NMSSM without flavor violation
Superpotential, Rotations and Interactions for eigenstates 'EWSB' including Renormalization Group Equations

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1 Superfields

1.1 Vector Superfields

SF	Spin $\frac{1}{2}$	Spin 1	$SU(N)$	Coupling	Name
$\hat{B}$	$\lambda_{\tilde{B}}$	B	$U(1)$	g_1	hypercharge
$\hat{W}$	$\lambda_{\tilde{W}}$	W	SU(2)	g_2	left
$\hat{g}$	$\lambda_{\tilde{g}}$	g	SU(3)	g_3	color

1.2 Chiral Superfields

SF	Spin 0	Spin $\frac{1}{2}$	Generations	$(U(1) \otimes \mathrm{SU}(2) \otimes \mathrm{SU}(3))$
$\hat{q}$	$\tilde{q}$	q	3	$(\frac{1}{6}, \mathbf{2}, \mathbf{3})$
$\hat{l}$	$\tilde{l}$	l	3	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{H}_d$	H_d	$\tilde{H}_d$	1	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{H}_u$	H_u	$\tilde{H}_u$	1	$(\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{d}$	$\tilde{d}_R^*$	d_R^*	3	$(\frac{1}{3}, \mathbf{1}, \overline{\mathbf{3}})$
$\hat{u}$	$\tilde{u}_R^*$	u_R^*	3	$(-\frac{2}{3}, \mathbf{1}, \overline{\mathbf{3}})$
$\hat{e}$	$\tilde{e}_R^*$	e_R^*	3	$(1, \mathbf{1}, \mathbf{1})$
$\hat{s}$	S	$\tilde{S}$	1	$(0, \mathbf{1}, \mathbf{1})$

2 Superpotential and Lagrangian

2.1 Superpotential

$$W = -Y_d\,\hat{d}\,\hat{q}\,\hat{H}_d\; - Y_e\,\hat{e}\,\hat{l}\,\hat{H}_d\; + \lambda\,\hat{H}_u\,\hat{H}_d\,\hat{s}\; + \frac{1}{3}\kappa\,\hat{s}\,\hat{s}\,\hat{s}\; + Y_u\,\hat{u}\,\hat{q}\,\hat{H}_u \tag{1}$$

2.2 Softbreaking terms

$$\begin{aligned}
-L_{SB,W} = &+ \frac{1}{3}S^3 T_\kappa - H_d^0 H_u^0 S T_\lambda + H_d^- H_u^+ S T_\lambda + H_d^0 \tilde{d}^*_{R,i\alpha}\delta_{\alpha\beta}\delta_{ij}\tilde{d}_{L,j\beta}T_{d,ij} \\
&- H_d^- \tilde{d}^*_{R,i\alpha}\delta_{\alpha\beta}\delta_{ij}\tilde{u}_{L,j\beta}T_{d,ij} + H_d^0 \tilde{e}^*_{R,i}\delta_{ij}\tilde{e}_{L,j}T_{e,ij} - H_d^-\tilde{e}^*_{R,i}\delta_{ij}\tilde{\nu}_j T_{e,ij} \\
&- H_u^+ \tilde{u}^*_{R,i\alpha}\delta_{\alpha\beta}\delta_{ij}\tilde{d}_{L,j\beta}T_{u,ij} + H_u^0 \tilde{u}^*_{R,i\alpha}\delta_{\alpha\beta}\delta_{ij}\tilde{u}_{L,j\beta}T_{u,ij} + \text{h.c.} \qquad (2) \\
-L_{SB,\phi} = &+ m_{H_d}^2|H_d^0|^2 + m_{H_d}^2|H_d^-|^2 + m_{H_u}^2|H_u^0|^2 + m_{H_u}^2|H_u^+|^2 + m_S^2|S|^2 + \tilde{d}^*_{L,i\alpha}\delta_{\alpha\beta}m^2_{q,ij}\tilde{d}_{L,j\beta} \\
&+ \tilde{d}^*_{R,i\alpha}\delta_{\alpha\beta}m^2_{d,ij}\tilde{d}_{R,j\beta} + \tilde{e}^*_{L,i}m^2_{l,ij}\tilde{e}_{L,j} + \tilde{e}^*_{R,i}m^2_{e,ij}\tilde{e}_{R,j} + \tilde{u}^*_{L,i\alpha}\delta_{\alpha\beta}m^2_{q,ij}\tilde{u}_{L,j\beta} \\
&+ \tilde{u}^*_{R,i\alpha}\delta_{\alpha\beta}m^2_{u,ij}\tilde{u}_{R,j\beta} + \tilde{\nu}_i^* m^2_{l,ij}\tilde{\nu}_j \qquad (3) \\
-L_{SB,\lambda} = &\frac{1}{2}\left(\lambda_{\tilde{B}}^2 M_1\delta_{ij} + M_2\delta_{ij}\lambda_{\tilde{W},i}\lambda_{\tilde{W},j} + M_3\delta_{ij}\lambda_{\tilde{g},\alpha}\lambda_{\tilde{g},\beta} + \text{h.c.}\right) \qquad (4)
\end{aligned}$$

