e^\dagger \epsilon_e (129g_1^2 - 180\text{Tr}(\epsilon_d \epsilon_d^\dagger) - 180\text{Tr}(Y_u Y_u^\dagger) + 225g_2^2 - 320\lambda_2 - 60\text{Tr}(\epsilon_e \epsilon_e^\dagger)) \\
& + 10Y_e (24\lambda_1 \lambda_6 + 12\lambda_3 \lambda_6 + 12\lambda_4 \lambda_6 + 12\lambda_5 \lambda_6 + 24\lambda_2 \lambda_7 + 12\lambda_3 \lambda_7 + 12\lambda_4 \lambda_7 + 12\lambda_5 \lambda_7 \\
& + 5(32g_3^2 + 9g_2^2 + g_1^2)\text{Tr}(\epsilon_d Y_d^\dagger) + 15(g_1^2 + g_2^2)\text{Tr}(\epsilon_e Y_e^\dagger) + 17g_1^2 \text{Tr}(\epsilon_u Y_u^\dagger) + 45g_2^2 \text{Tr}(\epsilon_u Y_u^\dagger) \\
& + 160g_3^2 \text{Tr}(\epsilon_u Y_u^\dagger) - 18\text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d Y_d^\dagger) + 6\text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_d^\dagger) - 36\text{Tr}(\epsilon_d Y_d^\dagger \epsilon_d \epsilon_d^\dagger) \\
& - 54\text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger) + 6\text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) + 6\text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger) - 6\text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e Y_e^\dagger) \\
& - 12\text{Tr}(\epsilon_e Y_e^\dagger \epsilon_e \epsilon_e^\dagger) - 18\text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger) + 6\text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger) - 54\text{Tr}(\epsilon_u Y_u^\dagger \epsilon_u \epsilon_u^\dagger) \\
& - 54\text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger)) \\
& + \epsilon_e \left( \frac{1449}{200} g_1^4 + \frac{27}{20} g_1^2 g_2^2 - \frac{21}{4} g_2^4 + 6\lambda_2^2 + \lambda_3^2 + \lambda_3 \lambda_4 + \lambda_4^2 + \frac{3}{2} \lambda_5^2 + \frac{3}{2} \lambda_6^2 + \frac{9}{2} \lambda_7^2 \right. \\
& + \frac{5}{8} (32g_3^2 + 9g_2^2 + g_1^2) \text{Tr}(\epsilon_d \epsilon_d^\dagger) + \frac{15}{8} (g_1^2 + g_2^2) \text{Tr}(\epsilon_e \epsilon_e^\dagger) + \frac{17}{8} g_1^2 \text{Tr}(Y_u Y_u^\dagger) + \frac{45}{8} g_2^2 \text{Tr}(Y_u Y_u^\dagger) \\
& + 20g_3^2 \text{Tr}(Y_u Y_u^\dagger) - \frac{27}{4} \text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger) - \frac{9}{2} \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger) - \frac{9}{4} \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) \\
& + 3\text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) - \frac{9}{4} \text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger) + \frac{3}{2} \text{Tr}(\epsilon_d Y_u^\dagger Y_u \epsilon_d^\dagger) - \frac{9}{4} \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger) \\
& - \frac{3}{2} \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger) - \frac{3}{4} \text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger) + 3\text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) - \frac{9}{2} \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger) \\
& \left. - \frac{9}{4} \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) - \frac{9}{4} \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) - \frac{27}{4} \text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger) \right)
\end{aligned} \tag{35}$$

### 3.4 Scalar Mass Terms

$$\begin{aligned}
\beta_{m_{12}}^{(1)} = & -12\lambda_6 m_1^2 - \frac{9}{10}g_1^2 m_{12} - \frac{9}{2}g_2^2 m_{12} + 2\lambda_3 m_{12} + 4\lambda_4 m_{12} + 6\lambda_5 m_{12} - 12\lambda_7 m_2^2 - 6m_1^2 \text{Tr}(\epsilon_d Y_d^\dagger) \\
& - 2m_1^2 \text{Tr}(\epsilon_e Y_e^\dagger) - 6m_2^2 \text{Tr}(\epsilon_u Y_u^\dagger) - 6m_2^2 \text{Tr}(Y_d \epsilon_d^\dagger) + 6m_{12} \text{Tr}(Y_d Y_d^\dagger) - 2m_2^2 \text{Tr}(Y_e \epsilon_e^\dagger) \\
& + 2m_{12} \text{Tr}(Y_e Y_e^\dagger) - 6m_1^2 \text{Tr}(Y_u \epsilon_u^\dagger) + 6m_{12} \text{Tr}(Y_u Y_u^\dagger)
\end{aligned} \tag{36}$$

$$\begin{aligned}
\beta_{m_{12}}^{(2)} = & -\frac{72}{5}g_1^2 \lambda_6 m_1^2 - 72g_2^2 \lambda_6 m_1^2 + 72\lambda_1 \lambda_6 m_1^2 + 24\lambda_3 \lambda_6 m_1^2 + 24\lambda_4 \lambda_6 m_1^2 + 24\lambda_5 \lambda_6 m_1^2 + 12\lambda_3 \lambda_7 m_1^2 \\
& + 12\lambda_4 \lambda_7 m_1^2 + 12\lambda_5 \lambda_7 m_1^2 + \frac{1377}{400}g_1^4 m_{12} + \frac{9}{8}g_1^2 g_2^2 m_{12} - \frac{243}{16}g_2^4 m_{12} + 6\lambda_1^2 m_{12} + 6\lambda_2^2 m_{12} + \frac{12}{5}g_1^2 \lambda_3 m_{12} \\
& + 12g_2^2 \lambda_3 m_{12} - 12\lambda_1 \lambda_3 m_{12} - 12\lambda_2 \lambda_3 m_{12} + \frac{24}{5}g_1^2 \lambda_4 m_{12} + 24g_2^2 \lambda_4 m_{12} - 12\lambda_1 \lambda_4 m_{12} - 12\lambda_2 \lambda_4 m_{12} \\
& - 6\lambda_3 \lambda_4 m_{12} + \frac{36}{5}g_1^2 \lambda_5 m_{12} + 36g_2^2 \lambda_5 m_{12} - 12\lambda_1 \lambda_5 m_{12} - 12\lambda_2 \lambda_5 m_{12} - 12\lambda_3 \lambda_5 m_{12} - 12\lambda_4 \lambda_5 m_{12} \\
& + 3\lambda_5^2 m_{12} - 12\lambda_6^2 m_{12} - 36\lambda_6 \lambda_7 m_{12} - 12\lambda_7^2 m_{12} + 12\lambda_3 \lambda_6 m_2^2 + 12\lambda_4 \lambda_6 m_2^2 + 12\lambda_5 \lambda_6 m_2^2 \\
& - \frac{72}{5}g_1^2 \lambda_7 m_2^2 - 72g_2^2 \lambda_7 m_2^2 + 72\lambda_2 \lambda_7 m_2^2 + 24\lambda_3 \lambda_7 m_2^2 + 24\lambda_4 \lambda_7 m_2^2 + 24\lambda_5 \lambda_7 m_2^2 \\
& + \left( -12\lambda_4 m_{12} - 18\lambda_5 m_{12} + 20g_3^2 m_{12} - 6\lambda_3 m_{12} + 72\lambda_7 m_2^2 + \frac{45}{8}g_2^2 m_{12} + \frac{5}{8}g_1^2 m_{12} \right) \text{Tr}(\epsilon_d \epsilon_d^\dagger) \\
& + 6 \left( 2\lambda_4 (m_1^2 + m_2^2) + 3 \left( -2(\lambda_6 + \lambda_7) m_{12} + \lambda_5 (m_1^2 + m_2^2) \right) + \lambda_3 (m_1^2 + m_2^2) \right) \text{Tr}(\epsilon_d Y_d^\dagger) + \frac{15}{8}g_1^2 m_{12} \text{Tr}(\epsilon_e \epsilon_e^\dagger) \\
& + \frac{15}{8}g_2^2 m_{12} \text{Tr}(\epsilon_e \epsilon_e^\dagger) - 2\lambda_3 m_{12} \text{Tr}(\epsilon_e \epsilon_e^\dagger) - 4\lambda_4 m_{12} \text{Tr}(\epsilon_e \epsilon_e^\dagger) - 6\lambda_5 m_{12} \text{Tr}(\epsilon_e \epsilon_e^\dagger) \\
& + 24\lambda_7 m_2^2 \text{Tr}(\epsilon_e \epsilon_e^\dagger) + 2\lambda_3 m_1^2 \text{Tr}(\epsilon_e Y_e^\dagger) + 4\lambda_4 m_1^2 \text{Tr}(\epsilon_e Y_e^\dagger) + 6\lambda_5 m_1^2 \text{Tr}(\epsilon_e Y_e^\dagger) \\
& - 12\lambda_6 m_{12} \text{Tr}(\epsilon_e Y_e^\dagger) - 12\lambda_7 m_{12} \text{Tr}(\epsilon_e Y_e^\dagger) + 2\lambda_3 m_2^2 \text{Tr}(\epsilon_e Y_e^\dagger) + 4\lambda_4 m_2^2 \text{Tr}(\epsilon_e Y_e^\dagger) \\
& + 6\lambda_5 m_2^2 \text{Tr}(\epsilon_e Y_e^\dagger) + 72\lambda_6 m_1^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger) + \frac{17}{8}g_1^2 m_{12} \text{Tr}(\epsilon_u \epsilon_u^\dagger) + \frac{45}{8}g_2^2 m_{12} \text{Tr}(\epsilon_u \epsilon_u^\dagger) \\
& + 20g_3^2 m_{12} \text{Tr}(\epsilon_u \epsilon_u^\dagger) - 6\lambda_3 m_{12} \text{Tr}(\epsilon_u \epsilon_u^\dagger) - 12\lambda_4 m_{12} \text{Tr}(\epsilon_u \epsilon_u^\dagger) - 18\lambda_5 m_{12} \text{Tr}(\epsilon_u \epsilon_u^\dagger) \\
& + 6\lambda_3 m_1^2 \text{Tr}(\epsilon_u Y_u^\dagger) + 12\lambda_4 m_1^2 \text{Tr}(\epsilon_u Y_u^\dagger) + 18\lambda_5 m_1^2 \text{Tr}(\epsilon_u Y_u^\dagger) - 36\lambda_6 m_{12} \text{Tr}(\epsilon_u Y_u^\dagger) \\
& - 36\lambda_7 m_{12} \text{Tr}(\epsilon_u Y_u^\dagger) + 6\lambda_3 m_2^2 \text{Tr}(\epsilon_u Y_u^\dagger) + 12\lambda_4 m_2^2 \text{Tr}(\epsilon_u Y_u^\dagger) + 18\lambda_5 m_2^2 \text{Tr}(\epsilon_u Y_u^\dagger) \\
& + 6\lambda_3 m_1^2 \text{Tr}(Y_d \epsilon_d^\dagger) + 12\lambda_4 m_1^2 \text{Tr}(Y_d \epsilon_d^\dagger) + 18\lambda_5 m_1^2 \text{Tr}(Y_d \epsilon_d^\dagger) + 6\lambda_3 m_2^2 \text{Tr}(Y_d \epsilon_d^\dagger) \\
& + 12\lambda_4 m_2^2 \text{Tr}(Y_d \epsilon_d^\dagger) + 18\lambda_5 m_2^2 \text{Tr}(Y_d \epsilon_d^\dagger) + 72\lambda_6 m_1^2 \text{Tr}(Y_d Y_d^\dagger) + \frac{5}{8}g_1^2 m_{12} \text{Tr}(Y_d Y_d^\dagger) \\
& + \frac{45}{8}g_2^2 m_{12} \text{Tr}(Y_d Y_d^\dagger) + 20g_3^2 m_{12} \text{Tr}(Y_d Y_d^\dagger) - 6\lambda_3 m_{12} \text{Tr}(Y_d Y_d^\dagger) - 12\lambda_4 m_{12} \text{Tr}(Y_d Y_d^\dagger) \\
& - 18\lambda_5 m_{12} \text{Tr}(Y_d Y_d^\dagger) + 2\lambda_3 m_1^2 \text{Tr}(Y_e \epsilon_e^\dagger) + 4\lambda_4 m_1^2 \text{Tr}(Y_e \epsilon_e^\dagger) + 6\lambda_5 m_1^2 \text{Tr}(Y_e \epsilon_e^\dagger) \\
& + 2\lambda_3 m_2^2 \text{Tr}(Y_e \epsilon_e^\dagger) + 4\lambda_4 m_2^2 \text{Tr}(Y_e \epsilon_e^\dagger) + 6\lambda_5 m_2^2 \text{Tr}(Y_e \epsilon_e^\dagger) + 24\lambda_6 m_1^2 \text{Tr}(Y_e Y_e^\dagger)
\end{aligned}$$

$$\begin{aligned}
& + \frac{15}{8}g_1^2m_{12}\text{Tr}(Y_eY_e^\dagger) + \frac{15}{8}g_2^2m_{12}\text{Tr}(Y_eY_e^\dagger) - 2\lambda_3m_{12}\text{Tr}(Y_eY_e^\dagger) - 4\lambda_4m_{12}\text{Tr}(Y_eY_e^\dagger) \\
& - 6\lambda_5m_{12}\text{Tr}(Y_eY_e^\dagger) + 6\lambda_3m_1^2\text{Tr}(Y_u\epsilon_u^\dagger) + 12\lambda_4m_1^2\text{Tr}(Y_u\epsilon_u^\dagger) + 18\lambda_5m_1^2\text{Tr}(Y_u\epsilon_u^\dagger) \\
& + 6\lambda_3m_2^2\text{Tr}(Y_u\epsilon_u^\dagger) + 12\lambda_4m_2^2\text{Tr}(Y_u\epsilon_u^\dagger) + 18\lambda_5m_2^2\text{Tr}(Y_u\epsilon_u^\dagger) + \frac{17}{8}g_1^2m_{12}\text{Tr}(Y_uY_u^\dagger) \\
& + \frac{45}{8}g_2^2m_{12}\text{Tr}(Y_uY_u^\dagger) + 20g_3^2m_{12}\text{Tr}(Y_uY_u^\dagger) - 6\lambda_3m_{12}\text{Tr}(Y_uY_u^\dagger) - 12\lambda_4m_{12}\text{Tr}(Y_uY_u^\dagger) \\
& - 18\lambda_5m_{12}\text{Tr}(Y_uY_u^\dagger) + 72\lambda_7m_2^2\text{Tr}(Y_uY_u^\dagger) - \frac{27}{4}m_{12}\text{Tr}(\epsilon_d\epsilon_d^\dagger\epsilon_d\epsilon_d^\dagger) - 9m_{12}\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d Y_d^\dagger) \\
& - \frac{33}{2}m_{12}\text{Tr}(\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger) + 12m_1^2\text{Tr}(\epsilon_d\epsilon_u^\dagger\epsilon_u Y_d^\dagger) + 12m_2^2\text{Tr}(\epsilon_d\epsilon_u^\dagger Y_u\epsilon_d^\dagger) \\
& + 6m_{12}\text{Tr}(\epsilon_d\epsilon_u^\dagger Y_u Y_d^\dagger) - \frac{9}{2}m_{12}\text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger) + 12m_2^2\text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) \\
& - 24m_{12}\text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u Y_d^\dagger) + \frac{3}{2}m_{12}\text{Tr}(\epsilon_d Y_u^\dagger Y_u \epsilon_d^\dagger) + 12m_2^2\text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger) \\
& - \frac{9}{4}m_{12}\text{Tr}(\epsilon_e\epsilon_e^\dagger\epsilon_e\epsilon_e^\dagger) - 3m_{12}\text{Tr}(\epsilon_e\epsilon_e^\dagger Y_e Y_e^\dagger) - \frac{3}{2}m_{12}\text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger) \\
& + 12m_1^2\text{Tr}(\epsilon_u\epsilon_u^\dagger Y_d \epsilon_u^\dagger) + 6m_{12}\text{Tr}(\epsilon_u\epsilon_u^\dagger Y_d Y_u^\dagger) - \frac{27}{4}m_{12}\text{Tr}(\epsilon_u\epsilon_u^\dagger\epsilon_u\epsilon_u^\dagger) \\
& - 9m_{12}\text{Tr}(\epsilon_u\epsilon_u^\dagger Y_u Y_u^\dagger) + \frac{3}{2}m_{12}\text{Tr}(\epsilon_u Y_d^\dagger Y_d \epsilon_u^\dagger) + 12m_1^2\text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger) \\
& - \frac{9}{2}m_{12}\text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) + 12m_1^2\text{Tr}(Y_d\epsilon_u^\dagger Y_u Y_d^\dagger) - \frac{27}{4}m_{12}\text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) \\
& + 12m_2^2\text{Tr}(Y_d Y_u^\dagger Y_u \epsilon_d^\dagger) - \frac{33}{2}m_{12}\text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) - \frac{9}{4}m_{12}\text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) - \frac{27}{4}m_{12}\text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger)
\end{aligned} \tag{37}$$

$$\begin{aligned}
\beta_{m_1^2}^{(1)} &= -\frac{9}{10}g_1^2m_1^2 - \frac{9}{2}g_2^2m_1^2 + 12\lambda_1m_1^2 - 6\lambda_6m_{12} + 4\lambda_3m_2^2 + 2\lambda_4m_2^2 + 6m_1^2\text{Tr}(\epsilon_u\epsilon_u^\dagger) - 6m_{12}\text{Tr}(\epsilon_u Y_u^\dagger) \\
& + 6m_1^2\text{Tr}(Y_d Y_d^\dagger) + 2m_1^2\text{Tr}(Y_e Y_e^\dagger)
\end{aligned} \tag{38}$$

$$\begin{aligned}
\beta_{m_1^2}^{(2)} &= +\frac{1737}{400}g_1^4m_1^2 + \frac{9}{8}g_1^2g_2^2m_1^2 - \frac{123}{16}g_2^4m_1^2 + \frac{72}{5}g_1^2\lambda_1m_1^2 + 72g_2^2\lambda_1m_1^2 - 60\lambda_1^2m_1^2 - 2\lambda_3^2m_1^2 \\
& - 2\lambda_3\lambda_4m_1^2 - 2\lambda_4^2m_1^2 - 3\lambda_5^2m_1^2 - 27\lambda_6^2m_1^2 + 3\lambda_7^2m_1^2 - \frac{36}{5}g_1^2\lambda_6m_{12} - 36g_2^2\lambda_6m_{12} \\
& + 36\lambda_1\lambda_6m_{12} + 12\lambda_3\lambda_6m_{12} + 12\lambda_4\lambda_6m_{12} + 12\lambda_5\lambda_6m_{12} + 6\lambda_3\lambda_7m_{12} + 6\lambda_4\lambda_7m_{12} + 6\lambda_5\lambda_7m_{12} + \frac{9}{10}g_1^4m_2^2 \\
& + \frac{15}{2}g_2^4m_2^2 + \frac{24}{5}g_1^2\lambda_3m_2^2 + 24g_2^2\lambda_3m_2^2 - 8\lambda_3^2m_2^2 + \frac{12}{5}g_1^2\lambda_4m_2^2 + 12g_2^2\lambda_4m_2^2 - 8\lambda_3\lambda_4m_2^2 \\
& - 8\lambda_4^2m_2^2 - 12\lambda_5^2m_2^2 - 18\lambda_6^2m_2^2 - 18\lambda_7^2m_2^2 + 6\left(-2(2\lambda_3 + \lambda_4)m_2^2 + 3\lambda_6m_{12}\right)\text{Tr}(\epsilon_d\epsilon_d^\dagger) \\
& - 6\left(-\left(2\lambda_3 + 6\lambda_1 + \lambda_4\right)m_{12} + 3\lambda_6\left(m_1^2 + m_2^2\right)\right)\text{Tr}(\epsilon_d Y_d^\dagger) + 6\lambda_6m_{12}\text{Tr}(\epsilon_e\epsilon_e^\dagger) - 8\lambda_3m_2^2\text{Tr}(\epsilon_e\epsilon_e^\dagger) \\
& - 4\lambda_4m_2^2\text{Tr}(\epsilon_e\epsilon_e^\dagger) - 6\lambda_6m_1^2\text{Tr}(\epsilon_e Y_e^\dagger) + 12\lambda_1m_{12}\text{Tr}(\epsilon_e Y_e^\dagger) + 4\lambda_3m_{12}\text{Tr}(\epsilon_e Y_e^\dagger) \\
& + 2\lambda_4m_{12}\text{Tr}(\epsilon_e Y_e^\dagger) - 6\lambda_6m_2^2\text{Tr}(\epsilon_e Y_e^\dagger) + \frac{17}{4}g_1^2m_1^2\text{Tr}(\epsilon_u\epsilon_u^\dagger) + \frac{45}{4}g_2^2m_1^2\text{Tr}(\epsilon_u\epsilon_u^\dagger)
\end{aligned}$$

$$\begin{aligned}
& + 40g_3^2 m_1^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger) - 72\lambda_1 m_1^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger) + 18\lambda_6 m_{12} \text{Tr}(\epsilon_u \epsilon_u^\dagger) - 18\lambda_6 m_1^2 \text{Tr}(\epsilon_u Y_u^\dagger) \\
& + 36\lambda_1 m_{12} \text{Tr}(\epsilon_u Y_u^\dagger) + 12\lambda_3 m_{12} \text{Tr}(\epsilon_u Y_u^\dagger) + 6\lambda_4 m_{12} \text{Tr}(\epsilon_u Y_u^\dagger) - 18\lambda_6 m_2^2 \text{Tr}(\epsilon_u Y_u^\dagger) \\
& - 18\lambda_6 m_1^2 \text{Tr}(Y_d \epsilon_d^\dagger) - 18\lambda_6 m_2^2 \text{Tr}(Y_d \epsilon_d^\dagger) + \frac{5}{4} g_1^2 m_1^2 \text{Tr}(Y_d Y_d^\dagger) + \frac{45}{4} g_2^2 m_1^2 \text{Tr}(Y_d Y_d^\dagger) \\
& + 40g_3^2 m_1^2 \text{Tr}(Y_d Y_d^\dagger) - 72\lambda_1 m_1^2 \text{Tr}(Y_d Y_d^\dagger) + 18\lambda_6 m_{12} \text{Tr}(Y_d Y_d^\dagger) - 6\lambda_6 m_1^2 \text{Tr}(Y_e \epsilon_e^\dagger) \\
& - 6\lambda_6 m_2^2 \text{Tr}(Y_e \epsilon_e^\dagger) + \frac{15}{4} g_1^2 m_1^2 \text{Tr}(Y_e Y_e^\dagger) + \frac{15}{4} g_2^2 m_1^2 \text{Tr}(Y_e Y_e^\dagger) - 24\lambda_1 m_1^2 \text{Tr}(Y_e Y_e^\dagger) \\
& + 6\lambda_6 m_{12} \text{Tr}(Y_e Y_e^\dagger) - 18\lambda_6 m_1^2 \text{Tr}(Y_u \epsilon_u^\dagger) - 18\lambda_6 m_2^2 \text{Tr}(Y_u \epsilon_u^\dagger) + 18\lambda_6 m_{12} \text{Tr}(Y_u Y_u^\dagger) \\
& - 24\lambda_3 m_2^2 \text{Tr}(Y_u Y_u^\dagger) - 12\lambda_4 m_2^2 \text{Tr}(Y_u Y_u^\dagger) - 9m_1^2 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger) \\
& - \frac{9}{2} m_1^2 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) + 12m_{12} \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u Y_d^\dagger) + 6m_1^2 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) \\
& - 12m_2^2 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) - \frac{9}{2} m_1^2 \text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger) - 3m_1^2 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger) \\
& - \frac{3}{2} m_1^2 \text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger) + 6m_1^2 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) - 12m_2^2 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) \\
& - \frac{27}{2} m_1^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger \epsilon_u \epsilon_u^\dagger) - 9m_2^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger) - 21m_1^2 \text{Tr}(\epsilon_u Y_d^\dagger Y_d \epsilon_u^\dagger) \\
& + 12m_{12} \text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger) - \frac{9}{2} m_1^2 \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) - \frac{27}{2} m_1^2 \text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) \\
& - \frac{9}{2} m_1^2 \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) - \frac{9}{2} m_1^2 \text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger)
\end{aligned} \tag{39}$$