2.3 Gauge fixing terms

2.3.1 Gauge fixing terms for eigenstates 'GaugeES'

$$L_{GF} = -\frac{1}{2}|\partial_\mu B|^2\xi_B^{-1} - \frac{1}{2}|\partial_\mu g|^2\xi_g^{-1} - \frac{1}{2}|\partial_\mu W|^2\xi_W^{-1} \tag{5}$$

2.3.2 Gauge fixing terms for eigenstates 'EWSB'

$$\begin{aligned} L_{GF} = &-\frac{1}{2}|\partial_\mu g|^2\xi_g^{-1} - \frac{1}{2}|\partial_\mu \gamma|^2\xi_\gamma^{-1} - |-\frac{i}{2}g_2\Big(H_d^- v_d - v_u H_u^{+,*}\Big)\xi_{W^-} + \partial_\mu W^-|^2\xi_{W^-}^{-1} \\ &-\frac{1}{2}|\frac{1}{2}\Big(2\partial_\mu Z + \Big(\sigma_d v_d - \sigma_u v_u\Big)\xi_Z\Big(g_1\sin\Theta_W + g_2\cos\Theta_W\Big)\Big)|^2\xi_Z^{-1} \end{aligned} \tag{6}$$

2.4 Fields integrated out

None

3 Renormalization Group Equations

3.1 Anomalous Dimensions

$$\gamma_{\hat{q}}^{(1)} = -\frac{1}{30}\Big(45g_2^2 + 80g_3^2 + g_1^2\Big)\mathbf{1} + Y_d^\dagger Y_d + Y_u^\dagger Y_u \tag{7}$$

$$\begin{aligned} \gamma_{\hat{q}}^{(2)} = &+\Big(8g_2^2g_3^2 + \frac{15}{4}g_2^4 + \frac{1}{90}g_1^2\Big(16g_3^2 + 9g_2^2\Big) + \frac{199}{900}g_1^4 - \frac{8}{9}g_3^4\Big)\mathbf{1} + \frac{4}{5}g_1^2Y_u^\dagger Y_u - |\lambda|^2Y_u^\dagger Y_u \\ &- 2Y_d^\dagger Y_dY_d^\dagger Y_d - 2Y_u^\dagger Y_uY_u^\dagger Y_u + Y_d^\dagger Y_d\Big(-3\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) + \frac{2}{5}g_1^2 - |\lambda|^2 - \mathrm{Tr}\Big(Y_eY_e^\dagger\Big)\Big) \\ &- 3Y_u^\dagger Y_u\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) \end{aligned} \tag{8}$$

$$\gamma_{\hat{l}}^{(1)} = -\frac{3}{10}\Big(5g_2^2 + g_1^2\Big)\mathbf{1} + Y_e^\dagger Y_e \tag{9}$$

$$\begin{aligned} \gamma_{\hat{l}}^{(2)} = &+\frac{3}{100}\Big(125g_2^4 + 30g_1^2g_2^2 + 69g_1^4\Big)\mathbf{1} - 2Y_e^\dagger Y_eY_e^\dagger Y_e \\ &+ Y_e^\dagger Y_e\Big(-3\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) + \frac{6}{5}g_1^2 - |\lambda|^2 - \mathrm{Tr}\Big(Y_eY_e^\dagger\Big)\Big) \end{aligned} \tag{10}$$

$$\gamma_{\hat{H}_d}^{(1)} = 3\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) - \frac{3}{10}g_1^2 - \frac{3}{2}g_2^2 + |\lambda|^2 + \mathrm{Tr}\Big(Y_eY_e^\dagger\Big) \tag{11}$$

$$\begin{aligned} \gamma_{\hat{H}_d}^{(2)} = &+\frac{207}{100}g_1^4 + \frac{9}{10}g_1^2g_2^2 + \frac{15}{4}g_2^4 - 2\lambda|\kappa|^2\lambda^* - 3\lambda^2\lambda^{*,2} - \frac{2}{5}g_1^2\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) + 16g_3^2\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) \\ &+ \frac{6}{5}g_1^2\mathrm{Tr}\Big(Y_eY_e^\dagger\Big) - 3|\lambda|^2\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) - 9\mathrm{Tr}\Big(Y_dY_d^\dagger Y_dY_d^\dagger\Big) - 3\mathrm{Tr}\Big(Y_dY_u^\dagger Y_uY_d^\dagger\Big) \\ &- 3\mathrm{Tr}\Big(Y_eY_e^\dagger Y_eY_e^\dagger\Big) \end{aligned} \tag{12}$$

$$\gamma_{\hat{H}_u}^{(1)} = -\frac{3}{10}\Big(-10\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) + 5g_2^2 + g_1^2\Big) + |\lambda|^2 \tag{13}$$