$$\begin{aligned}
\beta_{m_2^2}^{(1)} & = +4\lambda_3 m_1^2 + 2\lambda_4 m_1^2 - 6\lambda_7 m_{12} - \frac{9}{10} g_1^2 m_2^2 - \frac{9}{2} g_2^2 m_2^2 + 12\lambda_2 m_2^2 + 6m_2^2 \text{Tr}(\epsilon_d \epsilon_d^\dagger) - 6m_{12} \text{Tr}(\epsilon_d Y_d^\dagger) \\
& + 2m_2^2 \text{Tr}(\epsilon_e \epsilon_e^\dagger) - 2m_{12} \text{Tr}(\epsilon_e Y_e^\dagger) + 6m_2^2 \text{Tr}(Y_u Y_u^\dagger)
\end{aligned} \tag{40}$$

$$\begin{aligned}
\beta_{m_2^2}^{(2)} & = +\frac{9}{10} g_1^4 m_1^2 + \frac{15}{2} g_2^4 m_1^2 + \frac{24}{5} g_1^2 \lambda_3 m_1^2 + 24g_2^2 \lambda_3 m_1^2 - 8\lambda_3^2 m_1^2 + \frac{12}{5} g_1^2 \lambda_4 m_1^2 + 12g_2^2 \lambda_4 m_1^2 \\
& - 8\lambda_3 \lambda_4 m_1^2 - 8\lambda_4^2 m_1^2 - 12\lambda_5^2 m_1^2 - 18\lambda_6^2 m_1^2 - 18\lambda_7^2 m_1^2 + 6\lambda_3 \lambda_6 m_{12} + 6\lambda_4 \lambda_6 m_{12} + 6\lambda_5 \lambda_6 m_{12} \\
& - \frac{36}{5} g_1^2 \lambda_7 m_{12} - 36g_2^2 \lambda_7 m_{12} + 36\lambda_2 \lambda_7 m_{12} + 12\lambda_3 \lambda_7 m_{12} + 12\lambda_4 \lambda_7 m_{12} + 12\lambda_5 \lambda_7 m_{12} + \frac{1737}{400} g_1^4 m_2^2 \\
& + \frac{9}{8} g_1^2 g_2^2 m_2^2 - \frac{123}{16} g_2^4 m_2^2 + \frac{72}{5} g_1^2 \lambda_2 m_2^2 + 72g_2^2 \lambda_2 m_2^2 - 60\lambda_2^2 m_2^2 - 2\lambda_3^2 m_2^2 - 2\lambda_3 \lambda_4 m_2^2 \\
& - 2\lambda_4^2 m_2^2 - 3\lambda_5^2 m_2^2 + 3\lambda_6^2 m_2^2 - 27\lambda_7^2 m_2^2 \\
& + \left(18\lambda_7 m_{12} + \frac{1}{4} (160g_3^2 - 288\lambda_2 + 45g_2^2 + 5g_1^2) m_2^2\right) \text{Tr}(\epsilon_d \epsilon_d^\dagger) \\
& - 6 \left( - (2\lambda_3 + 6\lambda_2 + \lambda_4) m_{12} + 3\lambda_7 (m_1^2 + m_2^2) \right) \text{Tr}(\epsilon_d Y_d^\dagger) + 6\lambda_7 m_{12} \text{Tr}(\epsilon_e \epsilon_e^\dagger) + \frac{15}{4} g_1^2 m_2^2 \text{Tr}(\epsilon_e \epsilon_e^\dagger) \\
& + \frac{15}{4} g_2^2 m_2^2 \text{Tr}(\epsilon_e \epsilon_e^\dagger) - 24\lambda_2 m_2^2 \text{Tr}(\epsilon_e \epsilon_e^\dagger) - 6\lambda_7 m_1^2 \text{Tr}(\epsilon_e Y_e^\dagger) + 12\lambda_2 m_{12} \text{Tr}(\epsilon_e Y_e^\dagger) \\
& + 4\lambda_3 m_{12} \text{Tr}(\epsilon_e Y_e^\dagger) + 2\lambda_4 m_{12} \text{Tr}(\epsilon_e Y_e^\dagger) - 6\lambda_7 m_2^2 \text{Tr}(\epsilon_e Y_e^\dagger) - 24\lambda_3 m_1^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger)
\end{aligned}$$

$$\begin{aligned}
& -12\lambda_4 m_1^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger) + 18\lambda_7 m_{12} \text{Tr}(\epsilon_u \epsilon_u^\dagger) - 18\lambda_7 m_1^2 \text{Tr}(\epsilon_u Y_u^\dagger) + 36\lambda_2 m_{12} \text{Tr}(\epsilon_u Y_u^\dagger) \\
& + 12\lambda_3 m_{12} \text{Tr}(\epsilon_u Y_u^\dagger) + 6\lambda_4 m_{12} \text{Tr}(\epsilon_u Y_u^\dagger) - 18\lambda_7 m_2^2 \text{Tr}(\epsilon_u Y_u^\dagger) - 18\lambda_7 m_1^2 \text{Tr}(Y_d \epsilon_d^\dagger) \\
& - 18\lambda_7 m_2^2 \text{Tr}(Y_d \epsilon_d^\dagger) - 24\lambda_3 m_1^2 \text{Tr}(Y_d Y_d^\dagger) - 12\lambda_4 m_1^2 \text{Tr}(Y_d Y_d^\dagger) + 18\lambda_7 m_{12} \text{Tr}(Y_d Y_d^\dagger) \\
& - 6\lambda_7 m_1^2 \text{Tr}(Y_e \epsilon_e^\dagger) - 6\lambda_7 m_2^2 \text{Tr}(Y_e \epsilon_e^\dagger) - 8\lambda_3 m_1^2 \text{Tr}(Y_e Y_e^\dagger) - 4\lambda_4 m_1^2 \text{Tr}(Y_e Y_e^\dagger) \\
& + 6\lambda_7 m_{12} \text{Tr}(Y_e Y_e^\dagger) - 18\lambda_7 m_1^2 \text{Tr}(Y_u \epsilon_u^\dagger) - 18\lambda_7 m_2^2 \text{Tr}(Y_u \epsilon_u^\dagger) + 18\lambda_7 m_{12} \text{Tr}(Y_u Y_u^\dagger) \\
& + \frac{17}{4} g_1^2 m_2^2 \text{Tr}(Y_u Y_u^\dagger) + \frac{45}{4} g_2^2 m_2^2 \text{Tr}(Y_u Y_u^\dagger) + 40 g_3^2 m_2^2 \text{Tr}(Y_u Y_u^\dagger) - 72\lambda_2 m_2^2 \text{Tr}(Y_u Y_u^\dagger) \\
& - \frac{27}{2} m_2^2 \text{Tr}(\epsilon_d \epsilon_d^\dagger \epsilon_d \epsilon_d^\dagger) - 9m_2^2 \text{Tr}(\epsilon_d \epsilon_d^\dagger Y_d Y_d^\dagger) - \frac{9}{2} m_2^2 \text{Tr}(\epsilon_d \epsilon_u^\dagger \epsilon_u \epsilon_d^\dagger) \\
& - 12m_1^2 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) + 6m_2^2 \text{Tr}(\epsilon_d \epsilon_u^\dagger Y_u Y_d^\dagger) - \frac{9}{2} m_2^2 \text{Tr}(\epsilon_d Y_d^\dagger Y_d \epsilon_d^\dagger) \\
& + 12m_{12} \text{Tr}(\epsilon_d Y_u^\dagger \epsilon_u \epsilon_d^\dagger) - 21m_2^2 \text{Tr}(\epsilon_d Y_u^\dagger Y_u \epsilon_d^\dagger) + 12m_{12} \text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger) \\
& - \frac{9}{2} m_2^2 \text{Tr}(\epsilon_e \epsilon_e^\dagger \epsilon_e \epsilon_e^\dagger) - 3m_2^2 \text{Tr}(\epsilon_e \epsilon_e^\dagger Y_e Y_e^\dagger) - \frac{3}{2} m_2^2 \text{Tr}(\epsilon_e Y_e^\dagger Y_e \epsilon_e^\dagger) \\
& - 12m_1^2 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) + 6m_2^2 \text{Tr}(\epsilon_u \epsilon_d^\dagger Y_d Y_u^\dagger) - 9m_2^2 \text{Tr}(\epsilon_u \epsilon_u^\dagger Y_u Y_u^\dagger) \\
& - \frac{9}{2} m_2^2 \text{Tr}(\epsilon_u Y_u^\dagger Y_u \epsilon_u^\dagger) - \frac{9}{2} m_2^2 \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) - \frac{27}{2} m_2^2 \text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger)
\end{aligned} \tag{41}$$

### 3.5 Vacuum expectation values

$$\begin{aligned}
\beta_{v_1}^{(1)} &= \frac{1}{20} \left( 9g_1^2 v_1 + 45g_2^2 v_1 + 3g_1^2 v_1 \text{Xi} + 15g_2^2 v_1 \text{Xi} - 30v_2 \text{Tr}(\epsilon_d Y_d^\dagger) - 10v_2 \text{Tr}(\epsilon_e Y_e^\dagger) \right. \\
& - 60v_1 \text{Tr}(\epsilon_u \epsilon_u^\dagger) - 30v_2 \text{Tr}(\epsilon_u Y_u^\dagger) - 30v_2 \text{Tr}(Y_d \epsilon_d^\dagger) - 60v_1 \text{Tr}(Y_d Y_d^\dagger) - 10v_2 \text{Tr}(Y_e \epsilon_e^\dagger) \\
& \left. - 20v_1 \text{Tr}(Y_e Y_e^\dagger) - 30v_2 \text{Tr}(Y_u \epsilon_u^\dagger) \right) \\
\beta_{v_1}^{(2)} &= -\frac{1359}{800} g_1^4 v_1 - \frac{27}{80} g_1^2 g_2^2 v_1 + \frac{249}{32} g_2^4 v_1 - 6\lambda_1^2 v_1 - \lambda_3^2 v_1 - \lambda_3 \lambda_4 v_1 - \lambda_4^2 v_1 - \frac{3}{2} \lambda_5^2 v_1 \\
& - \frac{9}{2} \lambda_6^2 v_1 - \frac{3}{2} \lambda_7^2 v_1 - 3\lambda_1 \lambda_6 v_2 - \frac{3}{2} \lambda_3 \lambda_6 v_2 - \frac{3}{2} \lambda_4 \lambda_6 v_2 - \frac{3}{2} \lambda_5 \lambda_6 v_2 - 3\lambda_2 \lambda_7 v_2 - \frac{3}{2} \lambda_3 \lambda_7 v_2 \\
& - \frac{3}{2} \lambda_4 \lambda_7 v_2 - \frac{3}{2} \lambda_5 \lambda_7 v_2 + \frac{9}{400} g_1^4 v_1 \text{Xi} + \frac{9}{40} g_1^2 g_2^2 v_1 \text{Xi} + \frac{45}{16} g_2^4 v_1 \text{Xi} + \frac{9}{400} g_1^4 v_1 \text{Xi}^2 + \frac{9}{40} g_1^2 g_2^2 v_1 \text{Xi}^2 \\
& - \frac{9}{16} g_2^4 v_1 \text{Xi}^2 - \frac{1}{80} \left( 45g_2^2 (2 - 2iv_1 \text{Xi} + 5v_2) + 800g_3^2 v_2 + g_1^2 (18 - 18iv_1 \text{Xi} + 25v_2) \right) \text{Tr}(\epsilon_d Y_d^\dagger) \\
& - \frac{3}{80} \left( 5g_2^2 (2 - 2iv_1 \text{Xi} + 5v_2) + g_1^2 (2 - 2iv_1 \text{Xi} + 25v_2) \right) \text{Tr}(\epsilon_e Y_e^\dagger) - \frac{17}{8} g_1^2 v_1 \text{Tr}(\epsilon_u \epsilon_u^\dagger) - \frac{45}{8} g_2^2 v_1 \text{Tr}(\epsilon_u \epsilon_u^\dagger) \\
& - 20g_3^2 v_1 \text{Tr}(\epsilon_u \epsilon_u^\dagger) - \frac{9}{20} g_1^2 v_1 \text{Xi} \text{Tr}(\epsilon_u \epsilon_u^\dagger) - \frac{9}{4} g_2^2 v_1 \text{Xi} \text{Tr}(\epsilon_u \epsilon_u^\dagger) - \frac{17}{16} g_1^2 v_2 \text{Tr}(\epsilon_u Y_u^\dagger)
\end{aligned} \tag{42}$$

$$\begin{aligned}
& -\frac{45}{16}g_2^2v_2\text{Tr}(\epsilon_u Y_u^\dagger) - 10g_3^2v_2\text{Tr}(\epsilon_u Y_u^\dagger) - \frac{9}{40} + \frac{9i}{40}g_1^2v_1\text{XiTr}(\epsilon_u Y_u^\dagger) - \frac{9}{8} + \frac{9i}{8}g_2^2v_1\text{XiTr}(\epsilon_u Y_u^\dagger) \\
& -\frac{5}{16}g_1^2v_2\text{Tr}(Y_d\epsilon_d^\dagger) - \frac{45}{16}g_2^2v_2\text{Tr}(Y_d\epsilon_d^\dagger) - 10g_3^2v_2\text{Tr}(Y_d\epsilon_d^\dagger) - \frac{9}{40} - \frac{9i}{40}g_1^2v_1\text{XiTr}(Y_d\epsilon_d^\dagger) \\
& -\frac{9}{8} - \frac{9i}{8}g_2^2v_1\text{XiTr}(Y_d\epsilon_d^\dagger) - \frac{5}{8}g_1^2v_1\text{Tr}(Y_d Y_d^\dagger) - \frac{45}{8}g_2^2v_1\text{Tr}(Y_d Y_d^\dagger) - 20g_3^2v_1\text{Tr}(Y_d Y_d^\dagger) \\
& -\frac{9}{20}g_1^2v_1\text{XiTr}(Y_d Y_d^\dagger) - \frac{9}{4}g_2^2v_1\text{XiTr}(Y_d Y_d^\dagger) - \frac{15}{16}g_1^2v_2\text{Tr}(Y_e\epsilon_e^\dagger) - \frac{15}{16}g_2^2v_2\text{Tr}(Y_e\epsilon_e^\dagger) \\
& -\frac{3}{40} - \frac{3i}{40}g_1^2v_1\text{XiTr}(Y_e\epsilon_e^\dagger) - \frac{3}{8} - \frac{3i}{8}g_2^2v_1\text{XiTr}(Y_e\epsilon_e^\dagger) - \frac{15}{8}g_1^2v_1\text{Tr}(Y_e Y_e^\dagger) - \frac{15}{8}g_2^2v_1\text{Tr}(Y_e Y_e^\dagger) \\
& -\frac{3}{20}g_1^2v_1\text{XiTr}(Y_e Y_e^\dagger) - \frac{3}{4}g_2^2v_1\text{XiTr}(Y_e Y_e^\dagger) - \frac{17}{16}g_1^2v_2\text{Tr}(Y_u\epsilon_u^\dagger) - \frac{45}{16}g_2^2v_2\text{Tr}(Y_u\epsilon_u^\dagger) \\
& -10g_3^2v_2\text{Tr}(Y_u\epsilon_u^\dagger) - \frac{9}{40} - \frac{9i}{40}g_1^2v_1\text{XiTr}(Y_u\epsilon_u^\dagger) - \frac{9}{8} - \frac{9i}{8}g_2^2v_1\text{XiTr}(Y_u\epsilon_u^\dagger) + \frac{9}{8}v_2\text{Tr}(\epsilon_d\epsilon_d^\dagger\epsilon_d Y_d^\dagger) \\
& +\frac{27}{8}v_2\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d\epsilon_d^\dagger) + \frac{9}{2}v_1\text{Tr}(\epsilon_d\epsilon_d^\dagger Y_d Y_d^\dagger) + \frac{9}{4}v_1\text{Tr}(\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger) \\
& -\frac{3}{8}v_2\text{Tr}(\epsilon_d\epsilon_u^\dagger\epsilon_u Y_d^\dagger) - \frac{3}{8}v_2\text{Tr}(\epsilon_d\epsilon_u^\dagger Y_u\epsilon_d^\dagger) - 3v_1\text{Tr}(\epsilon_d\epsilon_u^\dagger Y_u Y_d^\dagger) \\
& +\frac{9}{4}v_2\text{Tr}(\epsilon_d Y_d^\dagger\epsilon_d\epsilon_d^\dagger) + \frac{9}{4}v_1\text{Tr}(\epsilon_d Y_d^\dagger Y_d\epsilon_d^\dagger) + \frac{27}{8}v_2\text{Tr}(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger) \\
& -\frac{3}{8}v_2\text{Tr}(\epsilon_d Y_u^\dagger\epsilon_u\epsilon_d^\dagger) - \frac{3}{8}v_2\text{Tr}(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger) + \frac{3}{8}v_2\text{Tr}(\epsilon_e\epsilon_e^\dagger\epsilon_e Y_e^\dagger) \\
& +\frac{9}{8}v_2\text{Tr}(\epsilon_e\epsilon_e^\dagger Y_e\epsilon_e^\dagger) + \frac{3}{2}v_1\text{Tr}(\epsilon_e\epsilon_e^\dagger Y_e Y_e^\dagger) + \frac{3}{4}v_2\text{Tr}(\epsilon_e Y_e^\dagger\epsilon_e\epsilon_e^\dagger) \\
& +\frac{3}{4}v_1\text{Tr}(\epsilon_e Y_e^\dagger Y_e\epsilon_e^\dagger) + \frac{9}{8}v_2\text{Tr}(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger) - \frac{3}{8}v_2\text{Tr}(\epsilon_u\epsilon_d^\dagger Y_d\epsilon_u^\dagger) \\
& -3v_1\text{Tr}(\epsilon_u\epsilon_d^\dagger Y_d Y_u^\dagger) + \frac{27}{4}v_1\text{Tr}(\epsilon_u\epsilon_u^\dagger\epsilon_u\epsilon_u^\dagger) + \frac{27}{8}v_2\text{Tr}(\epsilon_u\epsilon_u^\dagger Y_u\epsilon_u^\dagger) + \frac{9}{2}v_1\text{Tr}(\epsilon_u\epsilon_u^\dagger Y_u Y_u^\dagger) \\
& -\frac{3}{2}v_1\text{Tr}(\epsilon_u Y_d^\dagger Y_d\epsilon_u^\dagger) - \frac{3}{8}v_2\text{Tr}(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger) + \frac{27}{8}v_2\text{Tr}(\epsilon_u Y_u^\dagger\epsilon_u\epsilon_u^\dagger) \\
& +\frac{9}{4}v_1\text{Tr}(\epsilon_u Y_u^\dagger Y_u\epsilon_u^\dagger) + \frac{27}{8}v_2\text{Tr}(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger) + \frac{27}{8}v_2\text{Tr}(Y_d\epsilon_d^\dagger Y_d Y_d^\dagger) \\
& -\frac{3}{8}v_2\text{Tr}(Y_d\epsilon_u^\dagger Y_u Y_d^\dagger) + \frac{27}{4}v_1\text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) - \frac{3}{8}v_2\text{Tr}(Y_d Y_u^\dagger Y_u\epsilon_d^\dagger) \\
& +\frac{9}{4}v_1\text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) + \frac{9}{8}v_2\text{Tr}(Y_e\epsilon_e^\dagger Y_e Y_e^\dagger) + \frac{9}{4}v_1\text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) \\
& +\frac{9}{4}v_2\text{Tr}(Y_u\epsilon_u^\dagger Y_u Y_u^\dagger) + \frac{9}{8}v_2\text{Tr}(Y_u Y_u^\dagger Y_u\epsilon_u^\dagger)
\end{aligned} \tag{43}$$

$$\begin{aligned}
\beta_{v_2}^{(1)} &= \frac{1}{20}\left(9g_1^2v_2 + 45g_2^2v_2 + 3g_1^2v_2\text{Xi} + 15g_2^2v_2\text{Xi} - 60v_2\text{Tr}(\epsilon_d\epsilon_d^\dagger) - 30v_1\text{Tr}(\epsilon_d Y_d^\dagger)\right) \\
& -20v_2\text{Tr}(\epsilon_e\epsilon_e^\dagger) - 10v_1\text{Tr}(\epsilon_e Y_e^\dagger) - 30v_1\text{Tr}(\epsilon_u Y_u^\dagger) - 30v_1\text{Tr}(Y_d\epsilon_d^\dagger) - 10v_1\text{Tr}(Y_e\epsilon_e^\dagger) \\
& -30v_1\text{Tr}(Y_u\epsilon_u^\dagger) - 60v_2\text{Tr}(Y_u Y_u^\dagger)
\end{aligned} \tag{44}$$

$$\beta_{v_2}^{(2)} = -3\lambda_1\lambda_6v_1 - \frac{3}{2}\lambda_3\lambda_6v_1 - \frac{3}{2}\lambda_4\lambda_6v_1 - \frac{3}{2}\lambda_5\lambda_6v_1 - 3\lambda_2\lambda_7v_1 - \frac{3}{2}\lambda_3\lambda_7v_1 - \frac{3}{2}\lambda_4\lambda_7v_1 - \frac{3}{2}\lambda_5\lambda_7v_1$$