$$\gamma^{(2)}_{\hat{H}_u} = +\frac{207}{100}g_1^4 + \frac{9}{10}g_1^2g_2^2 + \frac{15}{4}g_2^4 - 2\lambda|\kappa|^2\lambda^* - 3\lambda^2\lambda^{*,2} - |\lambda|^2\Big(3\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) + \mathrm{Tr}\Big(Y_eY_e^\dagger\Big)\Big)$$
$$+ \frac{4}{5}g_1^2\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) + 16g_3^2\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) - 3\mathrm{Tr}\Big(Y_dY_u^\dagger Y_uY_d^\dagger\Big) - 9\mathrm{Tr}\Big(Y_uY_u^\dagger Y_uY_u^\dagger\Big) \tag{14}$$
$$\gamma^{(1)}_{\hat{d}} = 2Y_d^*Y_d^T - \frac{2}{15}\Big(20g_3^2 + g_1^2\Big)\mathbf{1} \tag{15}$$
$$\gamma^{(2)}_{\hat{d}} = +\frac{2}{225}\Big(-100g_3^4 + 101g_1^4 + 80g_1^2g_3^2\Big)\mathbf{1} - 2\Big(Y_d^*Y_d^TY_d^*Y_d^T + Y_d^*Y_u^TY_u^*Y_d^T\Big)$$
$$+ Y_d^*Y_d^T\Big(-2|\lambda|^2 - 2\mathrm{Tr}\Big(Y_eY_e^\dagger\Big) + 6g_2^2 - 6\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) + \frac{2}{5}g_1^2\Big) \tag{16}$$
$$\gamma^{(1)}_{\hat{u}} = 2Y_u^*Y_u^T - \frac{8}{15}\Big(5g_3^2 + g_1^2\Big)\mathbf{1} \tag{17}$$
$$\gamma^{(2)}_{\hat{u}} = +\frac{8}{225}\Big(107g_1^4 - 25g_3^4 + 80g_1^2g_3^2\Big)\mathbf{1}$$
$$- \frac{2}{5}\Big(5\Big(Y_u^*Y_d^TY_d^*Y_u^T + Y_u^*Y_u^TY_u^*Y_u^T\Big) + Y_u^*Y_u^T\Big(-15g_2^2 + 15\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) + 5|\lambda|^2 + g_1^2\Big)\Big) \tag{18}$$
$$\gamma^{(1)}_{\hat{e}} = 2Y_e^*Y_e^T - \frac{6}{5}g_1^2\mathbf{1} \tag{19}$$
$$\gamma^{(2)}_{\hat{e}} = -2Y_e^*Y_e^TY_e^*Y_e^T + \frac{234}{25}g_1^4\mathbf{1} + Y_e^*Y_e^T\Big(-2|\lambda|^2 - 2\mathrm{Tr}\Big(Y_eY_e^\dagger\Big) + 6g_2^2 - 6\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) - \frac{6}{5}g_1^2\Big) \tag{20}$$
$$\gamma^{(1)}_{\hat{s}} = 2\Big(|\kappa|^2 + |\lambda|^2\Big) \tag{21}$$
$$\gamma^{(2)}_{\hat{s}} = -8\kappa^2\kappa^{*,2} - 8\lambda|\kappa|^2\lambda^*$$
$$- \frac{2}{5}|\lambda|^2\Big(10\lambda\lambda^* - 15g_2^2 + 15\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) + 15\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) - 3g_1^2 + 5\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)\Big) \tag{22}$$

3.2 Gauge Couplings

$$\beta^{(1)}_{g_1} = \frac{33}{5}g_1^3 \tag{23}$$
$$\beta^{(2)}_{g_1} = \frac{1}{25}g_1^3\Big(-130\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) + 135g_2^2 + 199g_1^2 - 30|\lambda|^2 + 440g_3^2 - 70\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) - 90\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)\Big) \tag{24}$$
$$\beta^{(1)}_{g_2} = g_2^3 \tag{25}$$
$$\beta^{(2)}_{g_2} = \frac{1}{5}g_2^3\Big(-10|\lambda|^2 - 10\mathrm{Tr}\Big(Y_eY_e^\dagger\Big) + 120g_3^2 + 125g_2^2 - 30\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) - 30\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) + 9g_1^2\Big) \tag{26}$$
$$\beta^{(1)}_{g_3} = -3g_3^3 \tag{27}$$
$$\beta^{(2)}_{g_3} = \frac{1}{5}g_3^3\Big(11g_1^2 - 20\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) - 20\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) + 45g_2^2 + 70g_3^2\Big) \tag{28}$$