$$\begin{aligned}
& -\frac{1359}{800}g_1^4v_2 - \frac{27}{80}g_1^2g_2^2v_2 + \frac{249}{32}g_2^4v_2 - 6\lambda_2^2v_2 - \lambda_3^2v_2 - \lambda_3\lambda_4v_2 - \lambda_4^2v_2 - \frac{3}{2}\lambda_5^2v_2 \\
& -\frac{3}{2}\lambda_6^2v_2 - \frac{9}{2}\lambda_7^2v_2 + \frac{9}{400}g_1^4v_2\text{Xi} + \frac{9}{40}g_1^2g_2^2v_2\text{Xi} + \frac{45}{16}g_2^4v_2\text{Xi} + \frac{9}{400}g_1^4v_2\text{Xi}^2 + \frac{9}{40}g_1^2g_2^2v_2\text{Xi}^2 \\
& -\frac{9}{16}g_2^4v_2\text{Xi}^2 - \frac{1}{40}v_2\left(45g_2^2(2\text{Xi} + 5) + 800g_3^2 + g_1^2(18\text{Xi} + 25)\right)\text{Tr}\left(\epsilon_d\epsilon_d^\dagger\right) \\
& -\frac{1}{80}\left(45g_2^2(2 + 2iv_2\text{Xi} + 5v_1) + 800g_3^2v_1 + g_1^2(18 + 18iv_2\text{Xi} + 25v_1)\right)\text{Tr}\left(\epsilon_dY_d^\dagger\right) - \frac{15}{8}g_1^2v_2\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) \\
& -\frac{15}{8}g_2^2v_2\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - \frac{3}{20}g_1^2v_2\text{Xi}\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - \frac{3}{4}g_2^2v_2\text{Xi}\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\right) - \frac{15}{16}g_1^2v_1\text{Tr}\left(\epsilon_eY_e^\dagger\right) \\
& -\frac{15}{16}g_2^2v_1\text{Tr}\left(\epsilon_eY_e^\dagger\right) - \frac{3}{40} - \frac{3i}{40}g_1^2v_2\text{Xi}\text{Tr}\left(\epsilon_eY_e^\dagger\right) - \frac{3}{8} - \frac{3i}{8}g_2^2v_2\text{Xi}\text{Tr}\left(\epsilon_eY_e^\dagger\right) - \frac{17}{16}g_1^2v_1\text{Tr}\left(\epsilon_uY_u^\dagger\right) \\
& -\frac{45}{16}g_2^2v_1\text{Tr}\left(\epsilon_uY_u^\dagger\right) - 10g_3^2v_1\text{Tr}\left(\epsilon_uY_u^\dagger\right) - \frac{9}{40} - \frac{9i}{40}g_1^2v_2\text{Xi}\text{Tr}\left(\epsilon_uY_u^\dagger\right) - \frac{9}{8} - \frac{9i}{8}g_2^2v_2\text{Xi}\text{Tr}\left(\epsilon_uY_u^\dagger\right) \\
& -\frac{5}{16}g_1^2v_1\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - \frac{45}{16}g_2^2v_1\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - 10g_3^2v_1\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - \frac{9}{40} + \frac{9i}{40}g_1^2v_2\text{Xi}\text{Tr}\left(Y_d\epsilon_d^\dagger\right) \\
& -\frac{9}{8} + \frac{9i}{8}g_2^2v_2\text{Xi}\text{Tr}\left(Y_d\epsilon_d^\dagger\right) - \frac{15}{16}g_1^2v_1\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - \frac{15}{16}g_2^2v_1\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - \frac{3}{40} + \frac{3i}{40}g_1^2v_2\text{Xi}\text{Tr}\left(Y_e\epsilon_e^\dagger\right) \\
& -\frac{3}{8} + \frac{3i}{8}g_2^2v_2\text{Xi}\text{Tr}\left(Y_e\epsilon_e^\dagger\right) - \frac{17}{16}g_1^2v_1\text{Tr}\left(Y_u\epsilon_u^\dagger\right) - \frac{45}{16}g_2^2v_1\text{Tr}\left(Y_u\epsilon_u^\dagger\right) - 10g_3^2v_1\text{Tr}\left(Y_u\epsilon_u^\dagger\right) \\
& -\frac{9}{40} + \frac{9i}{40}g_1^2v_2\text{Xi}\text{Tr}\left(Y_u\epsilon_u^\dagger\right) - \frac{9}{8} + \frac{9i}{8}g_2^2v_2\text{Xi}\text{Tr}\left(Y_u\epsilon_u^\dagger\right) - \frac{17}{8}g_1^2v_2\text{Tr}\left(Y_uY_u^\dagger\right) - \frac{45}{8}g_2^2v_2\text{Tr}\left(Y_uY_u^\dagger\right) \\
& -20g_3^2v_2\text{Tr}\left(Y_uY_u^\dagger\right) - \frac{9}{20}g_1^2v_2\text{Xi}\text{Tr}\left(Y_uY_u^\dagger\right) - \frac{9}{4}g_2^2v_2\text{Xi}\text{Tr}\left(Y_uY_u^\dagger\right) + \frac{27}{4}v_2\text{Tr}\left(\epsilon_d\epsilon_d^\dagger\epsilon_d\epsilon_d^\dagger\right) \\
& +\frac{27}{8}v_1\text{Tr}\left(\epsilon_d\epsilon_d^\dagger Y_d\epsilon_d^\dagger\right) + \frac{9}{2}v_2\text{Tr}\left(\epsilon_d\epsilon_d^\dagger Y_d Y_d^\dagger\right) + \frac{9}{4}v_2\text{Tr}\left(\epsilon_d\epsilon_u^\dagger\epsilon_u\epsilon_d^\dagger\right) \\
& -\frac{3}{8}v_1\text{Tr}\left(\epsilon_d\epsilon_u^\dagger\epsilon_u Y_d^\dagger\right) - \frac{3}{8}v_1\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_u\epsilon_d^\dagger\right) - 3v_2\text{Tr}\left(\epsilon_d\epsilon_u^\dagger Y_u Y_d^\dagger\right) \\
& +\frac{27}{8}v_1\text{Tr}\left(\epsilon_d Y_d^\dagger\epsilon_d\epsilon_d^\dagger\right) + \frac{9}{4}v_2\text{Tr}\left(\epsilon_d Y_d^\dagger Y_d\epsilon_d^\dagger\right) + \frac{27}{8}v_1\text{Tr}\left(\epsilon_d Y_d^\dagger Y_d Y_d^\dagger\right) \\
& -\frac{3}{8}v_1\text{Tr}\left(\epsilon_d Y_u^\dagger\epsilon_u\epsilon_d^\dagger\right) - \frac{3}{2}v_2\text{Tr}\left(\epsilon_d Y_u^\dagger Y_u\epsilon_d^\dagger\right) - \frac{3}{8}v_1\text{Tr}\left(\epsilon_d Y_u^\dagger Y_u Y_d^\dagger\right) \\
& +\frac{9}{4}v_2\text{Tr}\left(\epsilon_e\epsilon_e^\dagger\epsilon_e\epsilon_e^\dagger\right) + \frac{9}{8}v_1\text{Tr}\left(\epsilon_e\epsilon_e^\dagger Y_e\epsilon_e^\dagger\right) + \frac{3}{2}v_2\text{Tr}\left(\epsilon_e\epsilon_e^\dagger Y_e Y_e^\dagger\right) + \frac{9}{8}v_1\text{Tr}\left(\epsilon_e Y_e^\dagger\epsilon_e\epsilon_e^\dagger\right) \\
& +\frac{3}{4}v_2\text{Tr}\left(\epsilon_e Y_e^\dagger Y_e\epsilon_e^\dagger\right) + \frac{9}{8}v_1\text{Tr}\left(\epsilon_e Y_e^\dagger Y_e Y_e^\dagger\right) - \frac{3}{8}v_1\text{Tr}\left(\epsilon_u\epsilon_d^\dagger Y_d\epsilon_u^\dagger\right) \\
& -3v_2\text{Tr}\left(\epsilon_u\epsilon_d^\dagger Y_d Y_u^\dagger\right) + \frac{9}{8}v_1\text{Tr}\left(\epsilon_u\epsilon_u^\dagger\epsilon_u Y_u^\dagger\right) + \frac{27}{8}v_1\text{Tr}\left(\epsilon_u\epsilon_u^\dagger Y_u\epsilon_u^\dagger\right) \\
& +\frac{9}{2}v_2\text{Tr}\left(\epsilon_u\epsilon_u^\dagger Y_u Y_u^\dagger\right) - \frac{3}{8}v_1\text{Tr}\left(\epsilon_u Y_d^\dagger Y_d Y_u^\dagger\right) + \frac{9}{4}v_1\text{Tr}\left(\epsilon_u Y_u^\dagger\epsilon_u\epsilon_u^\dagger\right) \\
& +\frac{9}{4}v_2\text{Tr}\left(\epsilon_u Y_u^\dagger Y_u\epsilon_u^\dagger\right) + \frac{27}{8}v_1\text{Tr}\left(\epsilon_u Y_u^\dagger Y_u Y_u^\dagger\right) + \frac{9}{4}v_1\text{Tr}\left(Y_d\epsilon_d^\dagger Y_d Y_d^\dagger\right) \\
& -\frac{3}{8}v_1\text{Tr}\left(Y_d\epsilon_u^\dagger Y_u Y_d^\dagger\right) + \frac{9}{8}v_1\text{Tr}\left(Y_d Y_d^\dagger Y_d\epsilon_d^\dagger\right) - \frac{3}{8}v_1\text{Tr}\left(Y_d Y_u^\dagger Y_u\epsilon_d^\dagger\right) \\
& +\frac{9}{4}v_2\text{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger\right) + \frac{3}{4}v_1\text{Tr}\left(Y_e\epsilon_e^\dagger Y_e Y_e^\dagger\right) + \frac{3}{8}v_1\text{Tr}\left(Y_e Y_e^\dagger Y_e\epsilon_e^\dagger\right)
\end{aligned}$$

$$+ \frac{27}{8} v_1 \text{Tr} \left( Y_u \epsilon_u^\dagger Y_u Y_u^\dagger \right) + \frac{27}{4} v_2 \text{Tr} \left( Y_u Y_u^\dagger Y_u Y_u^\dagger \right) \quad (45)$$

## 4 Field Rotations

### 4.1 Rotations in gauge sector for eigenstates 'EWSB'

$$\begin{pmatrix} B_\rho \\ W_{3\rho} \end{pmatrix} = Z^{\gamma Z} \begin{pmatrix} \gamma_\rho \\ Z_\rho \end{pmatrix} \quad (46)$$

$$\begin{pmatrix} W_{1\rho} \\ W_{2\rho} \end{pmatrix} = Z^W \begin{pmatrix} W_\rho^- \\ W_\rho^- \end{pmatrix} \quad (47)$$

$$(48)$$

The mixing matrices are parametrized by

$$Z^{\gamma Z} = \begin{pmatrix} \cos \Theta_W & -\sin \Theta_W \\ \sin \Theta_W & \cos \Theta_W \end{pmatrix} \quad (49)$$

$$Z^W = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -i\frac{1}{\sqrt{2}} & i\frac{1}{\sqrt{2}} \end{pmatrix} \quad (50)$$

$$(51)$$

### 4.2 Rotations in Mass sector for eigenstates 'EWSB'

#### 4.2.1 Mass Matrices for Scalars

- Mass matrix for Higgs, Basis:  $(\phi_1, \phi_2), (\phi_1, \phi_2)$

$$m_h^2 = \begin{pmatrix} m_{\phi_1\phi_1} & m_{\phi_2\phi_1} \\ m_{\phi_1\phi_2} & m_{\phi_2\phi_2} \end{pmatrix} \quad (52)$$

$$m_{\phi_1\phi_1} = \frac{1}{4} \left( 12(\lambda_1 v_1^2 + v_1 v_2 \Re(\lambda_6)) + v_2^2 (2(\lambda_3 + \lambda_4) + 2\Re(\lambda_5)) \right) + m_1^2 \quad (53)$$

$$m_{\phi_1\phi_2} = \frac{1}{4} \left( 3(2v_1^2 \Re(\lambda_6) + v_2^2 \lambda_7^*) - 4\Re(m_{12}) + v_2 (2v_1 (2(\lambda_3 + \lambda_4) + 2\Re(\lambda_5)) + 3\lambda_7 v_2) \right) \quad (54)$$

$$m_{\phi_2\phi_2} = \frac{1}{2} \left( 6\lambda_2 v_2^2 + 6v_1 v_2 \Re(\lambda_7) + v_1^2 (\lambda_3 + \lambda_4 + \Re(\lambda_5)) \right) + m_2^2 \quad (55)$$

This matrix is diagonalized by  $Z^H$ :

$$Z^H m_h^2 Z^{H,\dagger} = m_{2,h}^{dia} \quad (56)$$

with

$$\phi_1 = \sum_j Z_{j1}^H h_j, \quad \phi_2 = \sum_j Z_{j2}^H h_j \quad (57)$$

- **Mass matrix for Pseudo-Scalar Higgs**, Basis:  $(\sigma_1, \sigma_2), (\sigma_1, \sigma_2)$

$$m_{A^0}^2 = \begin{pmatrix} m_{\sigma_1\sigma_1} & m_{\sigma_2\sigma_1} \\ m_{\sigma_1\sigma_2} & m_{\sigma_2\sigma_2} \end{pmatrix} + \xi_Z m^2(Z) \quad (58)$$

$$m_{\sigma_1\sigma_1} = \frac{1}{4} \left( 4(\lambda_1 v_1^2 + v_1 v_2 \Re(\lambda_6)) + v_2^2 (2(\lambda_3 + \lambda_4) - 2\Re(\lambda_5)) \right) + m_1^2 \quad (59)$$

$$m_{\sigma_1\sigma_2} = \frac{1}{4} \left( 2v_1^2 \Re(\lambda_6) + 2v_2^2 \Re(\lambda_7) - 4\Re(m_{12}) + 4v_1 v_2 \Re(\lambda_5) \right) \quad (60)$$

$$m_{\sigma_2\sigma_2} = \frac{1}{2} \left( 2\lambda_2 v_2^2 + 2v_1 v_2 \Re(\lambda_7) + v_1^2 (-\Re(\lambda_5) + \lambda_3 + \lambda_4) \right) + m_2^2 \quad (61)$$

Gauge fixing contributions:

$$m^2(\xi_Z) = \begin{pmatrix} \frac{1}{4} v_1^2 (g_1 \sin \Theta_W + g_2 \cos \Theta_W)^2 & \frac{1}{4} v_1 v_2 (g_1 \sin \Theta_W + g_2 \cos \Theta_W)^2 \\ \frac{1}{4} v_1 v_2 (g_1 \sin \Theta_W + g_2 \cos \Theta_W)^2 & \frac{1}{4} v_2^2 (g_1 \sin \Theta_W + g_2 \cos \Theta_W)^2 \end{pmatrix} \quad (62)$$

This matrix is diagonalized by  $Z^A$ :

$$Z^A m_{A^0}^2 Z^{A,\dagger} = m_{2,A^0}^{dia} \quad (63)$$

with

$$\sigma_1 = \sum_j Z_{j1}^A A_j^0, \quad \sigma_2 = \sum_j Z_{j2}^A A_j^0 \quad (64)$$

- **Mass matrix for Charged Higgs**, Basis:  $(H_1^{+,*}, H_2^{+,*}), (H_1^+, H_2^+)$

$$m_{H^-}^2 = \begin{pmatrix} \frac{1}{2} \lambda_3 v_2^2 + \lambda_1 v_1^2 + v_1 v_2 \Re(\lambda_6) + m_1^2 & m_{H_2^{+,*} H_1^+}^* \\ \frac{1}{2} (-2m_{12} + \lambda_6 v_1^2 + \lambda_7 v_2^2 + v_1 v_2 (\lambda_4 + \lambda_5^*)) & \frac{1}{2} \lambda_3 v_1^2 + \lambda_2 v_2^2 + v_1 v_2 \Re(\lambda_7) + m_2^2 \end{pmatrix} + \xi_{W^-} m^2(W^-) \quad (65)$$

Gauge fixing contributions:

$$m^2(\xi_{W^-}) = \begin{pmatrix} \frac{1}{4} g_2^2 v_1^2 & \frac{1}{4} g_2^2 v_1 v_2 \\ \frac{1}{4} g_2^2 v_1 v_2 & \frac{1}{4} g_2^2 v_2^2 \end{pmatrix} \quad (66)$$

This matrix is diagonalized by  $Z^+$ :

$$Z^+ m_{H^-}^2 Z^{+,\dagger} = m_{2,H^-}^{dia} \quad (67)$$

with

$$H_1^+ = \sum_j Z_{j1}^+ H_j^+, \quad H_2^+ = \sum_j Z_{j2}^+ H_j^+ \quad (68)$$

#### 4.2.2 Mass Matrices for Fermions

- **Mass matrix for Down-Quarks**, Basis:  $(d_{L,\alpha_1}), (d_{R,\beta_1}^*)$

$$m_d = \left( \frac{1}{\sqrt{2}} \delta_{\alpha_1\beta_1} (v_1 Y_d^T + v_2 \epsilon_d^T) \right) \quad (69)$$

This matrix is diagonalized by  $U_L^d$  and  $U_R^d$

$$U_L^{d,*} m_d U_R^{d,\dagger} = m_d^{dia} \quad (70)$$

with

$$d_{L,i\alpha} = \sum_{t_2} U_{L,ji}^{d,*} D_{L,j\alpha} \quad (71)$$

$$d_{R,i\alpha} = \sum_{t_2} U_{R,ij}^d D_{R,j\alpha}^* \quad (72)$$

- **Mass matrix for Up-Quarks**, Basis:  $(u_{L,\alpha_1}), (u_{R,\beta_1}^*)$

$$m_u = \left( -\frac{1}{\sqrt{2}} \delta_{\alpha_1\beta_1} (v_1 \epsilon_u^T + v_2 Y_u^T) \right) \quad (73)$$

This matrix is diagonalized by  $U_L^u$  and  $U_R^u$

$$U_L^{u,*} m_u U_R^{u,\dagger} = m_u^{dia} \quad (74)$$

with

$$u_{L,i\alpha} = \sum_{t_2} U_{L,ji}^{u,*} U_{L,j\alpha} \quad (75)$$

$$u_{R,i\alpha} = \sum_{t_2} U_{R,ij}^u U_{R,j\alpha}^* \quad (76)$$

- **Mass matrix for Leptons**, Basis:  $(e_L), (e_R^*)$

$$m_e = \left( \frac{1}{\sqrt{2}} (v_1 Y_e^T + v_2 \epsilon_e^T) \right) \quad (77)$$

This matrix is diagonalized by  $U_L^e$  and  $U_R^e$

$$U_L^{e,*} m_e U_R^{e,\dagger} = m_e^{dia} \quad (78)$$

with

$$e_{L,i} = \sum_{t_2} U_{L,ji}^{e,*} E_{L,j} \quad (79)$$

$$e_{R,i} = \sum_{t_2} U_{R,ij}^e E_{R,j}^* \quad (80)$$

## 5 Vacuum Expectation Values

$$H_1^0 = \frac{1}{\sqrt{2}}\phi_1 + \frac{1}{\sqrt{2}}v_1 + i\frac{1}{\sqrt{2}}\sigma_1 \quad (81)$$

$$H_2^0 = \frac{1}{\sqrt{2}}\phi_2 + \frac{1}{\sqrt{2}}v_2 + i\frac{1}{\sqrt{2}}\sigma_2 \quad (82)$$

## 6 Tadpole Equations

$$\frac{\partial V}{\partial \phi_1} = \frac{1}{4} \left( 4(\lambda_1 v_1^3 + m_1^2 v_1) + v_2 \left( 3v_1^2 (\lambda_6 + \lambda_6^*) - 4\Re(m_{12}) + v_2^2 \lambda_7^* + v_2 (\lambda_7 v_2 + v_1 (2(\lambda_3 + \lambda_4) + \lambda_5 + \lambda_5^*)) \right) \right) \quad (83)$$

$$\begin{aligned} \frac{\partial V}{\partial \phi_2} = & \frac{1}{4} \left( \lambda_6 v_1^3 + v_2 \left( (2(\lambda_3 + \lambda_4) + \lambda_5) v_1^2 + 3\lambda_7 v_1 v_2 + 4(\lambda_2 v_2^2 + m_2^2) \right) \right. \\ & \left. + v_1 \left( 3v_2^2 \lambda_7^* - 4\Re(m_{12}) + v_1^2 \lambda_6^* + v_1 v_2 \lambda_5^* \right) \right) \quad (84) \end{aligned}$$

## 7 Particle content for eigenstates 'EWSB'

Name	Type	complex/real	Generations	Indices
$h$	Scalar	real	2	generation, 2
$A^0$	Scalar	real	2	generation, 2
$H^-$	Scalar	complex	2	generation, 2
$\nu$	Fermion	Dirac	3	generation, 3
$d$	Fermion	Dirac	3	generation, 3, color, 3
$u$	Fermion	Dirac	3	generation, 3, color, 3
$e$	Fermion	Dirac	3	generation, 3
$g$	Vector	real	1	color, 8, lorentz, 4
$\gamma$	Vector	real	1	lorentz, 4
$Z$	Vector	real	1	lorentz, 4
$W^-$	Vector	complex	1	lorentz, 4
$\eta^G$	Ghost	real	1	color, 8
$\eta^\gamma$	Ghost	real	1	
$\eta^Z$	Ghost	real	1	
$\eta^-$	Ghost	complex	1	
$\eta^+$	Ghost	complex	1	

## 8 One Loop Self-Energy and One Loop Tadpoles for eigenstates 'EWSB'

### 8.1 One Loop Self-Energy

- Self-Energy for Higgs ( $h$ )

$$\begin{aligned}
\Pi_{i,j}(p^2) = & +2\left(-\frac{1}{2}\text{rMS} + B_0(p^2, m_Z^2, m_Z^2)\right)\Gamma_{\check{h}_j, Z, Z}^*\Gamma_{\check{h}_i, Z, Z} + 4\left(-\frac{1}{2}\text{rMS} + B_0(p^2, m_{W^-}^2, m_{W^-}^2)\right)\Gamma_{\check{h}_j, W^+, W^-}^*\Gamma_{\check{h}_i, W^+, W^-} \\
& - B_0(p^2, m_{\eta^-}^2, m_{\eta^-}^2)\Gamma_{\check{h}_i, \eta^-, \eta^-}\Gamma_{\check{h}_j, \eta^-, \eta^-} - B_0(p^2, m_{\eta^+}^2, m_{\eta^+}^2)\Gamma_{\check{h}_i, \eta^+, \eta^+}\Gamma_{\check{h}_j, \eta^+, \eta^+} \\
& - B_0(p^2, m_{\eta^z}^2, m_{\eta^z}^2)\Gamma_{\check{h}_i, \eta^z, \eta^z}\Gamma_{\check{h}_j, \eta^z, \eta^z} + 4\Gamma_{\check{h}_i, \check{h}_j, W^+, W^-} - \left(-\frac{1}{2}\text{rMS}m_{W^-}^2 + A_0(m_{W^-}^2)\right) \\
& + 2\Gamma_{\check{h}_i, \check{h}_j, Z, Z}\left(-\frac{1}{2}\text{rMS}m_Z^2 + A_0(m_Z^2)\right) - \frac{1}{2}\sum_{a=1}^2 A_0(m_{A_a^0}^2)\Gamma_{\check{h}_i, \check{h}_j, A_a^0, A_a^0} \\
& - \sum_{a=1}^2 A_0(m_{H_a^-}^2)\Gamma_{\check{h}_i, \check{h}_j, H_a^+, H_a^-} - \frac{1}{2}\sum_{a=1}^2 A_0(m_{h_a}^2)\Gamma_{\check{h}_i, \check{h}_j, h_a, h_a} \\
& + \frac{1}{2}\sum_{a=1}^2\sum_{b=1}^2 B_0(p^2, m_{A_a^0}^2, m_{A_b^0}^2)\Gamma_{\check{h}_j, A_a^0, A_b^0}^*\Gamma_{\check{h}_i, A_a^0, A_b^0} \\
& + \sum_{a=1}^2\sum_{b=1}^2 B_0(p^2, m_{H_a^-}^2, m_{H_b^-}^2)\Gamma_{\check{h}_j, H_a^+, H_b^-}^*\Gamma_{\check{h}_i, H_a^+, H_b^-} \\
& + \sum_{a=1}^2\sum_{b=1}^2 B_0(p^2, m_{h_a}^2, m_{A_b^0}^2)\Gamma_{\check{h}_j, h_a, A_b^0}^*\Gamma_{\check{h}_i, h_a, A_b^0} + \frac{1}{2}\sum_{a=1}^2\sum_{b=1}^2 B_0(p^2, m_{h_a}^2, m_{h_b}^2)\Gamma_{\check{h}_j, h_a, h_b}^*\Gamma_{\check{h}_i, h_a, h_b} \\
& - 6\sum_{a=1}^3 m_{d_a}\sum_{b=1}^3 B_0(p^2, m_{d_a}^2, m_{d_b}^2)m_{d_b}\left(\Gamma_{\check{h}_j, \bar{d}_a, d_b}^{L*}\Gamma_{\check{h}_i, \bar{d}_a, d_b}^R + \Gamma_{\check{h}_j, \bar{d}_a, d_b}^{R*}\Gamma_{\check{h}_i, \bar{d}_a, d_b}^L\right) \\
& + 3\sum_{a=1}^3\sum_{b=1}^3 G_0(p^2, m_{d_a}^2, m_{d_b}^2)\left(\Gamma_{\check{h}_j, \bar{d}_a, d_b}^{L*}\Gamma_{\check{h}_i, \bar{d}_a, d_b}^L + \Gamma_{\check{h}_j, \bar{d}_a, d_b}^{R*}\Gamma_{\check{h}_i, \bar{d}_a, d_b}^R\right) \\
& - 2\sum_{a=1}^3 m_{e_a}\sum_{b=1}^3 B_0(p^2, m_{e_a}^2, m_{e_b}^2)m_{e_b}\left(\Gamma_{\check{h}_j, \bar{e}_a, e_b}^{L*}\Gamma_{\check{h}_i, \bar{e}_a, e_b}^R + \Gamma_{\check{h}_j, \bar{e}_a, e_b}^{R*}\Gamma_{\check{h}_i, \bar{e}_a, e_b}^L\right) \\
& + \sum_{a=1}^3\sum_{b=1}^3 G_0(p^2, m_{e_a}^2, m_{e_b}^2)\left(\Gamma_{\check{h}_j, \bar{e}_a, e_b}^{L*}\Gamma_{\check{h}_i, \bar{e}_a, e_b}^L + \Gamma_{\check{h}_j, \bar{e}_a, e_b}^{R*}\Gamma_{\check{h}_i, \bar{e}_a, e_b}^R\right) \\
& - 6\sum_{a=1}^3 m_{u_a}\sum_{b=1}^3 B_0(p^2, m_{u_a}^2, m_{u_b}^2)m_{u_b}\left(\Gamma_{\check{h}_j, \bar{u}_a, u_b}^{L*}\Gamma_{\check{h}_i, \bar{u}_a, u_b}^R + \Gamma_{\check{h}_j, \bar{u}_a, u_b}^{R*}\Gamma_{\check{h}_i, \bar{u}_a, u_b}^L\right) \\
& + 3\sum_{a=1}^3\sum_{b=1}^3 G_0(p^2, m_{u_a}^2, m_{u_b}^2)\left(\Gamma_{\check{h}_j, \bar{u}_a, u_b}^{L*}\Gamma_{\check{h}_i, \bar{u}_a, u_b}^L + \Gamma_{\check{h}_j, \bar{u}_a, u_b}^{R*}\Gamma_{\check{h}_i, \bar{u}_a, u_b}^R\right)
\end{aligned}$$

$$+ \sum_{b=1}^2 \Gamma_{\tilde{h}_j, Z, A_b^0}^* \Gamma_{\tilde{h}_i, Z, A_b^0} F_0(p^2, m_{A_b^0}^2, m_Z^2) + 2 \sum_{b=1}^2 \Gamma_{\tilde{h}_j, W^+, H_b^-}^* \Gamma_{\tilde{h}_i, W^+, H_b^-} F_0(p^2, m_{H_b^-}^2, m_{W^-}^2) \quad (85)$$