3.3 Gaugino Mass Parameters

$$\beta^{(1)}_{M_1} = \frac{66}{5}g_1^2M_1 \tag{29}$$

$$\beta^{(2)}_{M_1} = \frac{2}{25}g_1^2\Big(398g_1^2M_1 + 135g_2^2M_1 + 440g_3^2M_1 + 440g_3^2M_3 + 135g_2^2M_2 - 30\lambda^*\Big(M_1\lambda - T_\lambda\Big) - 70M_1\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)$$
$$- 90M_1\mathrm{Tr}\Big(Y_eY_e^\dagger\Big) - 130M_1\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) + 70\mathrm{Tr}\Big(Y_d^\dagger T_d\Big) + 90\mathrm{Tr}\Big(Y_e^\dagger T_e\Big) + 130\mathrm{Tr}\Big(Y_u^\dagger T_u\Big)\Big) \tag{30}$$
$$\beta^{(1)}_{M_2} = 2g_2^2M_2 \tag{31}$$
$$\beta^{(2)}_{M_2} = \frac{2}{5}g_2^2\Big(9g_1^2M_1 + 120g_3^2M_3 + 9g_1^2M_2 + 250g_2^2M_2 + 120g_3^2M_2 - 10\lambda^*\Big(M_2\lambda - T_\lambda\Big) - 30M_2\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)$$
$$- 10M_2\mathrm{Tr}\Big(Y_eY_e^\dagger\Big) - 30M_2\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) + 30\mathrm{Tr}\Big(Y_d^\dagger T_d\Big) + 10\mathrm{Tr}\Big(Y_e^\dagger T_e\Big) + 30\mathrm{Tr}\Big(Y_u^\dagger T_u\Big)\Big) \tag{32}$$
$$\beta^{(1)}_{M_3} = -6g_3^2M_3 \tag{33}$$
$$\beta^{(2)}_{M_3} = \frac{2}{5}g_3^2\Big(11g_1^2M_1 + 11g_1^2M_3 + 45g_2^2M_3 + 140g_3^2M_3 + 45g_2^2M_2 - 20M_3\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) - 20M_3\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)$$
$$+ 20\mathrm{Tr}\Big(Y_d^\dagger T_d\Big) + 20\mathrm{Tr}\Big(Y_u^\dagger T_u\Big)\Big) \tag{34}$$

3.4 Trilinear Superpotential Parameters

$$\beta^{(1)}_{Y_d} = 3Y_dY_d^\dagger Y_d + Y_d\Big(-3g_2^2 + 3\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) - \frac{16}{3}g_3^2 - \frac{7}{15}g_1^2 + |\lambda|^2 + \mathrm{Tr}\Big(Y_eY_e^\dagger\Big)\Big) + Y_dY_u^\dagger Y_u \tag{35}$$
$$\beta^{(2)}_{Y_d} = +\frac{4}{5}g_1^2Y_dY_u^\dagger Y_u - |\lambda|^2Y_dY_u^\dagger Y_u - 4Y_dY_d^\dagger Y_dY_d^\dagger Y_d - 2Y_dY_u^\dagger Y_uY_d^\dagger Y_d$$
$$- 2Y_dY_u^\dagger Y_uY_u^\dagger Y_u + Y_dY_d^\dagger Y_d\Big(-3|\lambda|^2 - 3\mathrm{Tr}\Big(Y_eY_e^\dagger\Big) + 6g_2^2 - 9\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) + \frac{4}{5}g_1^2\Big)$$
$$- 3Y_dY_u^\dagger Y_u\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)$$
$$+ Y_d\Big(\frac{287}{90}g_1^4 + g_1^2g_2^2 + \frac{15}{2}g_2^4 + \frac{8}{9}g_1^2g_3^2 + 8g_2^2g_3^2 - \frac{16}{9}g_3^4 - 2\lambda|\kappa|^2\lambda^* - 3\lambda^2\lambda^{*,2}$$
$$- \frac{2}{5}g_1^2\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) + 16g_3^2\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) + \frac{6}{5}g_1^2\mathrm{Tr}\Big(Y_eY_e^\dagger\Big) - 3|\lambda|^2\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) - 9\mathrm{Tr}\Big(Y_dY_d^\dagger Y_dY_d^\dagger\Big)$$
$$- 3\mathrm{Tr}\Big(Y_dY_u^\dagger Y_uY_d^\dagger\Big) - 3\mathrm{Tr}\Big(Y_eY_e^\dagger Y_eY_e^\dagger\Big)\Big) \tag{36}$$
$$\beta^{(1)}_{Y_e} = 3Y_eY_e^\dagger Y_e + Y_e\Big(-3g_2^2 + 3\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) - \frac{9}{5}g_1^2 + |\lambda|^2 + \mathrm{Tr}\Big(Y_eY_e^\dagger\Big)\Big) \tag{37}$$
$$\beta^{(2)}_{Y_e} = -4Y_eY_e^\dagger Y_eY_e^\dagger Y_e + Y_eY_e^\dagger Y_e\Big(-3|\lambda|^2 - 3\mathrm{Tr}\Big(Y_eY_e^\dagger\Big) + 6g_2^2 - 9\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)\Big)$$
$$+ Y_e\Big(\frac{27}{2}g_1^4 + \frac{9}{5}g_1^2g_2^2 + \frac{15}{2}g_2^4 - 2\lambda|\kappa|^2\lambda^* - 3\lambda^2\lambda^{*,2} - \frac{2}{5}g_1^2\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) + 16g_3^2\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)$$
$$+ \frac{6}{5}g_1^2\mathrm{Tr}\Big(Y_eY_e^\dagger\Big) - 3|\lambda|^2\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) - 9\mathrm{Tr}\Big(Y_dY_d^\dagger Y_dY_d^\dagger\Big) - 3\mathrm{Tr}\Big(Y_dY_u^\dagger Y_uY_d^\dagger\Big)$$
$$- 3\mathrm{Tr}\Big(Y_eY_e^\dagger Y_eY_e^\dagger\Big)\Big) \tag{38}$$
$$\beta^{(1)}_{\lambda} = 2\lambda|\kappa|^2 - 3g_2^2\lambda + 3\lambda\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) + 3\lambda\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) + 4\lambda^2\lambda^* - \frac{3}{5}g_1^2\lambda + \lambda\mathrm{Tr}\Big(Y_eY_e^\dagger\Big) \tag{39}$$
$$\beta^{(2)}_{\lambda} = -\frac{1}{50}\lambda\Big(-207g_1^4 - 90g_1^2g_2^2 - 375g_2^4 + 400\kappa^2\kappa^{*,2} + 600\lambda|\kappa|^2\lambda^* + 500\lambda^2\lambda^{*,2} + 20g_1^2\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)$$