• **Self-Energy for Pseudo-Scalar Higgs ( $A^0$ )**

$$\begin{aligned} \Pi_{i,j}(p^2) = & -B_0(p^2, m_{\eta^-}^2, m_{\eta^-}^2) \Gamma_{\tilde{A}_i^0, \eta^-, \eta^-} \Gamma_{\tilde{A}_j^0, \eta^-, \eta^-} - B_0(p^2, m_{\eta^+}^2, m_{\eta^+}^2) \Gamma_{\tilde{A}_i^0, \eta^+, \eta^+} \Gamma_{\tilde{A}_j^0, \eta^+, \eta^+} \\ & + 4\Gamma_{\tilde{A}_i^0, \tilde{A}_j^0, W^+, W^-} \left( -\frac{1}{2} \text{rMS} m_{W^-}^2 + A_0(m_{W^-}^2) \right) + 2\Gamma_{\tilde{A}_i^0, \tilde{A}_j^0, Z, Z} \left( -\frac{1}{2} \text{rMS} m_Z^2 + A_0(m_Z^2) \right) \\ & - \frac{1}{2} \sum_{a=1}^2 A_0(m_{A_a^0}^2) \Gamma_{\tilde{A}_i^0, \tilde{A}_j^0, A_a^0, A_a^0} - \sum_{a=1}^2 A_0(m_{H_a^-}^2) \Gamma_{\tilde{A}_i^0, \tilde{A}_j^0, H_a^+, H_a^-} \\ & - \frac{1}{2} \sum_{a=1}^2 A_0(m_{h_a}^2) \Gamma_{\tilde{A}_i^0, \tilde{A}_j^0, h_a, h_a} + \frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^2 B_0(p^2, m_{A_a^0}^2, m_{A_b^0}^2) \Gamma_{\tilde{A}_j^0, A_a^0, A_b^0}^* \Gamma_{\tilde{A}_i^0, A_a^0, A_b^0} \\ & + \sum_{a=1}^2 \sum_{b=1}^2 B_0(p^2, m_{H_a^-}^2, m_{H_b^-}^2) \Gamma_{\tilde{A}_j^0, H_a^+, H_b^-}^* \Gamma_{\tilde{A}_i^0, H_a^+, H_b^-} \\ & + \sum_{a=1}^2 \sum_{b=1}^2 B_0(p^2, m_{h_a}^2, m_{A_b^0}^2) \Gamma_{\tilde{A}_j^0, h_a, A_b^0}^* \Gamma_{\tilde{A}_i^0, h_a, A_b^0} \\ & + \frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^2 B_0(p^2, m_{h_a}^2, m_{h_b}^2) \Gamma_{\tilde{A}_j^0, h_a, h_b}^* \Gamma_{\tilde{A}_i^0, h_a, h_b} \\ & - 6 \sum_{a=1}^3 m_{d_a} \sum_{b=1}^3 B_0(p^2, m_{d_a}^2, m_{d_b}^2) m_{d_b} \left( \Gamma_{\tilde{A}_j^0, \bar{d}_a, d_b}^{L*} \Gamma_{\tilde{A}_i^0, \bar{d}_a, d_b}^R + \Gamma_{\tilde{A}_j^0, \bar{d}_a, d_b}^{R*} \Gamma_{\tilde{A}_i^0, \bar{d}_a, d_b}^L \right) \\ & + 3 \sum_{a=1}^3 \sum_{b=1}^3 G_0(p^2, m_{d_a}^2, m_{d_b}^2) \left( \Gamma_{\tilde{A}_j^0, \bar{d}_a, d_b}^{L*} \Gamma_{\tilde{A}_i^0, \bar{d}_a, d_b}^L + \Gamma_{\tilde{A}_j^0, \bar{d}_a, d_b}^{R*} \Gamma_{\tilde{A}_i^0, \bar{d}_a, d_b}^R \right) \\ & - 2 \sum_{a=1}^3 m_{e_a} \sum_{b=1}^3 B_0(p^2, m_{e_a}^2, m_{e_b}^2) m_{e_b} \left( \Gamma_{\tilde{A}_j^0, \bar{e}_a, e_b}^{L*} \Gamma_{\tilde{A}_i^0, \bar{e}_a, e_b}^R + \Gamma_{\tilde{A}_j^0, \bar{e}_a, e_b}^{R*} \Gamma_{\tilde{A}_i^0, \bar{e}_a, e_b}^L \right) \\ & + \sum_{a=1}^3 \sum_{b=1}^3 G_0(p^2, m_{e_a}^2, m_{e_b}^2) \left( \Gamma_{\tilde{A}_j^0, \bar{e}_a, e_b}^{L*} \Gamma_{\tilde{A}_i^0, \bar{e}_a, e_b}^L + \Gamma_{\tilde{A}_j^0, \bar{e}_a, e_b}^{R*} \Gamma_{\tilde{A}_i^0, \bar{e}_a, e_b}^R \right) \\ & - 6 \sum_{a=1}^3 m_{u_a} \sum_{b=1}^3 B_0(p^2, m_{u_a}^2, m_{u_b}^2) m_{u_b} \left( \Gamma_{\tilde{A}_j^0, \bar{u}_a, u_b}^{L*} \Gamma_{\tilde{A}_i^0, \bar{u}_a, u_b}^R + \Gamma_{\tilde{A}_j^0, \bar{u}_a, u_b}^{R*} \Gamma_{\tilde{A}_i^0, \bar{u}_a, u_b}^L \right) \\ & + 3 \sum_{a=1}^3 \sum_{b=1}^3 G_0(p^2, m_{u_a}^2, m_{u_b}^2) \left( \Gamma_{\tilde{A}_j^0, \bar{u}_a, u_b}^{L*} \Gamma_{\tilde{A}_i^0, \bar{u}_a, u_b}^L + \Gamma_{\tilde{A}_j^0, \bar{u}_a, u_b}^{R*} \Gamma_{\tilde{A}_i^0, \bar{u}_a, u_b}^R \right) \\ & + \sum_{b=1}^2 \Gamma_{\tilde{A}_j^0, Z, h_b}^* \Gamma_{\tilde{A}_i^0, Z, h_b} F_0(p^2, m_{h_b}^2, m_Z^2) + 2 \sum_{b=1}^2 \Gamma_{\tilde{A}_j^0, W^+, H_b^-}^* \Gamma_{\tilde{A}_i^0, W^+, H_b^-} F_0(p^2, m_{H_b^-}^2, m_{W^-}^2) \quad (86) \end{aligned}$$

• **Self-Energy for Charged Higgs ( $H^-$ )**

$$\begin{aligned}
\Pi_{i,j}(p^2) = & +4\left(-\frac{1}{2}\text{rMS} + B_0(p^2, 0, m_{W^-}^2)\right)\Gamma_{\check{H}_j^+, W^-, \gamma}^* \Gamma_{\check{H}_i^+, W^-, \gamma} + 4\left(-\frac{1}{2}\text{rMS} + B_0(p^2, m_{W^-}^2, m_Z^2)\right)\Gamma_{\check{H}_j^+, Z, W^-}^* \Gamma_{\check{H}_i^+, Z, W^-} \\
& - B_0(p^2, m_{\eta^Z}^2, m_{\eta^+}^2)\Gamma_{\check{H}_i^+, \eta^+, \eta^Z} \Gamma_{\check{H}_j^-, \eta^+, \eta^Z} - B_0(p^2, m_{\eta^-}^2, m_{\eta^Z}^2)\Gamma_{\check{H}_i^+, \eta^Z, \eta^-} \Gamma_{\check{H}_j^-, \eta^Z, \eta^-} \\
& + 4\Gamma_{\check{H}_i^-, \check{H}_j^+, W^+, W^-} \left(-\frac{1}{2}\text{rMS}m_{W^-}^2 + A_0(m_{W^-}^2)\right) + 2\Gamma_{\check{H}_i^-, \check{H}_j^+, Z, Z} \left(-\frac{1}{2}\text{rMS}m_Z^2 + A_0(m_Z^2)\right) \\
& - \frac{1}{2}\sum_{a=1}^2 A_0(m_{A_a^0}^2)\Gamma_{\check{H}_i^-, \check{H}_j^+, A_a^0, A_a^0} - \sum_{a=1}^2 A_0(m_{H_a^-}^2)\Gamma_{\check{H}_i^-, \check{H}_j^+, H_a^+, H_a^-} \\
& - \frac{1}{2}\sum_{a=1}^2 A_0(m_{h_a}^2)\Gamma_{\check{H}_i^-, \check{H}_j^+, h_a, h_a} + \sum_{a=1}^2 \sum_{b=1}^2 B_0(p^2, m_{H_a^-}^2, m_{A_b^0}^2)\Gamma_{\check{H}_j^+, H_a^-, A_b^0}^* \Gamma_{\check{H}_i^+, H_a^-, A_b^0} \\
& + \sum_{a=1}^2 \sum_{b=1}^2 B_0(p^2, m_{H_a^-}^2, m_{h_b}^2)\Gamma_{\check{H}_j^+, H_a^-, h_b} \Gamma_{\check{H}_i^+, H_a^-, h_b} \\
& - 6\sum_{a=1}^3 m_{u_a} \sum_{b=1}^3 B_0(p^2, m_{u_a}^2, m_{d_b}^2)m_{d_b} \left(\Gamma_{\check{H}_j^+, \bar{u}_a, d_b}^{L*} \Gamma_{\check{H}_i^+, \bar{u}_a, d_b}^R + \Gamma_{\check{H}_j^+, \bar{u}_a, d_b}^{R*} \Gamma_{\check{H}_i^+, \bar{u}_a, d_b}^L\right) \\
& + 3\sum_{a=1}^3 \sum_{b=1}^3 G_0(p^2, m_{u_a}^2, m_{d_b}^2) \left(\Gamma_{\check{H}_j^+, \bar{u}_a, d_b}^{L*} \Gamma_{\check{H}_i^+, \bar{u}_a, d_b}^L + \Gamma_{\check{H}_j^+, \bar{u}_a, d_b}^{R*} \Gamma_{\check{H}_i^+, \bar{u}_a, d_b}^R\right) \\
& - 2\sum_{a=1}^3 m_{\nu_a} \sum_{b=1}^3 B_0(p^2, m_{\nu_a}^2, m_{e_b}^2)m_{e_b} \left(\Gamma_{\check{H}_j^+, \bar{\nu}_a, e_b}^{L*} \Gamma_{\check{H}_i^+, \bar{\nu}_a, e_b}^R + \Gamma_{\check{H}_j^+, \bar{\nu}_a, e_b}^{R*} \Gamma_{\check{H}_i^+, \bar{\nu}_a, e_b}^L\right) \\
& + \sum_{a=1}^3 \sum_{b=1}^3 G_0(p^2, m_{\nu_a}^2, m_{e_b}^2) \left(\Gamma_{\check{H}_j^+, \bar{\nu}_a, e_b}^{L*} \Gamma_{\check{H}_i^+, \bar{\nu}_a, e_b}^L + \Gamma_{\check{H}_j^+, \bar{\nu}_a, e_b}^{R*} \Gamma_{\check{H}_i^+, \bar{\nu}_a, e_b}^R\right) \\
& + \sum_{b=1}^2 \Gamma_{\check{H}_j^+, W^-, A_b^0}^* \Gamma_{\check{H}_i^+, W^-, A_b^0} F_0(p^2, m_{A_b^0}^2, m_{W^-}^2) + \sum_{b=1}^2 \Gamma_{\check{H}_j^+, W^-, h_b}^* \Gamma_{\check{H}_i^+, W^-, h_b} F_0(p^2, m_{h_b}^2, m_{W^-}^2) \\
& + \sum_{b=1}^2 \Gamma_{\check{H}_j^+, \gamma, H_b^-}^* \Gamma_{\check{H}_i^+, \gamma, H_b^-} F_0(p^2, m_{H_b^-}^2, 0) + \sum_{b=1}^2 \Gamma_{\check{H}_j^+, Z, H_b^-}^* \Gamma_{\check{H}_i^+, Z, H_b^-} F_0(p^2, m_{H_b^-}^2, m_Z^2) \tag{87}
\end{aligned}$$

• Self-Energy for Down-Quarks (d)

$$\begin{aligned}
\Sigma_{i,j}^S(p^2) = & + \sum_{a=1}^2 \sum_{b=1}^3 B_0(p^2, m_{d_b}^2, m_{h_a}^2)\Gamma_{\check{d}_j^+, h_a, d_b}^{L*} m_{d_b} \Gamma_{\check{d}_i^+, h_a, d_b}^R \\
& + \sum_{a=1}^2 \sum_{b=1}^3 B_0(p^2, m_{u_b}^2, m_{H_a^-}^2)\Gamma_{\check{d}_j^+, H_a^-, u_b}^{L*} m_{u_b} \Gamma_{\check{d}_i^+, H_a^-, u_b}^R \\
& + \sum_{a=1}^3 m_{d_a} \sum_{b=1}^2 B_0(p^2, m_{d_a}^2, m_{A_b^0}^2)\Gamma_{\check{d}_j^+, d_a, A_b^0}^{L*} \Gamma_{\check{d}_i^+, d_a, A_b^0}^R
\end{aligned}$$



$$\begin{aligned}
& -\frac{16}{3} \sum_{b=1}^3 \left( -\frac{1}{2} \text{rMS} + B_0(p^2, m_{d_b}^2, 0) \right) \Gamma_{\tilde{d}_j, g, d_b}^{R*} m_{d_b} \Gamma_{\tilde{d}_i, g, d_b}^L - 4 \sum_{b=1}^3 \left( -\frac{1}{2} \text{rMS} + B_0(p^2, m_{d_b}^2, 0) \right) \Gamma_{\tilde{d}_j, \gamma, d_b}^{R*} m_{d_b} \Gamma_{\tilde{d}_i, \gamma, d_b}^L \\
& - 4 \sum_{b=1}^3 \left( -\frac{1}{2} \text{rMS} + B_0(p^2, m_{u_b}^2, m_{W^-}^2) \right) \Gamma_{\tilde{d}_j, W^-, u_b}^{R*} m_{u_b} \Gamma_{\tilde{d}_i, W^-, u_b}^L \\
& - 4 \sum_{b=1}^3 \left( -\frac{1}{2} \text{rMS} + B_0(p^2, m_{d_b}^2, m_Z^2) \right) \Gamma_{\tilde{d}_j, Z, d_b}^{R*} m_{d_b} \Gamma_{\tilde{d}_i, Z, d_b}^L \tag{88}
\end{aligned}$$

$$\begin{aligned}
\Sigma_{i,j}^R(p^2) &= -\frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, m_{h_a}^2) \Gamma_{\tilde{d}_j, h_a, d_b}^{R*} \Gamma_{\tilde{d}_i, h_a, d_b}^R \\
& - \frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, m_{H_a^-}^2) \Gamma_{\tilde{d}_j, H_a^-, u_b}^{R*} \Gamma_{\tilde{d}_i, H_a^-, u_b}^R \\
& - \frac{1}{2} \sum_{a=1}^3 \sum_{b=1}^2 B_1(p^2, m_{d_a}^2, m_{A_b^0}^2) \Gamma_{\tilde{d}_j, d_a, A_b^0}^{R*} \Gamma_{\tilde{d}_i, d_a, A_b^0}^R - \frac{4}{3} \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, 0) \Gamma_{\tilde{d}_j, g, d_b}^{L*} \Gamma_{\tilde{d}_i, g, d_b}^L \\
& - \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, 0) \Gamma_{\tilde{d}_j, \gamma, d_b}^{L*} \Gamma_{\tilde{d}_i, \gamma, d_b}^L - \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, m_{W^-}^2) \Gamma_{\tilde{d}_j, W^-, u_b}^{L*} \Gamma_{\tilde{d}_i, W^-, u_b}^L \\
& - \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, m_Z^2) \Gamma_{\tilde{d}_j, Z, d_b}^{L*} \Gamma_{\tilde{d}_i, Z, d_b}^L \tag{89}
\end{aligned}$$

$$\begin{aligned}
\Sigma_{i,j}^L(p^2) &= -\frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, m_{h_a}^2) \Gamma_{\tilde{d}_j, h_a, d_b}^{L*} \Gamma_{\tilde{d}_i, h_a, d_b}^L \\
& - \frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, m_{H_a^-}^2) \Gamma_{\tilde{d}_j, H_a^-, u_b}^{L*} \Gamma_{\tilde{d}_i, H_a^-, u_b}^L \\
& - \frac{1}{2} \sum_{a=1}^3 \sum_{b=1}^2 B_1(p^2, m_{d_a}^2, m_{A_b^0}^2) \Gamma_{\tilde{d}_j, d_a, A_b^0}^{L*} \Gamma_{\tilde{d}_i, d_a, A_b^0}^L - \frac{4}{3} \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, 0) \Gamma_{\tilde{d}_j, g, d_b}^{R*} \Gamma_{\tilde{d}_i, g, d_b}^R \\
& - \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, 0) \Gamma_{\tilde{d}_j, \gamma, d_b}^{R*} \Gamma_{\tilde{d}_i, \gamma, d_b}^R - \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, m_{W^-}^2) \Gamma_{\tilde{d}_j, W^-, u_b}^{R*} \Gamma_{\tilde{d}_i, W^-, u_b}^R \\
& - \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, m_Z^2) \Gamma_{\tilde{d}_j, Z, d_b}^{R*} \Gamma_{\tilde{d}_i, Z, d_b}^R \tag{90}
\end{aligned}$$

• Self-Energy for Up-Quarks ( $u$ )

$$\Sigma_{i,j}^S(p^2) = + \sum_{a=1}^2 \sum_{b=1}^3 B_0(p^2, m_{d_b}^2, m_{H_a^-}^2) \Gamma_{\tilde{u}_j, H_a^-, d_b}^{L*} m_{d_b} \Gamma_{\tilde{u}_i, H_a^-, d_b}^R$$

$$\begin{aligned}
& + \sum_{a=1}^2 \sum_{b=1}^3 B_0(p^2, m_{u_b}^2, m_{h_a}^2) \Gamma_{\tilde{u}_j, h_a, u_b}^{L*} m_{u_b} \Gamma_{\tilde{u}_i, h_a, u_b}^R \\
& + \sum_{a=1}^3 m_{u_a} \sum_{b=1}^2 B_0(p^2, m_{u_a}^2, m_{A_b^0}^2) \Gamma_{\tilde{u}_j, u_a, A_b^0}^{L*} \Gamma_{\tilde{u}_i, u_a, A_b^0}^R \\
& - \frac{16}{3} \sum_{b=1}^3 \left( -\frac{1}{2} \text{rMS} + B_0(p^2, m_{u_b}^2, 0) \right) \Gamma_{\tilde{u}_j, g, u_b}^{R*} m_{u_b} \Gamma_{\tilde{u}_i, g, u_b}^L - 4 \sum_{b=1}^3 \left( -\frac{1}{2} \text{rMS} + B_0(p^2, m_{u_b}^2, 0) \right) \Gamma_{\tilde{u}_j, \gamma, u_b}^{R*} m_{u_b} \Gamma_{\tilde{u}_i, \gamma, u_b}^L \\
& - 4 \sum_{b=1}^3 \left( -\frac{1}{2} \text{rMS} + B_0(p^2, m_{u_b}^2, m_Z^2) \right) \Gamma_{\tilde{u}_j, Z, u_b}^{R*} m_{u_b} \Gamma_{\tilde{u}_i, Z, u_b}^L \\
& - 4 \sum_{b=1}^3 \left( -\frac{1}{2} \text{rMS} + B_0(p^2, m_{d_b}^2, m_{W^-}^2) \right) \Gamma_{\tilde{u}_j, W^+, d_b}^{R*} m_{d_b} \Gamma_{\tilde{u}_i, W^+, d_b}^L \tag{91}
\end{aligned}$$

$$\begin{aligned}
\Sigma_{i,j}^R(p^2) &= -\frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, m_{H_a^-}^2) \Gamma_{\tilde{u}_j, H_a^+, d_b}^{R*} \Gamma_{\tilde{u}_i, H_a^+, d_b}^R \\
& - \frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, m_{h_a}^2) \Gamma_{\tilde{u}_j, h_a, u_b}^{R*} \Gamma_{\tilde{u}_i, h_a, u_b}^R \\
& - \frac{1}{2} \sum_{a=1}^3 \sum_{b=1}^2 B_1(p^2, m_{u_a}^2, m_{A_b^0}^2) \Gamma_{\tilde{u}_j, u_a, A_b^0}^{R*} \Gamma_{\tilde{u}_i, u_a, A_b^0}^R - \frac{4}{3} \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, 0) \Gamma_{\tilde{u}_j, g, u_b}^{L*} \Gamma_{\tilde{u}_i, g, u_b}^L \\
& - \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, 0) \Gamma_{\tilde{u}_j, \gamma, u_b}^{L*} \Gamma_{\tilde{u}_i, \gamma, u_b}^L - \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, m_Z^2) \Gamma_{\tilde{u}_j, Z, u_b}^{L*} \Gamma_{\tilde{u}_i, Z, u_b}^L \\
& - \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, m_{W^-}^2) \Gamma_{\tilde{u}_j, W^+, d_b}^{L*} \Gamma_{\tilde{u}_i, W^+, d_b}^L \tag{92}
\end{aligned}$$

$$\begin{aligned}
\Sigma_{i,j}^L(p^2) &= -\frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, m_{H_a^-}^2) \Gamma_{\tilde{u}_j, H_a^+, d_b}^{L*} \Gamma_{\tilde{u}_i, H_a^+, d_b}^L \\
& - \frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, m_{h_a}^2) \Gamma_{\tilde{u}_j, h_a, u_b}^{L*} \Gamma_{\tilde{u}_i, h_a, u_b}^L \\
& - \frac{1}{2} \sum_{a=1}^3 \sum_{b=1}^2 B_1(p^2, m_{u_a}^2, m_{A_b^0}^2) \Gamma_{\tilde{u}_j, u_a, A_b^0}^{L*} \Gamma_{\tilde{u}_i, u_a, A_b^0}^L - \frac{4}{3} \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, 0) \Gamma_{\tilde{u}_j, g, u_b}^{R*} \Gamma_{\tilde{u}_i, g, u_b}^R \\
& - \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, 0) \Gamma_{\tilde{u}_j, \gamma, u_b}^{R*} \Gamma_{\tilde{u}_i, \gamma, u_b}^R - \sum_{b=1}^3 B_1(p^2, m_{u_b}^2, m_Z^2) \Gamma_{\tilde{u}_j, Z, u_b}^{R*} \Gamma_{\tilde{u}_i, Z, u_b}^R \\
& - \sum_{b=1}^3 B_1(p^2, m_{d_b}^2, m_{W^-}^2) \Gamma_{\tilde{u}_j, W^+, d_b}^{R*} \Gamma_{\tilde{u}_i, W^+, d_b}^R \tag{93}
\end{aligned}$$

- Self-Energy for Leptons ( $e$ )

$$\begin{aligned}
\Sigma_{i,j}^S(p^2) = & + \sum_{a=1}^2 \sum_{b=1}^3 B_0(p^2, m_{e_b}^2, m_{h_a}^2) \Gamma_{\tilde{e}_j, h_a, e_b}^{L*} m_{e_b} \Gamma_{\tilde{e}_i, h_a, e_b}^R \\
& + \sum_{a=1}^2 \sum_{b=1}^3 B_0(p^2, m_{\nu_b}^2, m_{H_a^-}^2) \Gamma_{\tilde{e}_j, H_a^-, \nu_b}^{L*} m_{\nu_b} \Gamma_{\tilde{e}_i, H_a^-, \nu_b}^R \\
& + \sum_{a=1}^3 m_{e_a} \sum_{b=1}^2 B_0(p^2, m_{e_a}^2, m_{A_b^0}^2) \Gamma_{\tilde{e}_j, e_a, A_b^0}^{L*} \Gamma_{\tilde{e}_i, e_a, A_b^0}^R \\
& - 4 \sum_{b=1}^3 \left( -\frac{1}{2} \text{rMS} + B_0(p^2, m_{e_b}^2, 0) \right) \Gamma_{\tilde{e}_j, \gamma, e_b}^{R*} m_{e_b} \Gamma_{\tilde{e}_i, \gamma, e_b}^L \\
& - 4 \sum_{b=1}^3 \left( -\frac{1}{2} \text{rMS} + B_0(p^2, m_{\nu_b}^2, m_{W^-}^2) \right) \Gamma_{\tilde{e}_j, W^-, \nu_b}^{R*} m_{\nu_b} \Gamma_{\tilde{e}_i, W^-, \nu_b}^L \\
& - 4 \sum_{b=1}^3 \left( -\frac{1}{2} \text{rMS} + B_0(p^2, m_{e_b}^2, m_Z^2) \right) \Gamma_{\tilde{e}_j, Z, e_b}^{R*} m_{e_b} \Gamma_{\tilde{e}_i, Z, e_b}^L \tag{94}
\end{aligned}$$

$$\begin{aligned}
\Sigma_{i,j}^R(p^2) = & -\frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^3 B_1(p^2, m_{e_b}^2, m_{h_a}^2) \Gamma_{\tilde{e}_j, h_a, e_b}^{R*} \Gamma_{\tilde{e}_i, h_a, e_b}^R \\
& - \frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^3 B_1(p^2, m_{\nu_b}^2, m_{H_a^-}^2) \Gamma_{\tilde{e}_j, H_a^-, \nu_b}^{R*} \Gamma_{\tilde{e}_i, H_a^-, \nu_b}^R \\
& - \frac{1}{2} \sum_{a=1}^3 \sum_{b=1}^2 B_1(p^2, m_{e_a}^2, m_{A_b^0}^2) \Gamma_{\tilde{e}_j, e_a, A_b^0}^{R*} \Gamma_{\tilde{e}_i, e_a, A_b^0}^R - \sum_{b=1}^3 B_1(p^2, m_{e_b}^2, 0) \Gamma_{\tilde{e}_j, \gamma, e_b}^{L*} \Gamma_{\tilde{e}_i, \gamma, e_b}^L \\
& - \sum_{b=1}^3 B_1(p^2, m_{\nu_b}^2, m_{W^-}^2) \Gamma_{\tilde{e}_j, W^-, \nu_b}^{L*} \Gamma_{\tilde{e}_i, W^-, \nu_b}^L - \sum_{b=1}^3 B_1(p^2, m_{e_b}^2, m_Z^2) \Gamma_{\tilde{e}_j, Z, e_b}^{L*} \Gamma_{\tilde{e}_i, Z, e_b}^L \tag{95}
\end{aligned}$$

$$\begin{aligned}
\Sigma_{i,j}^L(p^2) = & -\frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^3 B_1(p^2, m_{e_b}^2, m_{h_a}^2) \Gamma_{\tilde{e}_j, h_a, e_b}^{L*} \Gamma_{\tilde{e}_i, h_a, e_b}^L \\
& - \frac{1}{2} \sum_{a=1}^2 \sum_{b=1}^3 B_1(p^2, m_{\nu_b}^2, m_{H_a^-}^2) \Gamma_{\tilde{e}_j, H_a^-, \nu_b}^{L*} \Gamma_{\tilde{e}_i, H_a^-, \nu_b}^L \\
& - \frac{1}{2} \sum_{a=1}^3 \sum_{b=1}^2 B_1(p^2, m_{e_a}^2, m_{A_b^0}^2) \Gamma_{\tilde{e}_j, e_a, A_b^0}^{L*} \Gamma_{\tilde{e}_i, e_a, A_b^0}^L - \sum_{b=1}^3 B_1(p^2, m_{e_b}^2, 0) \Gamma_{\tilde{e}_j, \gamma, e_b}^{R*} \Gamma_{\tilde{e}_i, \gamma, e_b}^R \\
& - \sum_{b=1}^3 B_1(p^2, m_{\nu_b}^2, m_{W^-}^2) \Gamma_{\tilde{e}_j, W^-, \nu_b}^{R*} \Gamma_{\tilde{e}_i, W^-, \nu_b}^R - \sum_{b=1}^3 B_1(p^2, m_{e_b}^2, m_Z^2) \Gamma_{\tilde{e}_j, Z, e_b}^{R*} \Gamma_{\tilde{e}_i, Z, e_b}^R \tag{96}
\end{aligned}$$

• Self-Energy for Z-Boson ( $Z$ )

$$\begin{aligned}
\Pi(p^2) = & +|\Gamma_{Z,\eta^-, \eta^-}|^2 B_{00}(p^2, m_{\eta^-}^2, m_{\eta^-}^2) + |\Gamma_{Z,\eta^+, \eta^+}|^2 B_{00}(p^2, m_{\eta^+}^2, m_{\eta^+}^2) \\
& - |\Gamma_{Z,W^+, W^-}|^2 \left(10B_{00}(p^2, m_{W^-}^2, m_{W^-}^2) + 2A_0(m_{W^-}^2) - 2\text{rMS}(2m_{W^-}^2 - \frac{1}{3}p^2) + B_0(p^2, m_{W^-}^2, m_{W^-}^2) \right) (2m_{W^-}^2 + 4p^2) \\
& + \frac{1}{2} \sum_{a=1}^2 A_0(m_{A_a^0}^2) \Gamma_{Z,Z,A_a^0, A_a^0} + \sum_{a=1}^2 A_0(m_{H_a^-}^2) \Gamma_{Z,Z,H_a^+, H_a^-} + \frac{1}{2} \sum_{a=1}^2 A_0(m_{h_a}^2) \Gamma_{Z,Z,h_a, h_a} \\
& - 4 \sum_{a=1}^2 \sum_{b=1}^2 |\Gamma_{Z,h_a, A_b^0}|^2 B_{00}(p^2, m_{A_b^0}^2, m_{h_a}^2) - 4 \sum_{a=1}^2 \sum_{b=1}^2 |\Gamma_{Z,H_a^+, H_b^-}|^2 B_{00}(p^2, m_{H_a^-}^2, m_{H_b^-}^2) \\
& + 3 \sum_{a=1}^3 \sum_{b=1}^3 \left[ \left( |\Gamma_{Z,\bar{d}_a, d_b}^L|^2 + |\Gamma_{Z,\bar{d}_a, d_b}^R|^2 \right) H_0(p^2, m_{d_a}^2, m_{d_b}^2) \right. \\
& \left. + 4B_0(p^2, m_{d_a}^2, m_{d_b}^2) m_{d_a} m_{d_b} \Re\left(\Gamma_{Z,\bar{d}_a, d_b}^{L*} \Gamma_{Z,\bar{d}_a, d_b}^R\right) \right] \\
& + \sum_{a=1}^3 \sum_{b=1}^3 \left[ \left( |\Gamma_{Z,\bar{e}_a, e_b}^L|^2 + |\Gamma_{Z,\bar{e}_a, e_b}^R|^2 \right) H_0(p^2, m_{e_a}^2, m_{e_b}^2) \right. \\
& \left. + 4B_0(p^2, m_{e_a}^2, m_{e_b}^2) m_{e_a} m_{e_b} \Re\left(\Gamma_{Z,\bar{e}_a, e_b}^{L*} \Gamma_{Z,\bar{e}_a, e_b}^R\right) \right] \\
& + 3 \sum_{a=1}^3 \sum_{b=1}^3 \left[ \left( |\Gamma_{Z,\bar{u}_a, u_b}^L|^2 + |\Gamma_{Z,\bar{u}_a, u_b}^R|^2 \right) H_0(p^2, m_{u_a}^2, m_{u_b}^2) \right. \\
& \left. + 4B_0(p^2, m_{u_a}^2, m_{u_b}^2) m_{u_a} m_{u_b} \Re\left(\Gamma_{Z,\bar{u}_a, u_b}^{L*} \Gamma_{Z,\bar{u}_a, u_b}^R\right) \right] \\
& + \sum_{a=1}^3 \sum_{b=1}^3 \left[ \left( |\Gamma_{Z,\bar{\nu}_a, \nu_b}^L|^2 + |\Gamma_{Z,\bar{\nu}_a, \nu_b}^R|^2 \right) H_0(p^2, m_{\nu_a}^2, m_{\nu_b}^2) \right. \\
& \left. + 4B_0(p^2, m_{\nu_a}^2, m_{\nu_b}^2) m_{\nu_a} m_{\nu_b} \Re\left(\Gamma_{Z,\bar{\nu}_a, \nu_b}^{L*} \Gamma_{Z,\bar{\nu}_a, \nu_b}^R\right) \right] \\
& + 2 \sum_{b=1}^2 |\Gamma_{Z,W^+, H_b^-}|^2 B_0(p^2, m_{W^-}^2, m_{H_b^-}^2) + \sum_{b=1}^2 |\Gamma_{Z,Z, h_b}|^2 B_0(p^2, m_Z^2, m_{h_b}^2) + 2\text{rMS} m_{W^-}^2 \Gamma_{Z,Z,W^+, W^-}^1 \\
& - A_0(m_{W^-}^2) \left( 4\Gamma_{Z,Z,W^+, W^-}^1 + \Gamma_{Z,Z,W^+, W^-}^2 + \Gamma_{Z,Z,W^+, W^-}^3 \right) \tag{97}
\end{aligned}$$

• **Self-Energy for W-Boson ( $W^-$ )**

$$\begin{aligned}
\Pi(p^2) = & 2\text{rMS} m_{W^-}^2 \Gamma_{W^-, W^+, W^+, W^-}^1 + 3 \sum_{a=1}^3 \sum_{b=1}^3 \left[ \left( |\Gamma_{W^+, \bar{u}_a, d_b}^L|^2 + |\Gamma_{W^+, \bar{u}_a, d_b}^R|^2 \right) H_0(p^2, m_{u_a}^2, m_{d_b}^2) \right. \\
& \left. + 4B_0(p^2, m_{u_a}^2, m_{d_b}^2) m_{d_b} m_{u_a} \Re\left(\Gamma_{W^+, \bar{u}_a, d_b}^{L*} \Gamma_{W^+, \bar{u}_a, d_b}^R\right) \right] - 4 \sum_{a=1}^2 \sum_{b=1}^2 |\Gamma_{W^+, H_a^-, A_b^0}|^2 B_{00}(p^2, m_{A_b^0}^2, m_{H_a^-}^2) - 4 \sum_{a=1}^2 \sum_{b=1}^2 |\Gamma_{W^+, H}
\end{aligned}$$

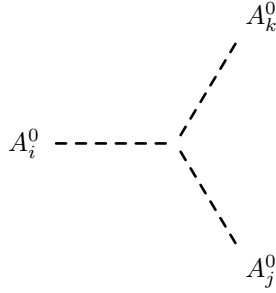
$$+ 4B_0\left(p^2, m_{\nu_a}^2, m_{e_b}^2\right) m_{e_b} m_{\nu_a} \Re\left(\Gamma_{W^+, \bar{\nu}_a, e_b}^{L*} \Gamma_{W^+, \bar{\nu}_a, e_b}^R\right)] + \sum_{b=1}^2 |\Gamma_{W^+, \gamma, H_b^-}|^2 B_0\left(p^2, 0, m_{H_b^-}^2\right) + \sum_{b=1}^2 |\Gamma_{W^+, W^-, h_b}|^2 B_0\left(p^2, m_{W^+}^2, m_{W^-}^2\right) \quad (98)$$

## 8.2 Tadpoles

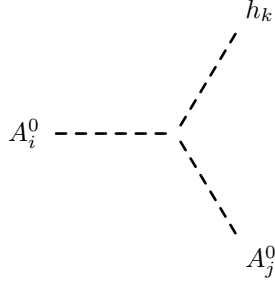
$$\begin{aligned} \delta t_h^{(1)} = & + A_0\left(m_{\eta^-}^2\right) \Gamma_{\tilde{h}_i, \eta^-, \eta^-} + A_0\left(m_{\eta^+}^2\right) \Gamma_{\tilde{h}_i, \eta^+, \eta^+} + A_0\left(m_{\eta^Z}^2\right) \Gamma_{\tilde{h}_i, \eta^Z, \eta^Z} \\ & + 4\Gamma_{\tilde{h}_i, W^+, W^-} \left(-\frac{1}{2} \text{rMS} m_{W^-}^2 + A_0\left(m_{W^-}^2\right)\right) + 2\Gamma_{\tilde{h}_i, Z, Z} \left(-\frac{1}{2} \text{rMS} m_Z^2 + A_0\left(m_Z^2\right)\right) - \frac{1}{2} \sum_{a=1}^2 A_0\left(m_{A_a^0}^2\right) \Gamma_{\tilde{h}_i, A_a^0, A_a^0} \\ & - \sum_{a=1}^2 A_0\left(m_{H_a^-}^2\right) \Gamma_{\tilde{h}_i, H_a^+, H_a^-} - \frac{1}{2} \sum_{a=1}^2 A_0\left(m_{h_a}^2\right) \Gamma_{\tilde{h}_i, h_a, h_a} \\ & + 6 \sum_{a=1}^3 A_0\left(m_{d_a}^2\right) m_{d_a} \left(\Gamma_{\tilde{h}_i, \bar{d}_a, d_a}^L + \Gamma_{\tilde{h}_i, \bar{d}_a, d_a}^R\right) \\ & + 2 \sum_{a=1}^3 A_0\left(m_{e_a}^2\right) m_{e_a} \left(\Gamma_{\tilde{h}_i, \bar{e}_a, e_a}^L + \Gamma_{\tilde{h}_i, \bar{e}_a, e_a}^R\right) \\ & + 6 \sum_{a=1}^3 A_0\left(m_{u_a}^2\right) m_{u_a} \left(\Gamma_{\tilde{h}_i, \bar{u}_a, u_a}^L + \Gamma_{\tilde{h}_i, \bar{u}_a, u_a}^R\right) \end{aligned} \quad (99)$$

## 9 Interactions for eigenstates 'EWSB'

### 9.1 Three Scalar-Interaction

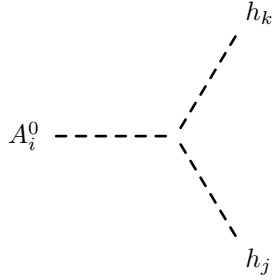


$$\begin{aligned} & \frac{1}{2} \left( Z_{i2}^A \left( Z_{j2}^A \left( 3v_1 \left( -\lambda_7^* + \lambda_7 \right) Z_{k2}^A + \left( -\lambda_5 v_1 - \lambda_7 v_2 + v_1 \lambda_5^* + v_2 \lambda_7^* \right) Z_{k1}^A \right) \right. \right. \\ & + Z_{j1}^A \left( \left( -\lambda_5 v_1 - \lambda_7 v_2 + v_1 \lambda_5^* + v_2 \lambda_7^* \right) Z_{k2}^A + \left( \lambda_5 v_2 + \lambda_6 v_1 - v_1 \lambda_6^* - v_2 \lambda_5^* \right) Z_{k1}^A \right) \right) \\ & + Z_{i1}^A \left( Z_{j1}^A \left( 3v_2 \left( -\lambda_6 + \lambda_6^* \right) Z_{k1}^A + \left( \lambda_5 v_2 + \lambda_6 v_1 - v_1 \lambda_6^* - v_2 \lambda_5^* \right) Z_{k2}^A \right) \right. \\ & \left. \left. + Z_{j2}^A \left( \left( -\lambda_5 v_1 - \lambda_7 v_2 + v_1 \lambda_5^* + v_2 \lambda_7^* \right) Z_{k2}^A + \left( \lambda_5 v_2 + \lambda_6 v_1 - v_1 \lambda_6^* - v_2 \lambda_5^* \right) Z_{k1}^A \right) \right) \right) \end{aligned} \quad (100)$$



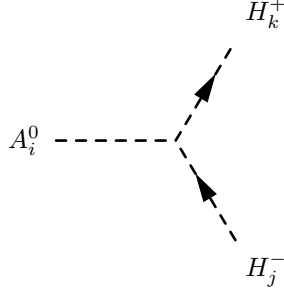
$$\begin{aligned}
& -\frac{i}{2} \left( Z_{i2}^A \left( Z_{j2}^A \left( (2\lambda_3 v_1 + 2\lambda_4 v_1 - \lambda_5 v_1 + \lambda_7 v_2 - v_1 \lambda_5^* + v_2 \lambda_7^*) Z_{k1}^H + (4\lambda_2 v_2 + \lambda_7 v_1 + v_1 \lambda_7^*) Z_{k2}^H \right) \right. \right. \\
& + Z_{j1}^A \left( (\lambda_5 v_1 + \lambda_7 v_2 + v_1 \lambda_5^* + v_2 \lambda_7^*) Z_{k2}^H + (\lambda_5 v_2 + \lambda_6 v_1 + v_1 \lambda_6^* + v_2 \lambda_5^*) Z_{k1}^H \right) \left. \right) \\
& + Z_{i1}^A \left( Z_{j1}^A \left( (2\lambda_3 v_2 + 2\lambda_4 v_2 - \lambda_5 v_2 + \lambda_6 v_1 + v_1 \lambda_6^* - v_2 \lambda_5^*) Z_{k2}^H + (4\lambda_1 v_1 + \lambda_6 v_2 + v_2 \lambda_6^*) Z_{k1}^H \right) \right. \\
& \left. \left. + Z_{j2}^A \left( (\lambda_5 v_1 + \lambda_7 v_2 + v_1 \lambda_5^* + v_2 \lambda_7^*) Z_{k2}^H + (\lambda_5 v_2 + \lambda_6 v_1 + v_1 \lambda_6^* + v_2 \lambda_5^*) Z_{k1}^H \right) \right) \right) \quad (101)
\end{aligned}$$


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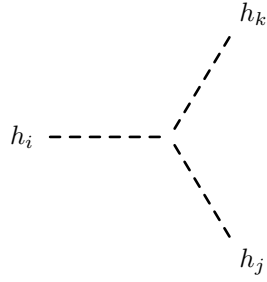
$$\begin{aligned}
& \frac{1}{2} \left( Z_{i2}^A \left( Z_{j2}^H \left( (-\lambda_5 v_1 + \lambda_7 v_2 + v_1 \lambda_5^* - v_2 \lambda_7^*) Z_{k1}^H + v_1 (-\lambda_7^* + \lambda_7) Z_{k2}^H \right) \right. \right. \\
& + Z_{j1}^H \left( (3\lambda_6 v_1 - 3v_1 \lambda_6^* - \lambda_5 v_2 + v_2 \lambda_5^*) Z_{k1}^H + (-\lambda_5 v_1 + \lambda_7 v_2 + v_1 \lambda_5^* - v_2 \lambda_7^*) Z_{k2}^H \right) \left. \right) \\
& + Z_{i1}^A \left( Z_{j1}^H \left( (\lambda_5 v_2 - \lambda_6 v_1 + v_1 \lambda_6^* - v_2 \lambda_5^*) Z_{k2}^H + v_2 (-\lambda_6 + \lambda_6^*) Z_{k1}^H \right) \right. \\
& \left. \left. + Z_{j2}^H \left( (-3\lambda_7 v_2 + 3v_2 \lambda_7^* + \lambda_5 v_1 - v_1 \lambda_5^*) Z_{k2}^H + (\lambda_5 v_2 - \lambda_6 v_1 + v_1 \lambda_6^* - v_2 \lambda_5^*) Z_{k1}^H \right) \right) \right) \quad (102)
\end{aligned}$$


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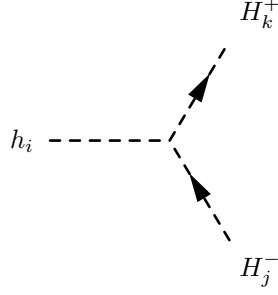
$$-\frac{1}{2} \left( -v_1 Z_{i2}^A + v_2 Z_{i1}^A \right) \left( Z_{j1}^+ \left( \left( -\lambda_4 + \lambda_5^* \right) Z_{k2}^+ + \left( -\lambda_6^* + \lambda_6 \right) Z_{k1}^+ \right) + Z_{j2}^+ \left( \left( -\lambda_5 + \lambda_4 \right) Z_{k1}^+ + \left( -\lambda_7^* + \lambda_7 \right) Z_{k2}^+ \right) \right) \quad (103)$$