$$
\begin{aligned}
&- 800g_3^2\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) - 60g_1^2\mathrm{Tr}\Big(Y_eY_e^\dagger\Big) \\
&- 30|\lambda|^2\Big(10g_2^2 - 15\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) - 15\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) + 2g_1^2 - 5\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)\Big) - 40g_1^2\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) \\
&- 800g_3^2\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) + 450\mathrm{Tr}\Big(Y_dY_d^\dagger Y_dY_d^\dagger\Big) + 300\mathrm{Tr}\Big(Y_dY_u^\dagger Y_uY_d^\dagger\Big) + 150\mathrm{Tr}\Big(Y_eY_e^\dagger Y_eY_e^\dagger\Big) \\
&+ 450\mathrm{Tr}\Big(Y_uY_u^\dagger Y_uY_u^\dagger\Big)\Big) \qquad (40)
\end{aligned}
$$

$$
\beta_\kappa^{(1)} = 6\kappa\Big(|\kappa|^2 + |\lambda|^2\Big) \qquad (41)
$$

$$
\begin{aligned}
\beta_\kappa^{(2)} = &-\frac{6}{5}\kappa\Big(20\kappa^2\kappa^{*,2} + 20\lambda|\kappa|^2\lambda^* \\
&+ |\lambda|^2\Big(10\lambda\lambda^* - 15g_2^2 + 15\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) + 15\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) - 3g_1^2 + 5\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)\Big)\Big) \qquad (42)
\end{aligned}
$$

$$
\beta_{Y_u}^{(1)} = 3Y_uY_u^\dagger Y_u + Y_u\Big(-3g_2^2 + 3\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) - \frac{13}{15}g_1^2 - \frac{16}{3}g_3^2 + |\lambda|^2\Big) + Y_uY_d^\dagger Y_d \qquad (43)
$$

$$
\begin{aligned}
\beta_{Y_u}^{(2)} = &+\frac{2}{5}g_1^2Y_uY_u^\dagger Y_u + 6g_2^2Y_uY_u^\dagger Y_u - 3|\lambda|^2Y_uY_u^\dagger Y_u - 2Y_uY_d^\dagger Y_dY_d^\dagger Y_d \\
&- 2Y_uY_d^\dagger Y_dY_u^\dagger Y_u - 4Y_uY_u^\dagger Y_uY_u^\dagger Y_u \\
&+ Y_uY_d^\dagger Y_d\Big(-3\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) + \frac{2}{5}g_1^2 - |\lambda|^2 - \mathrm{Tr}\Big(Y_eY_e^\dagger\Big)\Big) - 9Y_uY_u^\dagger Y_u\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) \\
&+ Y_u\Big(\frac{2743}{450}g_1^4 + g_1^2g_2^2 + \frac{15}{2}g_2^4 + \frac{136}{45}g_1^2g_3^2 + 8g_2^2g_3^2 - \frac{16}{9}g_3^4 - 2\lambda|\kappa|^2\lambda^* - 3\lambda^2\lambda^{*,2} \\
&- |\lambda|^2\Big(3\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) + \mathrm{Tr}\Big(Y_eY_e^\dagger\Big)\Big) + \frac{4}{5}g_1^2\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) + 16g_3^2\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) - 3\mathrm{Tr}\Big(Y_dY_u^\dagger Y_uY_d^\dagger\Big) \\
&- 9\mathrm{Tr}\Big(Y_uY_u^\dagger Y_uY_u^\dagger\Big)\Big) \qquad (44)
\end{aligned}
$$

3.5 Trilinear Soft-Breaking Parameters

$$
\begin{aligned}
\beta_{T_d}^{(1)} = &+4Y_dY_d^\dagger T_d + 2Y_dY_u^\dagger T_u + 5T_dY_d^\dagger Y_d + T_dY_u^\dagger Y_u - \frac{7}{15}g_1^2T_d - 3g_2^2T_d - \frac{16}{3}g_3^2T_d \\
&+ |\lambda|^2T_d + 3T_d\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) + T_d\mathrm{Tr}\Big(Y_eY_e^\dagger\Big) \\
&+ Y_d\Big(2\lambda^*T_\lambda + 2\mathrm{Tr}\Big(Y_e^\dagger T_e\Big) + 6g_2^2M_2 + 6\mathrm{Tr}\Big(Y_d^\dagger T_d\Big) + \frac{14}{15}g_1^2M_1 + \frac{32}{3}g_3^2M_3\Big) \qquad (45)
\end{aligned}
$$

$$
\begin{aligned}
\beta_{T_d}^{(2)} = &+\frac{6}{5}g_1^2Y_dY_d^\dagger T_d + 6g_2^2Y_dY_d^\dagger T_d - 4|\lambda|^2Y_dY_d^\dagger T_d - \frac{8}{5}g_1^2M_1Y_dY_u^\dagger Y_u \\
&+ \frac{8}{5}g_1^2Y_dY_u^\dagger T_u - 2|\lambda|^2Y_dY_u^\dagger T_u + \frac{6}{5}g_1^2T_dY_d^\dagger Y_d + 12g_2^2T_dY_d^\dagger Y_d \\
&- 5|\lambda|^2T_dY_d^\dagger Y_d + \frac{4}{5}g_1^2T_dY_u^\dagger Y_u - |\lambda|^2T_dY_u^\dagger Y_u - 6Y_dY_d^\dagger Y_dY_d^\dagger T_d \\
&- 8Y_dY_d^\dagger T_dY_d^\dagger Y_d - 2Y_dY_u^\dagger Y_uY_d^\dagger T_d - 4Y_dY_u^\dagger Y_uY_u^\dagger T_u - 4Y_dY_u^\dagger T_uY_d^\dagger Y_d \\
&- 4Y_dY_u^\dagger T_uY_u^\dagger Y_u - 6T_dY_d^\dagger Y_dY_d^\dagger Y_d - 4T_dY_u^\dagger Y_uY_d^\dagger Y_d - 2T_dY_u^\dagger Y_uY_u^\dagger Y_u
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{287}{90}g_1^4T_d+g_1^2g_2^2T_d+\frac{15}{2}g_2^4T_d+\frac{8}{9}g_1^2g_3^2T_d+8g_2^2g_3^2T_d-\frac{16}{9}g_3^4T_d-2\lambda|\kappa|^2\lambda^*T_d\\
&-3\lambda^2\lambda^{*,2}T_d-2\lambda^*Y_dY_u^\dagger Y_uT_\lambda-12Y_dY_d^\dagger T_d\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)\\
&-15T_dY_d^\dagger Y_d\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)-\frac{2}{5}g_1^2T_d\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)+16g_3^2T_d\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)\\
&-4Y_dY_d^\dagger T_d\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)-5T_dY_d^\dagger Y_d\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)+\frac{6}{5}g_1^2T_d\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)\\
&-6Y_dY_u^\dagger T_u\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)-3T_dY_u^\dagger Y_u\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)-3|\lambda|^2T_d\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)\\
&-\frac{2}{5}Y_dY_d^\dagger Y_d\Big(15\lambda^*T_\lambda+15\mathrm{Tr}\Big(Y_e^\dagger T_e\Big)+30g_2^2M_2+45\mathrm{Tr}\Big(Y_d^\dagger T_d\Big)+4g_1^2M_1\Big)\\
&-6Y_dY_u^\dagger Y_u\mathrm{Tr}\Big(Y_u^\dagger T_u\Big)-9T_d\mathrm{Tr}\Big(Y_dY_d^\dagger Y_dY_d^\dagger\Big)-3T_d\mathrm{Tr}\Big(Y_dY_u^\dagger Y_uY_d^\dagger\Big)\\
&-3T_d\mathrm{Tr}\Big(Y_eY_e^\dagger Y_eY_e^\dagger\Big)\\
&-\frac{2}{45}Y_d\Big(287g_1^4M_1+45g_1^2g_2^2M_1+40g_1^2g_3^2M_1+40g_1^2g_3^2M_3+360g_2^2g_3^2M_3-160g_3^4M_3\\
&+45g_1^2g_2^2M_2+675g_2^4M_2+360g_2^2g_3^2M_2+270\lambda\lambda^{*,2}T_\lambda+90\kappa^*\lambda^*\Big(\kappa T_\lambda+\lambda T_\kappa\Big)\\
&-18g_1^2M_1\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)+720g_3^2M_3\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)+54g_1^2M_1\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)+18g_1^2\mathrm{Tr}\Big(Y_d^\dagger T_d\Big)\\
&-720g_3^2\mathrm{Tr}\Big(Y_d^\dagger T_d\Big)-54g_1^2\mathrm{Tr}\Big(Y_e^\dagger T_e\Big)+135\lambda^*\Big(\lambda\mathrm{Tr}\Big(Y_u^\dagger T_u\Big)+T_\lambda\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)\Big)\\
&+810\mathrm{Tr}\Big(Y_dY_d^\dagger T_dY_d^\dagger\Big)+135\mathrm{Tr}\Big(Y_dY_u^\dagger T_uY_d^\dagger\Big)+270\mathrm{Tr}\Big(Y_eY_e^\dagger T_eY_e^\dagger\Big)+135\mathrm{Tr}\Big(Y_uY_d^\dagger T_dY_u^\dagger\Big)\Big) \qquad (46)\\
\beta_{T_e}^{(1)}&=+4Y_eY_e^\dagger T_e+5T_eY_e^\dagger Y_e-\frac{9}{5}g_1^2T_e-3g_2^2T_e+|\lambda|^2T_e+3T_e\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)+T_e\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)\\
&+Y_e\Big(2\lambda^*T_\lambda+2\mathrm{Tr}\Big(Y_e^\dagger T_e\Big)+6g_2^2M_2+6\mathrm{Tr}\Big(Y_d^\dagger T_d\Big)+\frac{18}{5}g_1^2M_1\Big) \qquad (47)\\
\beta_{T_e}^{(2)}&=+\frac{6}{5}g_1^2Y_eY_e^\dagger T_e+6g_2^2Y_eY_e^\dagger T_e-4|\lambda|^2Y_eY_e^\dagger T_e-\frac{6}{5}g_1^2T_eY_e^\dagger Y_e\\
&+12g_2^2T_eY_e^\dagger Y_e-5|\lambda|^2T_eY_e^\dagger Y_e-6Y_eY_e^\dagger Y_eY_e^\dagger T_e-8Y_eY_e^\dagger T_eY_e^\dagger Y_e\\
&-6T_eY_e^\dagger Y_eY_e^\dagger Y_e+\frac{27}{2}g_1^4T_e+\frac{9}{5}g_1^2g_2^2T_e+\frac{15}{2}g_2^4T_e-2\lambda|\kappa|^2\lambda^*T_e-3\lambda^2\lambda^{*,2}T_e\\
&-12Y_eY_e^\dagger T_e\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)-15T_eY_e^\dagger Y_e\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)-\frac{2}{5}g_1^2T_e\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)\\
&+16g_3^2T_e\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)-4Y_eY_e^\dagger T_e\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)-5T_eY_e^\dagger Y_e\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)\\
&+\frac{6}{5}g_1^2T_e\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)-3|\lambda|^2T_e\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)\\
&-6Y_eY_e^\dagger Y_e\Big(2g_2^2M_2+3\mathrm{Tr}\Big(Y_d^\dagger T_d\Big)+\lambda^*T_\lambda+\mathrm{Tr}\Big(Y_e^\dagger T_e\Big)\Big)-9T_e\mathrm{Tr}\Big(Y_dY_d^\dagger Y_dY_d^\dagger\Big)\\
&-3T_e\mathrm{Tr}\Big(Y_dY_u^\dagger Y_uY_d^\dagger\Big)-3T_e\mathrm{Tr}\Big(Y_eY_e^\dagger Y_eY_e^\dagger\Big)\\
&-\frac{2}{5}Y_e\Big(135g_1^4M_1+9g_1^2g_2^2M_1+9g_1^2g_2^2M_2+75g_2^4M_2+30\lambda\lambda^{*,2}T_\lambda+10\kappa^*\lambda^*\Big(\kappa T_\lambda+\lambda T_\kappa\Big)
\end{aligned}
$$