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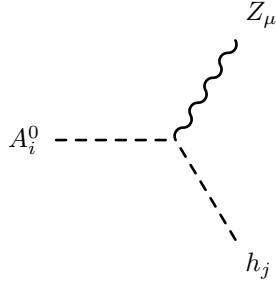
$$\begin{aligned} & -\frac{i}{2} \left( Z_{i2}^H \left( Z_{j2}^H \left( \left( 2\lambda_3 v_1 + 2\lambda_4 v_1 + 3\lambda_7 v_2 + 3v_2 \lambda_7^* + \lambda_5 v_1 + v_1 \lambda_5^* \right) Z_{k1}^H + 3 \left( 4\lambda_2 v_2 + \lambda_7 v_1 + v_1 \lambda_7^* \right) Z_{k2}^H \right) \right. \right. \\ & + Z_{j1}^H \left( \left( 2\lambda_3 v_2 + 2\lambda_4 v_2 + 3\lambda_6 v_1 + 3v_1 \lambda_6^* + \lambda_5 v_2 + v_2 \lambda_5^* \right) Z_{k1}^H \right. \\ & + \left. \left. \left( 2\lambda_3 v_1 + 2\lambda_4 v_1 + 3\lambda_7 v_2 + 3v_2 \lambda_7^* + \lambda_5 v_1 + v_1 \lambda_5^* \right) Z_{k2}^H \right) \right) \\ & + Z_{i1}^H \left( Z_{j1}^H \left( \left( 2\lambda_3 v_2 + 2\lambda_4 v_2 + 3\lambda_6 v_1 + 3v_1 \lambda_6^* + \lambda_5 v_2 + v_2 \lambda_5^* \right) Z_{k2}^H + 3 \left( 4\lambda_1 v_1 + \lambda_6 v_2 + v_2 \lambda_6^* \right) Z_{k1}^H \right) \right. \\ & + Z_{j2}^H \left( \left( 2\lambda_3 v_2 + 2\lambda_4 v_2 + 3\lambda_6 v_1 + 3v_1 \lambda_6^* + \lambda_5 v_2 + v_2 \lambda_5^* \right) Z_{k1}^H \right. \\ & + \left. \left. \left( 2\lambda_3 v_1 + 2\lambda_4 v_1 + 3\lambda_7 v_2 + 3v_2 \lambda_7^* + \lambda_5 v_1 + v_1 \lambda_5^* \right) Z_{k2}^H \right) \right) \right) \quad (104) \end{aligned}$$


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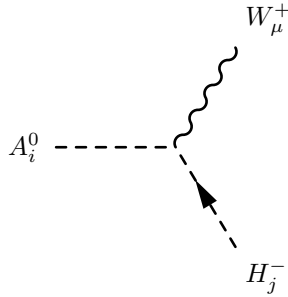


$$\begin{aligned}
& -\frac{i}{2} \left( Z_{i2}^H \left( Z_{j1}^+ \left( (2\lambda_3 v_2 + \lambda_6 v_1 + v_1 \lambda_6^*) Z_{k1}^+ + (2\lambda_7 v_2 + \lambda_4 v_1 + v_1 \lambda_5^*) Z_{k2}^+ \right) \right) \right. \\
& + Z_{j2}^+ \left( (2v_2 \lambda_7^* + (\lambda_4 + \lambda_5) v_1) Z_{k1}^+ + (4\lambda_2 v_2 + \lambda_7 v_1 + v_1 \lambda_7^*) Z_{k2}^+ \right) \\
& + Z_{i1}^H \left( Z_{j1}^+ \left( (2\lambda_6 v_1 + \lambda_4 v_2 + v_2 \lambda_5^*) Z_{k2}^+ + (4\lambda_1 v_1 + \lambda_6 v_2 + v_2 \lambda_6^*) Z_{k1}^+ \right) \right. \\
& \left. \left. + Z_{j2}^+ \left( (2\lambda_3 v_1 + \lambda_7 v_2 + v_2 \lambda_7^*) Z_{k2}^+ + (2v_1 \lambda_6^* + (\lambda_4 + \lambda_5) v_2) Z_{k1}^+ \right) \right) \right) \quad (105)
\end{aligned}$$

## 9.2 Two Scalar-One Vector Boson-Interaction



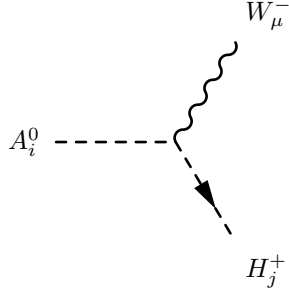
$$-\frac{1}{2} \left( g_1 \sin \Theta_W + g_2 \cos \Theta_W \right) \left( Z_{i1}^A Z_{j1}^H + Z_{i2}^A Z_{j2}^H \right) \left( -p_\mu^{h_j} + p_\mu^{A_i^0} \right) \quad (106)$$





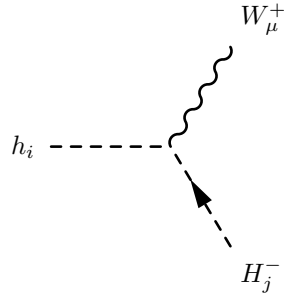
$$\frac{1}{2}g_2 \left( Z_{i1}^A Z_{j1}^+ + Z_{i2}^A Z_{j2}^+ \right) \left( -p_\mu^{H_j^-} + p_\mu^{A_i^0} \right) \quad (107)$$


---



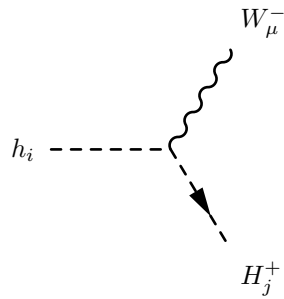
$$\frac{1}{2}g_2 \left( Z_{i1}^A Z_{j1}^+ + Z_{i2}^A Z_{j2}^+ \right) \left( -p_\mu^{H_j^+} + p_\mu^{A_i^0} \right) \quad (108)$$


---



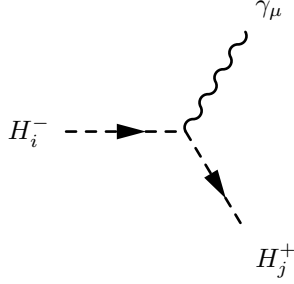
$$-\frac{i}{2}g_2 \left( Z_{i1}^H Z_{j1}^+ + Z_{i2}^H Z_{j2}^+ \right) \left( -p_\mu^{H_j^-} + p_\mu^{h_i} \right) \quad (109)$$


---



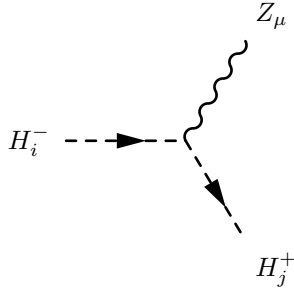
$$\frac{i}{2}g_2 \left( Z_{i1}^H Z_{j1}^+ + Z_{i2}^H Z_{j2}^+ \right) \left( -p_\mu^{H_j^+} + p_\mu^{h_i} \right) \quad (110)$$


---



$$\frac{i}{2} \left( g_1 \cos \Theta_W + g_2 \sin \Theta_W \right) \left( Z_{i1}^+ Z_{j1}^+ + Z_{i2}^+ Z_{j2}^+ \right) \left( -p_\mu^{H_j^+} + p_\mu^{H_i^-} \right) \quad (111)$$

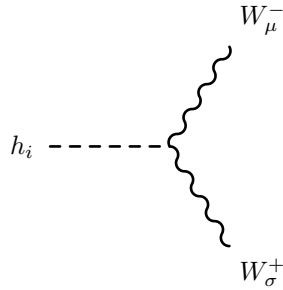

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$$\frac{i}{2} \left( -g_1 \sin \Theta_W + g_2 \cos \Theta_W \right) \left( Z_{i1}^+ Z_{j1}^+ + Z_{i2}^+ Z_{j2}^+ \right) \left( -p_\mu^{H_j^+} + p_\mu^{H_i^-} \right) \quad (112)$$

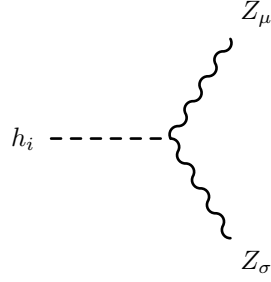

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### 9.3 One Scalar-Two Vector Boson-Interaction



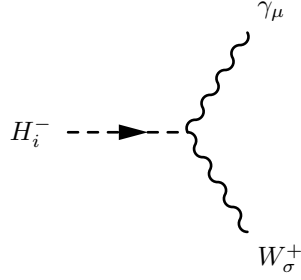
$$\frac{i}{2} g_2^2 \left( v_1 Z_{i1}^H + v_2 Z_{i2}^H \right) \left( g_{\sigma\mu} \right) \quad (113)$$


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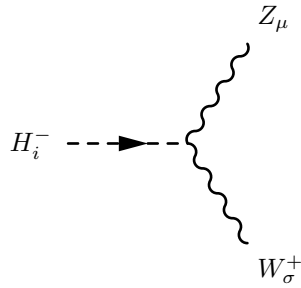
$$\frac{i}{2} \left( g_1 \sin \Theta_W + g_2 \cos \Theta_W \right)^2 \left( v_1 Z_{i1}^H + v_2 Z_{i2}^H \right) \left( g_{\sigma\mu} \right) \quad (114)$$


---



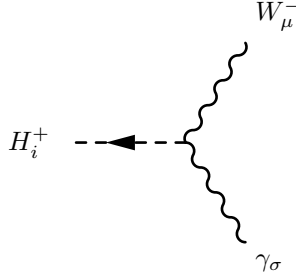
$$\frac{i}{2} g_1 g_2 \cos \Theta_W \left( v_1 Z_{i1}^+ + v_2 Z_{i2}^+ \right) \left( g_{\sigma\mu} \right) \quad (115)$$


---



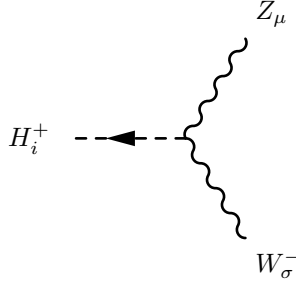
$$-\frac{i}{2} g_1 g_2 \sin \Theta_W \left( v_1 Z_{i1}^+ + v_2 Z_{i2}^+ \right) \left( g_{\sigma\mu} \right) \quad (116)$$


---



$$\frac{i}{2}g_1g_2 \cos \Theta_W \left( v_1Z_{i1}^+ + v_2Z_{i2}^+ \right) \left( g_{\sigma\mu} \right) \quad (117)$$

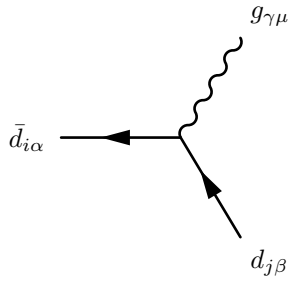

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$$-\frac{i}{2}g_1g_2 \sin \Theta_W \left( v_1Z_{i1}^+ + v_2Z_{i2}^+ \right) \left( g_{\sigma\mu} \right) \quad (118)$$


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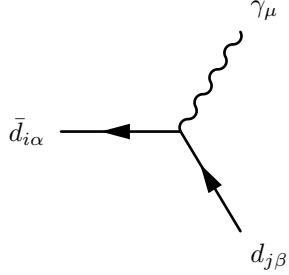
#### 9.4 Two Fermion-One Vector Boson-Interaction



$$-\frac{i}{2}g_3\delta_{ij}\lambda_{\alpha,\beta}^\gamma \left( \gamma_\mu \cdot \frac{1-\gamma_5}{2} \right) \quad (119)$$

$$+\frac{i}{2}g_3\delta_{ij}\lambda_{\alpha,\beta}^\gamma \left( \gamma_\mu \cdot \frac{1+\gamma_5}{2} \right) \quad (120)$$

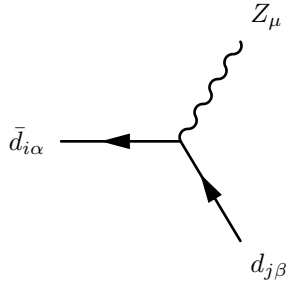

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$$-\frac{i}{6}\delta_{\alpha\beta}\delta_{ij}\left(-3g_2\sin\Theta_W+g_1\cos\Theta_W\right)\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (121)$$

$$+\frac{i}{3}g_1\cos\Theta_W\delta_{\alpha\beta}\delta_{ij}\left(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\right) \quad (122)$$

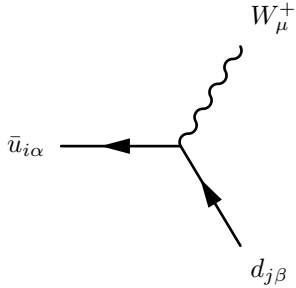

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$$\frac{i}{6}\delta_{\alpha\beta}\delta_{ij}\left(3g_2\cos\Theta_W+g_1\sin\Theta_W\right)\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (123)$$

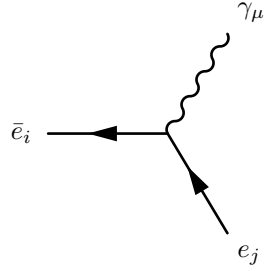
$$+\frac{i}{3}g_1\delta_{\alpha\beta}\delta_{ij}\sin\Theta_W\left(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\right) \quad (124)$$


---



$$-i\frac{1}{\sqrt{2}}g_2\delta_{\alpha\beta}\sum_{a=1}^3U_{L,ja}^{d,*}U_{L,ia}^u\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (125)$$

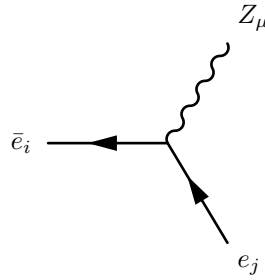

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$$\frac{i}{2}\delta_{ij}\left(g_1 \cos \Theta_W + g_2 \sin \Theta_W\right)\left(\gamma_\mu \cdot \frac{1-\gamma_5}{2}\right) \quad (126)$$

$$+ ig_1 \cos \Theta_W \delta_{ij}\left(\gamma_\mu \cdot \frac{1+\gamma_5}{2}\right) \quad (127)$$

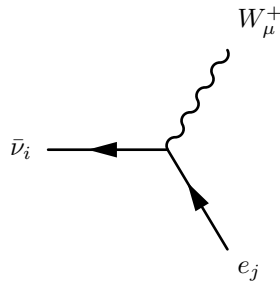

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$$\frac{i}{2}\delta_{ij}\left(-g_1 \sin \Theta_W + g_2 \cos \Theta_W\right)\left(\gamma_\mu \cdot \frac{1-\gamma_5}{2}\right) \quad (128)$$

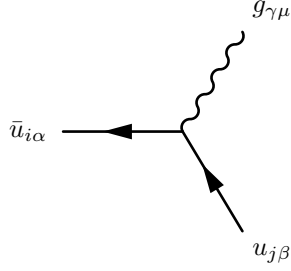
$$+ -ig_1 \delta_{ij} \sin \Theta_W \left(\gamma_\mu \cdot \frac{1+\gamma_5}{2}\right) \quad (129)$$


---



$$-i\frac{1}{\sqrt{2}}g_2 U_{L,i,j}^{e,*} \Theta_{i,3} \left(\gamma_\mu \cdot \frac{1-\gamma_5}{2}\right) \quad (130)$$

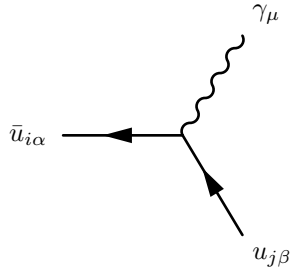

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$$-\frac{i}{2}g_3\delta_{ij}\lambda_{\alpha,\beta}^\gamma\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (131)$$

$$+\frac{i}{2}g_3\delta_{ij}\lambda_{\alpha,\beta}^\gamma\left(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\right) \quad (132)$$

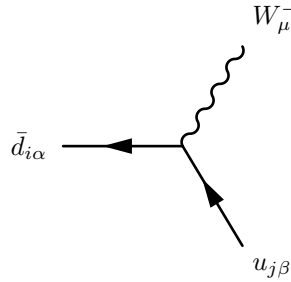

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$$-\frac{i}{6}\delta_{\alpha\beta}\delta_{ij}\left(3g_2\sin\Theta_W+g_1\cos\Theta_W\right)\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (133)$$

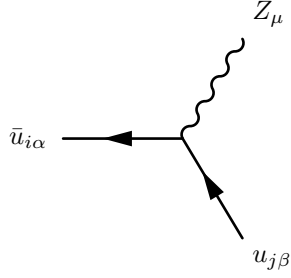
$$+\frac{2i}{3}g_1\cos\Theta_W\delta_{\alpha\beta}\delta_{ij}\left(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\right) \quad (134)$$


---



$$-i\frac{1}{\sqrt{2}}g_2\delta_{\alpha\beta}\sum_{a=1}^3U_{L,ja}^{u,*}U_{L,ia}^d\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (135)$$

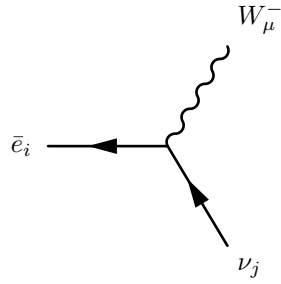

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$$-\frac{i}{6}\delta_{\alpha\beta}\delta_{ij}\left(3g_2\cos\Theta_W - g_1\sin\Theta_W\right)\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (136)$$

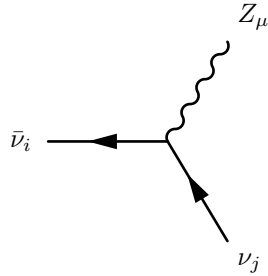
$$+\frac{2i}{3}g_1\delta_{\alpha\beta}\delta_{ij}\sin\Theta_W\left(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\right) \quad (137)$$


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$$-i\frac{1}{\sqrt{2}}g_2\Theta_{j,3}U_{L,ij}^e\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (138)$$


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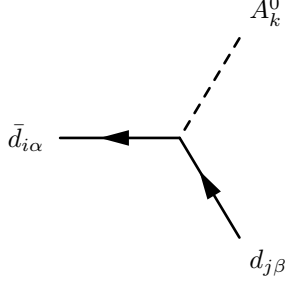


$$-\frac{i}{2}\delta_{ij}\left(g_1\sin\Theta_W + g_2\cos\Theta_W\right)\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (139)$$


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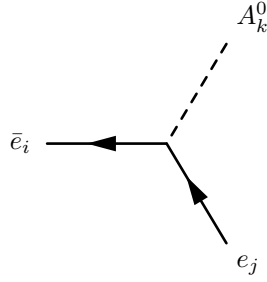
## 9.5 Two Fermion-One Scalar Boson-Interaction



$$-\frac{1}{\sqrt{2}}\delta_{\alpha\beta}\left(\sum_{b=1}^3 U_{L,jb}^{d,*} \sum_{a=1}^3 U_{R,ia}^{d,*} \epsilon_{d,ab} Z_{k2}^A + \sum_{b=1}^3 U_{L,jb}^{d,*} \sum_{a=1}^3 U_{R,ia}^{d,*} Y_{d,ab} Z_{k1}^A\right)\left(\frac{1-\gamma_5}{2}\right) \quad (140)$$

$$+\frac{1}{\sqrt{2}}\delta_{\alpha\beta}\left(\sum_{b=1}^3 \sum_{a=1}^3 \epsilon_{d,ab}^* U_{R,ja}^d U_{L,ib}^d Z_{k2}^A + \sum_{b=1}^3 \sum_{a=1}^3 Y_{d,ab}^* U_{R,ja}^d U_{L,ib}^d Z_{k1}^A\right)\left(\frac{1+\gamma_5}{2}\right) \quad (141)$$

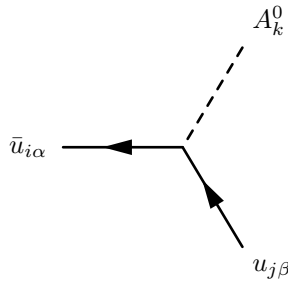

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$$-\frac{1}{\sqrt{2}}\left(\sum_{b=1}^3 U_{L,jb}^{e,*} \sum_{a=1}^3 U_{R,ia}^{e,*} \epsilon_{e,ab} Z_{k2}^A + \sum_{b=1}^3 U_{L,jb}^{e,*} \sum_{a=1}^3 U_{R,ia}^{e,*} Y_{e,ab} Z_{k1}^A\right)\left(\frac{1-\gamma_5}{2}\right) \quad (142)$$

$$+\frac{1}{\sqrt{2}}\left(\sum_{b=1}^3 \sum_{a=1}^3 \epsilon_{e,ab}^* U_{R,ja}^e U_{L,ib}^e Z_{k2}^A + \sum_{b=1}^3 \sum_{a=1}^3 Y_{e,ab}^* U_{R,ja}^e U_{L,ib}^e Z_{k1}^A\right)\left(\frac{1+\gamma_5}{2}\right) \quad (143)$$

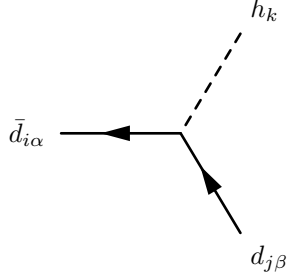

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$$- \frac{1}{\sqrt{2}} \delta_{\alpha\beta} \left( \sum_{b=1}^3 U_{L,jb}^{u,*} \sum_{a=1}^3 U_{R,ia}^{u,*} \epsilon_{u,ab} Z_{k1}^A + \sum_{b=1}^3 U_{L,jb}^{u,*} \sum_{a=1}^3 U_{R,ia}^{u,*} Y_{u,ab} Z_{k2}^A \right) \left( \frac{1-\gamma_5}{2} \right) \quad (144)$$

$$+ \frac{1}{\sqrt{2}} \delta_{\alpha\beta} \left( \sum_{b=1}^3 \sum_{a=1}^3 \epsilon_{u,ab}^* U_{R,ja}^u U_{L,ib}^u Z_{k1}^A + \sum_{b=1}^3 \sum_{a=1}^3 Y_{u,ab}^* U_{R,ja}^u U_{L,ib}^u Z_{k2}^A \right) \left( \frac{1+\gamma_5}{2} \right) \quad (145)$$

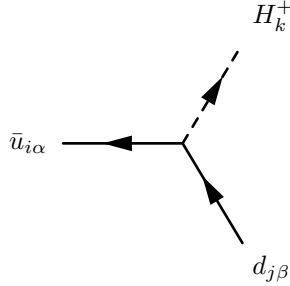

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$$- i \frac{1}{\sqrt{2}} \delta_{\alpha\beta} \left( \sum_{b=1}^3 U_{L,jb}^{d,*} \sum_{a=1}^3 U_{R,ia}^{d,*} \epsilon_{d,ab} Z_{k2}^H + \sum_{b=1}^3 U_{L,jb}^{d,*} \sum_{a=1}^3 U_{R,ia}^{d,*} Y_{d,ab} Z_{k1}^H \right) \left( \frac{1-\gamma_5}{2} \right) \quad (146)$$

$$+ -i \frac{1}{\sqrt{2}} \delta_{\alpha\beta} \left( \sum_{b=1}^3 \sum_{a=1}^3 \epsilon_{d,ab}^* U_{R,ja}^d U_{L,ib}^d Z_{k2}^H + \sum_{b=1}^3 \sum_{a=1}^3 Y_{d,ab}^* U_{R,ja}^d U_{L,ib}^d Z_{k1}^H \right) \left( \frac{1+\gamma_5}{2} \right) \quad (147)$$

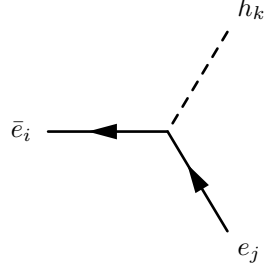

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$$- i \delta_{\alpha\beta} \left( \sum_{b=1}^3 U_{L,jb}^{d,*} \sum_{a=1}^3 U_{R,ia}^{u,*} \epsilon_{u,ab} Z_{k1}^+ + \sum_{b=1}^3 U_{L,jb}^{d,*} \sum_{a=1}^3 U_{R,ia}^{u,*} Y_{u,ab} Z_{k2}^+ \right) \left( \frac{1-\gamma_5}{2} \right) \quad (148)$$

$$+ -i \delta_{\alpha\beta} \left( \sum_{b=1}^3 \sum_{a=1}^3 \epsilon_{d,ab}^* U_{R,ja}^d U_{L,ib}^u Z_{k2}^+ + \sum_{b=1}^3 \sum_{a=1}^3 Y_{d,ab}^* U_{R,ja}^d U_{L,ib}^u Z_{k1}^+ \right) \left( \frac{1+\gamma_5}{2} \right) \quad (149)$$