$$
\begin{aligned}
&- 2g_1^2 M_1 \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) + 80 g_3^2 M_3 \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) + 6 g_1^2 M_1 \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) + 2 g_1^2 \mathrm{Tr}\Big(Y_d^\dagger T_d\Big) \\
&- 80 g_3^2 \mathrm{Tr}\Big(Y_d^\dagger T_d\Big) - 6 g_1^2 \mathrm{Tr}\Big(Y_e^\dagger T_e\Big) + 15 \lambda^* \Big(\lambda \mathrm{Tr}\Big(Y_u^\dagger T_u\Big) + T_\lambda \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big)\Big) + 90 \mathrm{Tr}\Big(Y_d Y_d^\dagger T_d Y_d^\dagger\Big) \\
&+ 15 \mathrm{Tr}\Big(Y_d Y_u^\dagger T_u Y_d^\dagger\Big) + 30 \mathrm{Tr}\Big(Y_e Y_e^\dagger T_e Y_e^\dagger\Big) + 15 \mathrm{Tr}\Big(Y_u Y_d^\dagger T_d Y_u^\dagger\Big)\Big) \qquad (48)
\end{aligned}
$$

$$
\begin{aligned}
\beta_{T_\lambda}^{(1)} = &+ \frac{6}{5} g_1^2 M_1 \lambda + 6 g_2^2 M_2 \lambda + 2\kappa^* \Big(2\lambda T_\kappa + \kappa T_\lambda\Big) \\
&+ T_\lambda \Big(12|\lambda|^2 - 3g_2^2 + 3\mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) + 3\mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) - \frac{3}{5} g_1^2 + \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big)\Big) + 6\lambda \mathrm{Tr}\Big(Y_d^\dagger T_d\Big) \\
&+ 2\lambda \mathrm{Tr}\Big(Y_e^\dagger T_e\Big) + 6\lambda \mathrm{Tr}\Big(Y_u^\dagger T_u\Big) \qquad (49)
\end{aligned}
$$