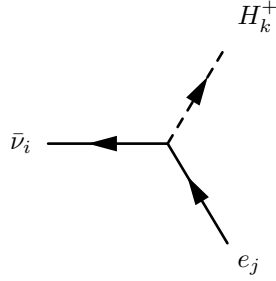

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$$-i \frac{1}{\sqrt{2}} \left( \sum_{b=1}^3 U_{L,jb}^{e,*} \sum_{a=1}^3 U_{R,ia}^{e,*} \epsilon_{e,ab} Z_{k2}^H + \sum_{b=1}^3 U_{L,jb}^{e,*} \sum_{a=1}^3 U_{R,ia}^{e,*} Y_{e,ab} Z_{k1}^H \right) \left( \frac{1-\gamma_5}{2} \right) \quad (150)$$

$$+ -i \frac{1}{\sqrt{2}} \left( \sum_{b=1}^3 \sum_{a=1}^3 \epsilon_{e,ab}^* U_{R,ja}^e U_{L,ib}^e Z_{k2}^H + \sum_{b=1}^3 \sum_{a=1}^3 Y_{e,ab}^* U_{R,ja}^e U_{L,ib}^e Z_{k1}^H \right) \left( \frac{1+\gamma_5}{2} \right) \quad (151)$$

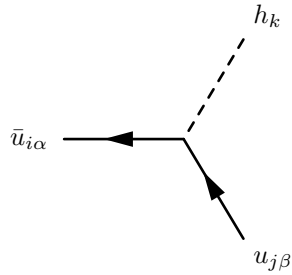

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$$(152)$$

$$+ -i \left( \sum_{a=1}^3 \epsilon_{e,ai}^* U_{R,ja}^e Z_{k2}^+ + \sum_{a=1}^3 Y_{e,ai}^* U_{R,ja}^e Z_{k1}^+ \right) \left( \frac{1+\gamma_5}{2} \right) \quad (153)$$

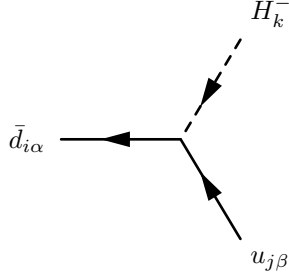

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$$i \frac{1}{\sqrt{2}} \delta_{\alpha\beta} \left( \sum_{b=1}^3 U_{L,jb}^{u,*} \sum_{a=1}^3 U_{R,ia}^{u,*} \epsilon_{u,ab} Z_{k1}^H + \sum_{b=1}^3 U_{L,jb}^{u,*} \sum_{a=1}^3 U_{R,ia}^{u,*} Y_{u,ab} Z_{k2}^H \right) \left( \frac{1-\gamma_5}{2} \right) \quad (154)$$

$$+ i \frac{1}{\sqrt{2}} \delta_{\alpha\beta} \left( \sum_{b=1}^3 \sum_{a=1}^3 \epsilon_{u,ab}^* U_{R,ja}^u U_{L,ib}^u Z_{k1}^H + \sum_{b=1}^3 \sum_{a=1}^3 Y_{u,ab}^* U_{R,ja}^u U_{L,ib}^u Z_{k2}^H \right) \left( \frac{1+\gamma_5}{2} \right) \quad (155)$$

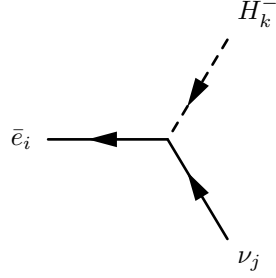

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$$- i \delta_{\alpha\beta} \left( \sum_{b=1}^3 U_{L,jb}^{u,*} \sum_{a=1}^3 U_{R,ia}^{d,*} \epsilon_{d,ab} Z_{k2}^+ + \sum_{b=1}^3 U_{L,jb}^{u,*} \sum_{a=1}^3 U_{R,ia}^{d,*} Y_{d,ab} Z_{k1}^+ \right) \left( \frac{1-\gamma_5}{2} \right) \quad (156)$$

$$+ -i \delta_{\alpha\beta} \left( \sum_{b=1}^3 \sum_{a=1}^3 \epsilon_{u,ab}^* U_{R,ja}^u U_{L,ib}^d Z_{k1}^+ + \sum_{b=1}^3 \sum_{a=1}^3 Y_{u,ab}^* U_{R,ja}^u U_{L,ib}^d Z_{k2}^+ \right) \left( \frac{1+\gamma_5}{2} \right) \quad (157)$$

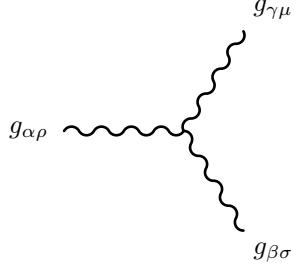

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$$- i \left( \sum_{a=1}^3 U_{R,ia}^{e,*} \epsilon_{e,aj} Z_{k2}^+ + \sum_{a=1}^3 U_{R,ia}^{e,*} Y_{e,aj} Z_{k1}^+ \right) \left( \frac{1-\gamma_5}{2} \right) \quad (158)$$

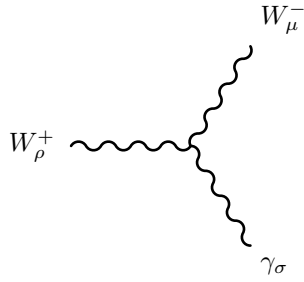

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## 9.6 Three Vector Boson-Interaction



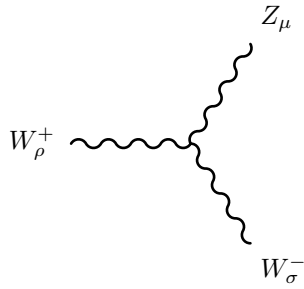
$$g_3 f_{\alpha,\beta,\gamma} \left( g_{\rho\mu} \left( -p_\sigma^{g_{\gamma\mu}} + p_\sigma^{g_{\alpha\rho}} \right) + g_{\rho\sigma} \left( -p_\mu^{g_{\alpha\rho}} + p_\mu^{g_{\beta\sigma}} \right) + g_{\sigma\mu} \left( -p_\rho^{g_{\beta\sigma}} + p_\rho^{g_{\gamma\mu}} \right) \right) \quad (159)$$


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$$i g_2 \sin \Theta_W \left( g_{\rho\mu} \left( -p_\sigma^{W_\mu^-} + p_\sigma^{W_\rho^+} \right) + g_{\rho\sigma} \left( -p_\mu^{W_\rho^+} + p_\mu^{\gamma_\sigma} \right) + g_{\sigma\mu} \left( -p_\rho^{\gamma_\sigma} + p_\rho^{W_\mu^-} \right) \right) \quad (160)$$

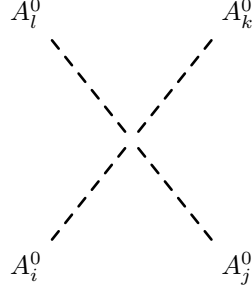

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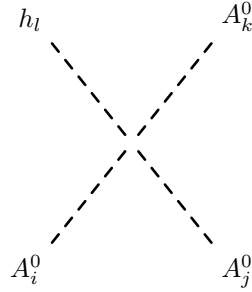
$$-i g_2 \cos \Theta_W \left( g_{\rho\mu} \left( -p_\sigma^{Z_\mu} + p_\sigma^{W_\rho^+} \right) + g_{\rho\sigma} \left( -p_\mu^{W_\rho^+} + p_\mu^{W_\sigma^-} \right) + g_{\sigma\mu} \left( -p_\rho^{W_\sigma^-} + p_\rho^{Z_\mu} \right) \right) \quad (161)$$


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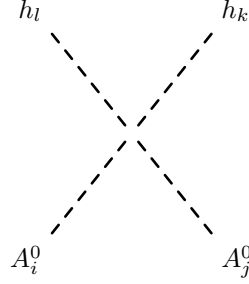
## 9.7 Four Scalar-Interaction



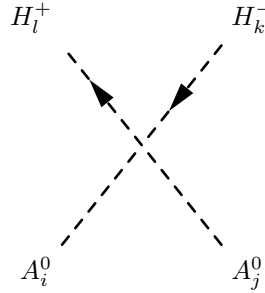
$$\begin{aligned}
& -\frac{i}{2} \left( Z_{i2}^A \left( Z_{j2}^A \left( 3Z_{k2}^A \left( 4\lambda_2 Z_{i2}^A + (\lambda_7 + \lambda_7^*) Z_{i1}^A \right) + Z_{k1}^A \left( (2\lambda_3 + 2\lambda_4 + \lambda_5 + \lambda_5^*) Z_{i1}^A + 3(\lambda_7 + \lambda_7^*) Z_{i2}^A \right) \right) \right) \right. \\
& + Z_{j1}^A \left( Z_{k1}^A \left( (2\lambda_3 + 2\lambda_4 + \lambda_5 + \lambda_5^*) Z_{i2}^A + 3(\lambda_6 + \lambda_6^*) Z_{i1}^A \right) + Z_{k2}^A \left( (2\lambda_3 + 2\lambda_4 + \lambda_5 + \lambda_5^*) Z_{i1}^A + 3(\lambda_7 + \lambda_7^*) Z_{i2}^A \right) \right) \\
& + Z_{i1}^A \left( Z_{j1}^A \left( 3Z_{k1}^A \left( 4\lambda_1 Z_{i1}^A + (\lambda_6 + \lambda_6^*) Z_{i2}^A \right) + Z_{k2}^A \left( (2\lambda_3 + 2\lambda_4 + \lambda_5 + \lambda_5^*) Z_{i2}^A + 3(\lambda_6 + \lambda_6^*) Z_{i1}^A \right) \right) \right. \\
& \left. \left. + Z_{j2}^A \left( Z_{k1}^A \left( (2\lambda_3 + 2\lambda_4 + \lambda_5 + \lambda_5^*) Z_{i2}^A + 3(\lambda_6 + \lambda_6^*) Z_{i1}^A \right) + Z_{k2}^A \left( (2\lambda_3 + 2\lambda_4 + \lambda_5 + \lambda_5^*) Z_{i1}^A + 3(\lambda_7 + \lambda_7^*) Z_{i2}^A \right) \right) \right) \right) \quad (162)
\end{aligned}$$



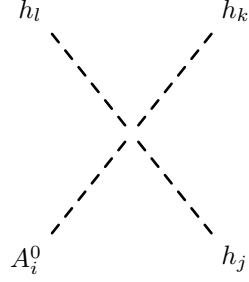
$$\begin{aligned}
& \frac{1}{2} \left( Z_{i2}^A \left( Z_{j2}^A \left( 3 \left( -\lambda_7^* + \lambda_7 \right) Z_{k2}^A Z_{i1}^H + Z_{k1}^A \left( \left( -\lambda_5 + \lambda_5^* \right) Z_{i1}^H + \left( -\lambda_7 + \lambda_7^* \right) Z_{i2}^H \right) \right) \right) \right. \\
& + Z_{j1}^A \left( Z_{k1}^A \left( \left( -\lambda_5^* + \lambda_5 \right) Z_{i2}^H + \left( -\lambda_6^* + \lambda_6 \right) Z_{i1}^H \right) + Z_{k2}^A \left( \left( -\lambda_5 + \lambda_5^* \right) Z_{i1}^H + \left( -\lambda_7 + \lambda_7^* \right) Z_{i2}^H \right) \right) \\
& + Z_{i1}^A \left( Z_{j1}^A \left( 3 \left( -\lambda_6 + \lambda_6^* \right) Z_{k1}^A Z_{i2}^H + Z_{k2}^A \left( \left( -\lambda_5^* + \lambda_5 \right) Z_{i2}^H + \left( -\lambda_6^* + \lambda_6 \right) Z_{i1}^H \right) \right) \right. \\
& \left. \left. + Z_{j2}^A \left( Z_{k1}^A \left( \left( -\lambda_5^* + \lambda_5 \right) Z_{i2}^H + \left( -\lambda_6^* + \lambda_6 \right) Z_{i1}^H \right) + Z_{k2}^A \left( \left( -\lambda_5 + \lambda_5^* \right) Z_{i1}^H + \left( -\lambda_7 + \lambda_7^* \right) Z_{i2}^H \right) \right) \right) \right) \quad (163)
\end{aligned}$$



$$\begin{aligned}
& -\frac{i}{2} \left( Z_{i2}^A \left( Z_{j2}^A \left( Z_{k1}^H \left( (2\lambda_3 + 2\lambda_4 - \lambda_5 - \lambda_5^*) Z_{l1}^H + (\lambda_7 + \lambda_7^*) Z_{l2}^H \right) + Z_{k2}^H \left( 4\lambda_2 Z_{l2}^H + (\lambda_7 + \lambda_7^*) Z_{l1}^H \right) \right) \right) \right. \\
& + Z_{j1}^A \left( Z_{k1}^H \left( (\lambda_5 + \lambda_5^*) Z_{l2}^H + (\lambda_6 + \lambda_6^*) Z_{l1}^H \right) + Z_{k2}^H \left( (\lambda_5 + \lambda_5^*) Z_{l1}^H + (\lambda_7 + \lambda_7^*) Z_{l2}^H \right) \right) \\
& + Z_{i1}^A \left( Z_{j1}^A \left( Z_{k1}^H \left( 4\lambda_1 Z_{l1}^H + (\lambda_6 + \lambda_6^*) Z_{l2}^H \right) + Z_{k2}^H \left( (2\lambda_3 + 2\lambda_4 - \lambda_5 - \lambda_5^*) Z_{l2}^H + (\lambda_6 + \lambda_6^*) Z_{l1}^H \right) \right) \right. \\
& \left. \left. + Z_{j2}^A \left( Z_{k1}^H \left( (\lambda_5 + \lambda_5^*) Z_{l2}^H + (\lambda_6 + \lambda_6^*) Z_{l1}^H \right) + Z_{k2}^H \left( (\lambda_5 + \lambda_5^*) Z_{l1}^H + (\lambda_7 + \lambda_7^*) Z_{l2}^H \right) \right) \right) \right) \quad (164)
\end{aligned}$$

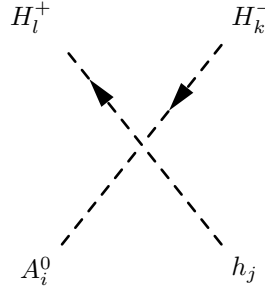


$$\begin{aligned}
& -\frac{i}{2} \left( Z_{i2}^A \left( 2Z_{j2}^A \left( Z_{k1}^+ \left( \lambda_3 Z_{l1}^+ + \lambda_7 Z_{l2}^+ \right) + Z_{k2}^+ \left( 2\lambda_2 Z_{l2}^+ + \lambda_7^* Z_{l1}^+ \right) \right) \right) \right. \\
& + Z_{j1}^A \left( Z_{k1}^+ \left( (\lambda_4 + \lambda_5^*) Z_{l2}^+ + (\lambda_6 + \lambda_6^*) Z_{l1}^+ \right) + Z_{k2}^+ \left( (\lambda_4 + \lambda_5) Z_{l1}^+ + (\lambda_7 + \lambda_7^*) Z_{l2}^+ \right) \right) \\
& + Z_{i1}^A \left( 2Z_{j1}^A \left( Z_{k1}^+ \left( 2\lambda_1 Z_{l1}^+ + \lambda_6 Z_{l2}^+ \right) + Z_{k2}^+ \left( \lambda_3 Z_{l2}^+ + \lambda_6^* Z_{l1}^+ \right) \right) \right. \\
& \left. \left. + Z_{j2}^A \left( Z_{k1}^+ \left( (\lambda_4 + \lambda_5^*) Z_{l2}^+ + (\lambda_6 + \lambda_6^*) Z_{l1}^+ \right) + Z_{k2}^+ \left( (\lambda_4 + \lambda_5) Z_{l1}^+ + (\lambda_7 + \lambda_7^*) Z_{l2}^+ \right) \right) \right) \right) \quad (165)
\end{aligned}$$



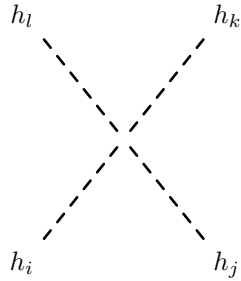
$$\begin{aligned}
& \frac{1}{2} \left( Z_{i2}^A \left( Z_{j2}^H \left( (-\lambda_7^* + \lambda_7) Z_{k2}^H Z_{l1}^H + Z_{k1}^H \left( (-\lambda_5 + \lambda_5^*) Z_{l1}^H + (-\lambda_7^* + \lambda_7) Z_{l2}^H \right) \right) \right) \right. \\
& + Z_{j1}^H \left( Z_{k1}^H \left( 3(-\lambda_6^* + \lambda_6) Z_{l1}^H + (-\lambda_5 + \lambda_5^*) Z_{l2}^H \right) + Z_{k2}^H \left( (-\lambda_5 + \lambda_5^*) Z_{l1}^H + (-\lambda_7^* + \lambda_7) Z_{l2}^H \right) \right) \left. \right) \\
& + Z_{i1}^A \left( Z_{j1}^H \left( (-\lambda_6 + \lambda_6^*) Z_{k1}^H Z_{l2}^H + Z_{k2}^H \left( (-\lambda_5^* + \lambda_5) Z_{l2}^H + (-\lambda_6 + \lambda_6^*) Z_{l1}^H \right) \right) \right) \\
& + Z_{j2}^H \left( Z_{k1}^H \left( (-\lambda_5^* + \lambda_5) Z_{l2}^H + (-\lambda_6 + \lambda_6^*) Z_{l1}^H \right) + Z_{k2}^H \left( 3(-\lambda_7 + \lambda_7^*) Z_{l2}^H + (-\lambda_5^* + \lambda_5) Z_{l1}^H \right) \right) \left. \right) \quad (166)
\end{aligned}$$


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$$\frac{1}{2} \left( -Z_{i1}^A Z_{j2}^H + Z_{i2}^A Z_{j1}^H \right) \left( Z_{k1}^+ \left( (-\lambda_4 + \lambda_5^*) Z_{l2}^+ + (-\lambda_6^* + \lambda_6) Z_{l1}^+ \right) + Z_{k2}^+ \left( (-\lambda_5 + \lambda_4) Z_{l1}^+ + (-\lambda_7^* + \lambda_7) Z_{l2}^+ \right) \right) \quad (167)$$

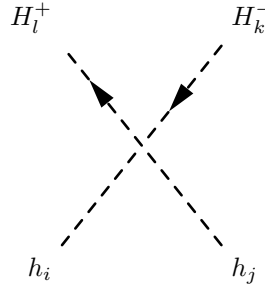

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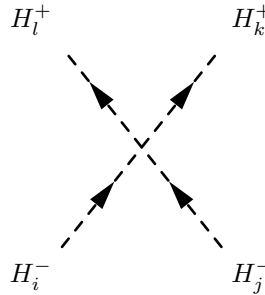
$$\begin{aligned}
& -\frac{i}{2} \left( Z_{i2}^H \left( Z_{j2}^H \left( 3Z_{k2}^H \left( 4\lambda_2 Z_{l2}^H + (\lambda_7 + \lambda_7^*) Z_{l1}^H \right) + Z_{k1}^H \left( (2\lambda_3 + 2\lambda_4 + \lambda_5 + \lambda_5^*) Z_{l1}^H + 3(\lambda_7 + \lambda_7^*) Z_{l2}^H \right) \right) \right. \right. \\
& + Z_{j1}^H \left( Z_{k1}^H \left( (2\lambda_3 + 2\lambda_4 + \lambda_5 + \lambda_5^*) Z_{l2}^H + 3(\lambda_6 + \lambda_6^*) Z_{l1}^H \right) + Z_{k2}^H \left( (2\lambda_3 + 2\lambda_4 + \lambda_5 + \lambda_5^*) Z_{l1}^H + 3(\lambda_7 + \lambda_7^*) Z_{l2}^H \right) \right) \left. \right) \\
& + Z_{i1}^H \left( Z_{j1}^H \left( 3Z_{k1}^H \left( 4\lambda_1 Z_{l1}^H + (\lambda_6 + \lambda_6^*) Z_{l2}^H \right) + Z_{k2}^H \left( (2\lambda_3 + 2\lambda_4 + \lambda_5 + \lambda_5^*) Z_{l2}^H + 3(\lambda_6 + \lambda_6^*) Z_{l1}^H \right) \right) \right. \\
& \left. \left. + Z_{j2}^H \left( Z_{k1}^H \left( (2\lambda_3 + 2\lambda_4 + \lambda_5 + \lambda_5^*) Z_{l2}^H + 3(\lambda_6 + \lambda_6^*) Z_{l1}^H \right) + Z_{k2}^H \left( (2\lambda_3 + 2\lambda_4 + \lambda_5 + \lambda_5^*) Z_{l1}^H + 3(\lambda_7 + \lambda_7^*) Z_{l2}^H \right) \right) \right) \right) \right) \quad (168)
\end{aligned}$$


---



$$\begin{aligned}
& -\frac{i}{2} \left( Z_{i2}^H \left( 2Z_{j2}^H \left( Z_{k1}^+ \left( \lambda_3 Z_{l1}^+ + \lambda_7 Z_{l2}^+ \right) + Z_{k2}^+ \left( 2\lambda_2 Z_{l2}^+ + \lambda_7^* Z_{l1}^+ \right) \right) \right. \right. \\
& + Z_{j1}^H \left( Z_{k1}^+ \left( (\lambda_4 + \lambda_5^*) Z_{l2}^+ + (\lambda_6 + \lambda_6^*) Z_{l1}^+ \right) + Z_{k2}^+ \left( (\lambda_4 + \lambda_5) Z_{l1}^+ + (\lambda_7 + \lambda_7^*) Z_{l2}^+ \right) \right) \left. \right) \\
& + Z_{i1}^H \left( 2Z_{j1}^H \left( Z_{k1}^+ \left( 2\lambda_1 Z_{l1}^+ + \lambda_6 Z_{l2}^+ \right) + Z_{k2}^+ \left( \lambda_3 Z_{l2}^+ + \lambda_6^* Z_{l1}^+ \right) \right) \right. \\
& \left. \left. + Z_{j2}^H \left( Z_{k1}^+ \left( (\lambda_4 + \lambda_5^*) Z_{l2}^+ + (\lambda_6 + \lambda_6^*) Z_{l1}^+ \right) + Z_{k2}^+ \left( (\lambda_4 + \lambda_5) Z_{l1}^+ + (\lambda_7 + \lambda_7^*) Z_{l2}^+ \right) \right) \right) \right) \quad (169)
\end{aligned}$$

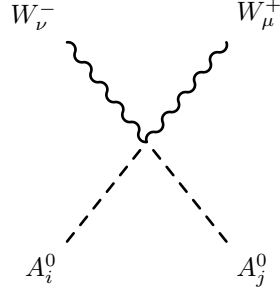

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$$\begin{aligned}
& -i \left( Z_{i1}^+ \left( Z_{j2}^+ \left( 2\lambda_6^* Z_{k1}^+ Z_{l1}^+ + (\lambda_3 + \lambda_4) Z_{k1}^+ Z_{l2}^+ + Z_{k2}^+ \left( 2\lambda_7 Z_{l2}^+ + (\lambda_3 + \lambda_4) Z_{l1}^+ \right) \right) \right. \right. \\
& + 2Z_{j1}^+ \left( Z_{k1}^+ \left( 2\lambda_1 Z_{l1}^+ + \lambda_6 Z_{l2}^+ \right) + Z_{k2}^+ \left( \lambda_5^* Z_{l2}^+ + \lambda_6 Z_{l1}^+ \right) \right) \left. \right) \\
& + Z_{i2}^+ \left( 2\lambda_6^* Z_{j1}^+ Z_{k1}^+ Z_{l1}^+ + Z_{j1}^+ \left( (\lambda_3 + \lambda_4) Z_{k1}^+ Z_{l2}^+ + Z_{k2}^+ \left( 2\lambda_7 Z_{l2}^+ + (\lambda_3 + \lambda_4) Z_{l1}^+ \right) \right) \right. \\
& \left. \left. + 2Z_{j2}^+ \left( Z_{k1}^+ \left( \lambda_5 Z_{l1}^+ + \lambda_7^* Z_{l2}^+ \right) + Z_{k2}^+ \left( 2\lambda_2 Z_{l2}^+ + \lambda_7^* Z_{l1}^+ \right) \right) \right) \right) \quad (170)
\end{aligned}$$

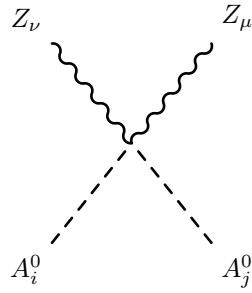

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## 9.8 Two Scalar-Two Vector Boson-Interaction



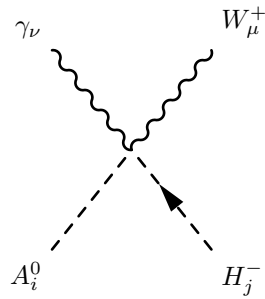
$$\frac{i}{2}g_2^2 \left( Z_{i1}^A Z_{j1}^A + Z_{i2}^A Z_{j2}^A \right) (g_{\mu\nu}) \quad (171)$$


---



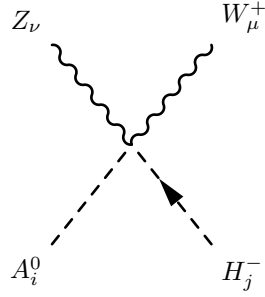
$$\frac{i}{2} \left( g_1 \sin \Theta_W + g_2 \cos \Theta_W \right)^2 \left( Z_{i1}^A Z_{j1}^A + Z_{i2}^A Z_{j2}^A \right) (g_{\mu\nu}) \quad (172)$$


---



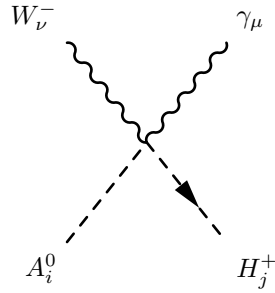
$$-\frac{1}{2}g_1g_2 \cos \Theta_W \left( Z_{i1}^A Z_{j1}^+ + Z_{i2}^A Z_{j2}^+ \right) (g_{\mu\nu}) \quad (173)$$


---



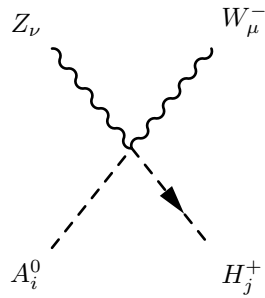
$$\frac{1}{2}g_1g_2 \sin \Theta_W \left( Z_{i1}^A Z_{j1}^+ + Z_{i2}^A Z_{j2}^+ \right) \left( g_{\mu\nu} \right) \quad (174)$$


---



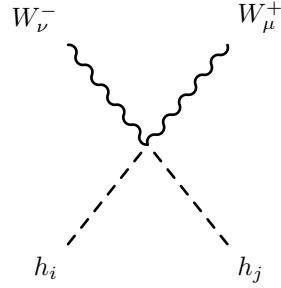
$$\frac{1}{2}g_1g_2 \cos \Theta_W \left( Z_{i1}^A Z_{j1}^+ + Z_{i2}^A Z_{j2}^+ \right) \left( g_{\mu\nu} \right) \quad (175)$$


---



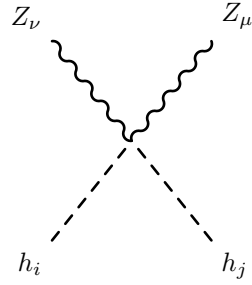
$$-\frac{1}{2}g_1g_2 \sin \Theta_W \left( Z_{i1}^A Z_{j1}^+ + Z_{i2}^A Z_{j2}^+ \right) \left( g_{\mu\nu} \right) \quad (176)$$


---



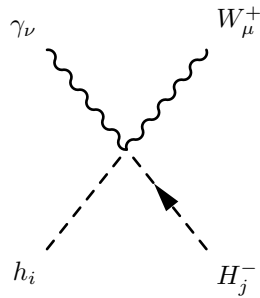
$$\frac{i}{2} g_2^2 \left( Z_{i1}^H Z_{j1}^H + Z_{i2}^H Z_{j2}^H \right) (g_{\mu\nu}) \quad (177)$$


---



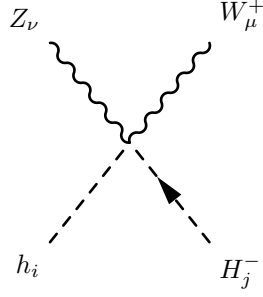
$$\frac{i}{2} \left( g_1 \sin \Theta_W + g_2 \cos \Theta_W \right)^2 \left( Z_{i1}^H Z_{j1}^H + Z_{i2}^H Z_{j2}^H \right) (g_{\mu\nu}) \quad (178)$$


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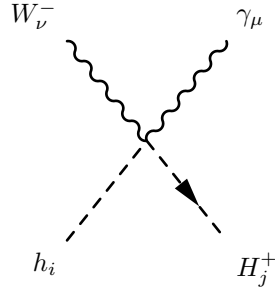
$$\frac{i}{2} g_1 g_2 \cos \Theta_W \left( Z_{i1}^H Z_{j1}^+ + Z_{i2}^H Z_{j2}^+ \right) (g_{\mu\nu}) \quad (179)$$


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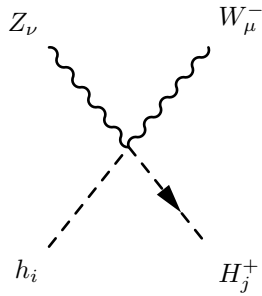
$$-\frac{i}{2}g_1g_2\sin\Theta_W\left(Z_{i1}^H Z_{j1}^+ + Z_{i2}^H Z_{j2}^+\right)\left(g_{\mu\nu}\right) \quad (180)$$


---



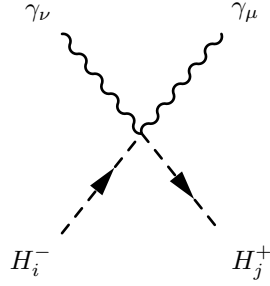
$$\frac{i}{2}g_1g_2\cos\Theta_W\left(Z_{i1}^H Z_{j1}^+ + Z_{i2}^H Z_{j2}^+\right)\left(g_{\mu\nu}\right) \quad (181)$$


---



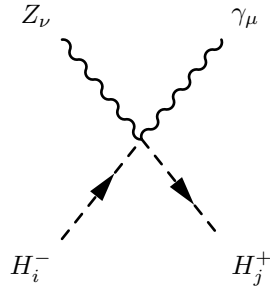
$$-\frac{i}{2}g_1g_2\sin\Theta_W\left(Z_{i1}^H Z_{j1}^+ + Z_{i2}^H Z_{j2}^+\right)\left(g_{\mu\nu}\right) \quad (182)$$


---



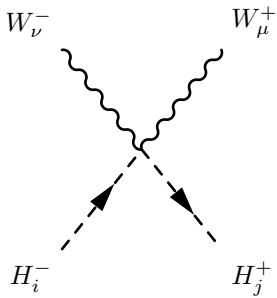
$$\frac{i}{2} (g_1 \cos \Theta_W + g_2 \sin \Theta_W)^2 (Z_{i1}^+ Z_{j1}^+ + Z_{i2}^+ Z_{j2}^+) (g_{\mu\nu}) \quad (183)$$


---



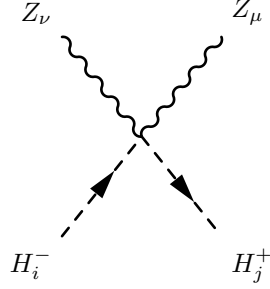
$$-\frac{i}{4} \left( -2g_1 g_2 \cos 2\Theta_W + (-g_2^2 + g_1^2) \sin 2\Theta_W \right) (Z_{i1}^+ Z_{j1}^+ + Z_{i2}^+ Z_{j2}^+) (g_{\mu\nu}) \quad (184)$$


---



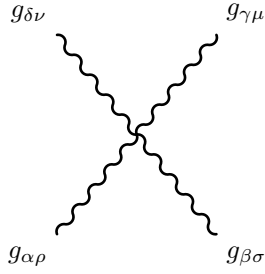
$$\frac{i}{2} g_2^2 (Z_{i1}^+ Z_{j1}^+ + Z_{i2}^+ Z_{j2}^+) (g_{\mu\nu}) \quad (185)$$


---



$$\frac{i}{2} \left( -g_1 \sin \Theta_W + g_2 \cos \Theta_W \right)^2 \left( Z_{i1}^+ Z_{j1}^+ + Z_{i2}^+ Z_{j2}^+ \right) (g_{\mu\nu}) \quad (186)$$

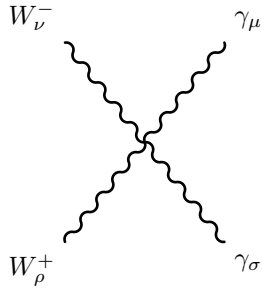
### 9.9 Four Vector Boson-Interaction



$$-ig_3^2 \left( \sum_{a=1}^8 f_{\alpha,\delta,a} f_{\beta,\gamma,a} + \sum_{a=1}^8 f_{\alpha,\gamma,a} f_{\beta,\delta,a} \right) (g_{\rho\sigma} g_{\mu\nu}) \quad (187)$$

$$+ ig_3^2 \left( -\sum_{a=1}^8 f_{\alpha,\beta,a} f_{\gamma,\delta,a} + \sum_{a=1}^8 f_{\alpha,\delta,a} f_{\beta,\gamma,a} \right) (g_{\rho\mu} g_{\sigma\nu}) \quad (188)$$

$$+ ig_3^2 \left( \sum_{a=1}^8 f_{\alpha,\gamma,a} f_{\beta,\delta,a} + \sum_{a=1}^8 f_{\alpha,\beta,a} f_{\gamma,\delta,a} \right) (g_{\rho\nu} g_{\sigma\mu}) \quad (189)$$

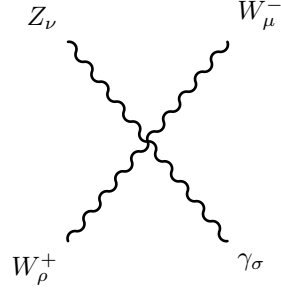


$$ig_2^2 \sin \Theta_W^2 (g_{\rho\sigma} g_{\mu\nu}) \quad (190)$$

$$+ ig_2^2 \sin \Theta_W^2 (g_{\rho\mu} g_{\sigma\nu}) \quad (191)$$

$$+ -2ig_2^2 \sin \Theta_W^2 (g_{\rho\nu} g_{\sigma\mu}) \quad (192)$$


---

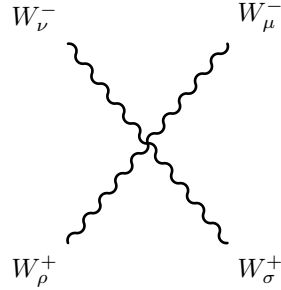


$$ig_2^2 \cos \Theta_W \sin \Theta_W (g_{\rho\sigma} g_{\mu\nu}) \quad (193)$$

$$+ -ig_2^2 \sin 2\Theta_W (g_{\rho\mu} g_{\sigma\nu}) \quad (194)$$

$$+ ig_2^2 \cos \Theta_W \sin \Theta_W (g_{\rho\nu} g_{\sigma\mu}) \quad (195)$$


---



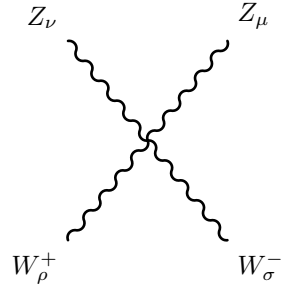
$$2ig_2^2 (g_{\rho\sigma} g_{\mu\nu}) \quad (196)$$

$$+ -ig_2^2 (g_{\rho\mu} g_{\sigma\nu}) \quad (197)$$

$$+ -ig_2^2 (g_{\rho\nu} g_{\sigma\mu}) \quad (198)$$


---



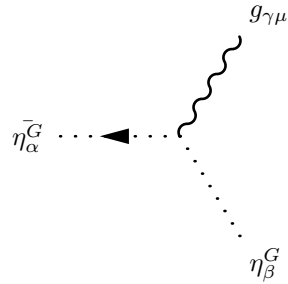


$$- 2ig_2^2 \cos \Theta_W^2 (g_{\rho\sigma}g_{\mu\nu}) \quad (199)$$

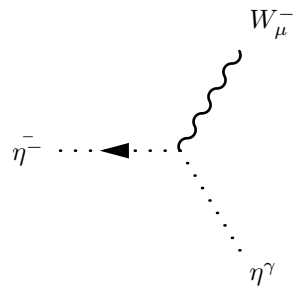
$$+ ig_2^2 \cos \Theta_W^2 (g_{\rho\mu}g_{\sigma\nu}) \quad (200)$$

$$+ ig_2^2 \cos \Theta_W^2 (g_{\rho\nu}g_{\sigma\mu}) \quad (201)$$

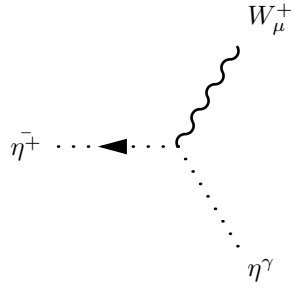
### 9.10 Two Ghosts-One Vector Boson-Interaction



$$g_3 f_{\alpha,\beta,\gamma} (p_\mu^{\eta_\beta^G}) \quad (202)$$

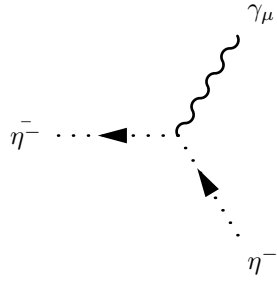


$$ig_2 \sin \Theta_W (p_\mu^{\eta^\gamma}) \quad (203)$$



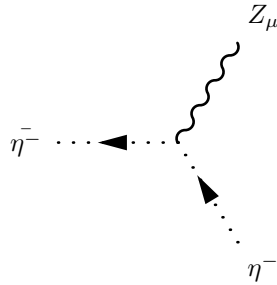
$$-ig_2 \sin \Theta_W (p_\mu^{\eta^\gamma}) \quad (204)$$


---



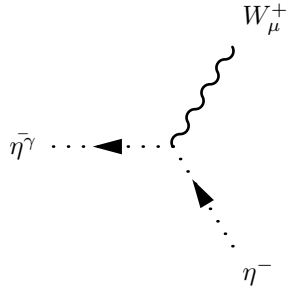
$$-ig_2 \sin \Theta_W (p_\mu^{\eta^-}) \quad (205)$$


---



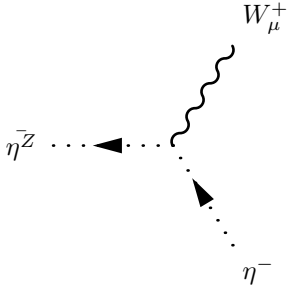
$$-ig_2 \cos \Theta_W (p_\mu^{\eta^-}) \quad (206)$$


---



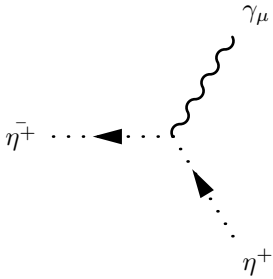
$$ig_2 \sin \Theta_W (p_\mu^{\eta^-}) \quad (207)$$


---



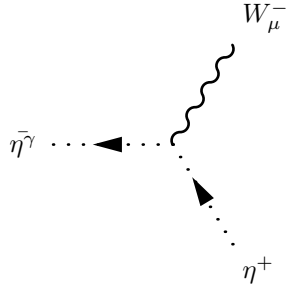
$$ig_2 \cos \Theta_W (p_\mu^{\eta^-}) \quad (208)$$


---



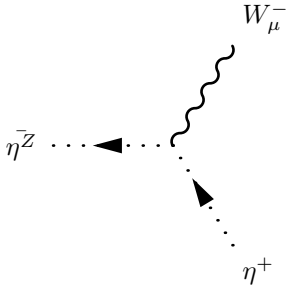
$$ig_2 \sin \Theta_W (p_\mu^{\eta^+}) \quad (209)$$


---



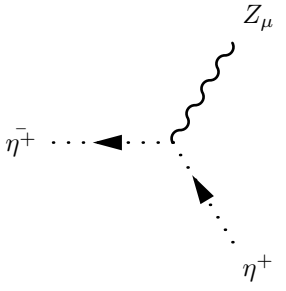
$$-ig_2 \sin \Theta_W (p_\mu^{\eta^+}) \quad (210)$$


---



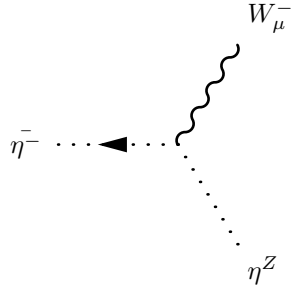
$$-ig_2 \cos \Theta_W (p_\mu^{\eta^+}) \quad (211)$$


---



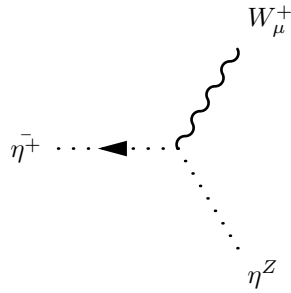
$$ig_2 \cos \Theta_W (p_\mu^{\eta^+}) \quad (212)$$


---



$$ig_2 \cos \Theta_W (p_\mu^{\eta^Z}) \quad (213)$$

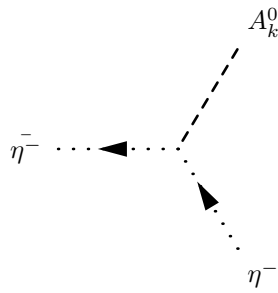

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$$-ig_2 \cos \Theta_W (p_\mu^{\eta^Z}) \quad (214)$$

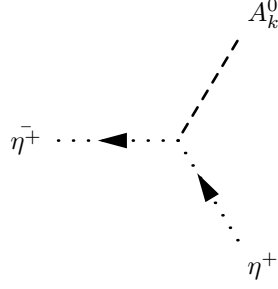

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### 9.11 Two Ghosts-One Scalar-Interaction



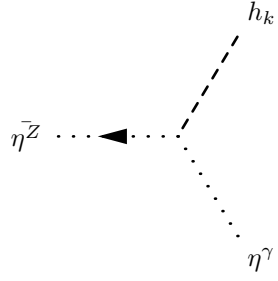
$$-\frac{1}{4} g_2^2 \xi_{W^-} (v_1 Z_{k1}^A + v_2 Z_{k2}^A) \quad (215)$$


---



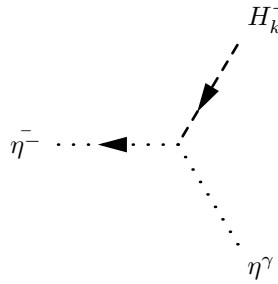
$$\frac{1}{4}g_2^2\xi_{W^-}\left(v_1Z_{k1}^A+v_2Z_{k2}^A\right) \quad (216)$$


---



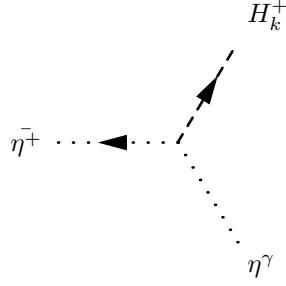
$$\frac{i}{8}\xi_Z\left(2g_1g_2\cos 2\Theta_W+\left(-g_2^2+g_1^2\right)\sin 2\Theta_W\right)\left(v_1Z_{k1}^H+v_2Z_{k2}^H\right) \quad (217)$$


---



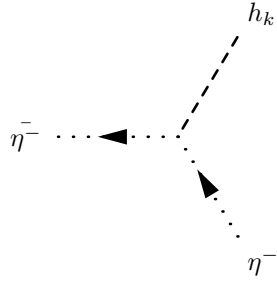
$$-\frac{i}{4}g_2\xi_{W^-}\left(g_1\cos \Theta_W+g_2\sin \Theta_W\right)\left(v_1Z_{k1}^++v_2Z_{k2}^+\right) \quad (218)$$


---



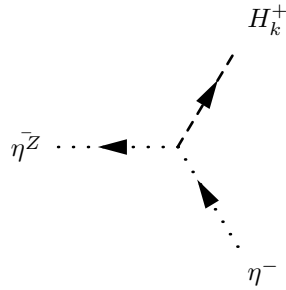
$$-\frac{i}{4}g_2\xi_{W^-}\left(g_1\cos\Theta_W+g_2\sin\Theta_W\right)\left(v_1Z_{k1}^++v_2Z_{k2}^+\right) \quad (219)$$


---



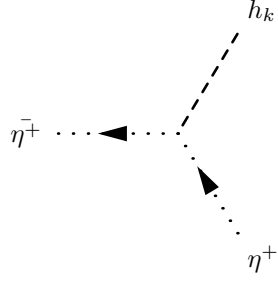
$$-\frac{i}{4}g_2^2\xi_{W^-}\left(v_1Z_{k1}^H+v_2Z_{k2}^H\right) \quad (220)$$


---



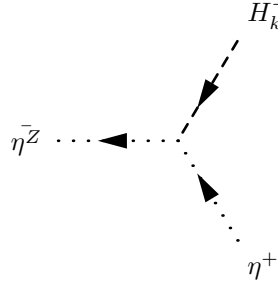
$$\frac{i}{4}g_2\xi_Z\left(g_1\sin\Theta_W+g_2\cos\Theta_W\right)\left(v_1Z_{k1}^++v_2Z_{k2}^+\right) \quad (221)$$


---



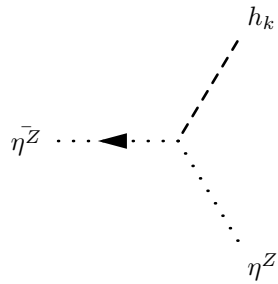
$$-\frac{i}{4}g_2^2\xi_{W^-}\left(v_1Z_{k1}^H+v_2Z_{k2}^H\right) \quad (222)$$


---



$$\frac{i}{4}g_2\xi_Z\left(g_1\sin\Theta_W+g_2\cos\Theta_W\right)\left(v_1Z_{k1}^++v_2Z_{k2}^+\right) \quad (223)$$

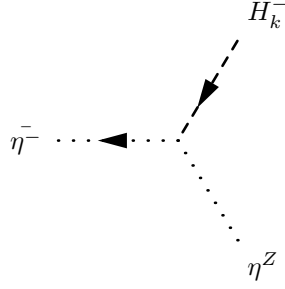

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$$-\frac{i}{4}\xi_Z\left(g_1\sin\Theta_W+g_2\cos\Theta_W\right)^2\left(v_1Z_{k1}^H+v_2Z_{k2}^H\right) \quad (224)$$

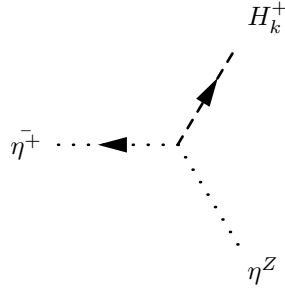

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$$-\frac{i}{4}g_2\xi_{W^-}\left(-g_1\sin\Theta_W+g_2\cos\Theta_W\right)\left(v_1Z_{k1}^++v_2Z_{k2}^+\right) \quad (225)$$


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$$-\frac{i}{4}g_2\xi_{W^-}\left(-g_1\sin\Theta_W+g_2\cos\Theta_W\right)\left(v_1Z_{k1}^++v_2Z_{k2}^+\right) \quad (226)$$


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## 10 Clebsch-Gordan Coefficients