$$
\begin{aligned}
\beta_{T_\lambda}^{(2)} = &- \frac{414}{25} g_1^4 M_1 \lambda - \frac{18}{5} g_1^2 g_2^2 M_1 \lambda - \frac{18}{5} g_1^2 g_2^2 M_2 \lambda - 30 g_2^4 M_2 \lambda + \frac{207}{50} g_1^4 T_\lambda + \frac{9}{5} g_1^2 g_2^2 T_\lambda \\
&+ \frac{15}{2} g_2^4 T_\lambda - 50 \lambda^2 \lambda^{*,2} T_\lambda - 8\kappa\kappa^{*,2} \Big(4\lambda T_\kappa + \kappa T_\lambda\Big) + \frac{4}{5} g_1^2 M_1 \lambda \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) \\
&- 32 g_3^2 M_3 \lambda \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) - \frac{2}{5} g_1^2 T_\lambda \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) + 16 g_3^2 T_\lambda \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) - \frac{12}{5} g_1^2 M_1 \lambda \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) \\
&+ \frac{6}{5} g_1^2 T_\lambda \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) - \frac{8}{5} g_1^2 M_1 \lambda \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) - 32 g_3^2 M_3 \lambda \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) + \frac{4}{5} g_1^2 T_\lambda \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) \\
&+ 16 g_3^2 T_\lambda \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) - \frac{4}{5} g_1^2 \lambda \mathrm{Tr}\Big(Y_d^\dagger T_d\Big) + 32 g_3^2 \lambda \mathrm{Tr}\Big(Y_d^\dagger T_d\Big) + \frac{12}{5} g_1^2 \lambda \mathrm{Tr}\Big(Y_e^\dagger T_e\Big) \\
&+ \frac{8}{5} g_1^2 \lambda \mathrm{Tr}\Big(Y_u^\dagger T_u\Big) + 32 g_3^2 \lambda \mathrm{Tr}\Big(Y_u^\dagger T_u\Big) \\
&- \frac{3}{5} |\lambda|^2 \Big(20\kappa^* \Big(2\lambda T_\kappa + 3\kappa T_\lambda\Big) + T_\lambda \Big(15 \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) - 30 g_2^2 + 45 \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) + 45 \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) - 6 g_1^2\Big) \\
&+ 2\lambda \Big(10 g_2^2 M_2 + 15 \mathrm{Tr}\Big(Y_d^\dagger T_d\Big) + 15 \mathrm{Tr}\Big(Y_u^\dagger T_u\Big) + 2 g_1^2 M_1 + 5 \mathrm{Tr}\Big(Y_e^\dagger T_e\Big)\Big)\Big) \\
&- 9 T_\lambda \mathrm{Tr}\Big(Y_d Y_d^\dagger Y_d Y_d^\dagger\Big) - 36 \lambda \mathrm{Tr}\Big(Y_d Y_d^\dagger T_d Y_d^\dagger\Big) - 6 T_\lambda \mathrm{Tr}\Big(Y_d Y_u^\dagger Y_u Y_d^\dagger\Big) - 12 \lambda \mathrm{Tr}\Big(Y_d Y_u^\dagger T_u Y_d^\dagger\Big) \\
&- 3 T_\lambda \mathrm{Tr}\Big(Y_e Y_e^\dagger Y_e Y_e^\dagger\Big) - 12 \lambda \mathrm{Tr}\Big(Y_e Y_e^\dagger T_e Y_e^\dagger\Big) - 12 \lambda \mathrm{Tr}\Big(Y_u Y_d^\dagger T_d Y_u^\dagger\Big) - 9 T_\lambda \mathrm{Tr}\Big(Y_u Y_u^\dagger Y_u Y_u^\dagger\Big) \\
&- 36 \lambda \mathrm{Tr}\Big(Y_u Y_u^\dagger T_u Y_u^\dagger\Big) \qquad (50)
\end{aligned}
$$

$$
\beta_{T_\kappa}^{(1)} = 6\Big(3|\kappa|^2 T_\kappa + \lambda^* \Big(2\kappa T_\lambda + \lambda T_\kappa\Big)\Big) \qquad (51)
$$

$$
\begin{aligned}
\beta_{T_\kappa}^{(2)} = &- \frac{6}{5} \Big(100 \kappa^2 \kappa^{*,2} T_\kappa + 10 \lambda \lambda^{*,2} \Big(4\kappa T_\lambda + \lambda T_\kappa\Big) \\
&+ \lambda^* \Big(\lambda T_\kappa \Big(-15 g_2^2 + 15 \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) + 15 \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) - 3 g_1^2 + 5 \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) + 60 |\kappa|^2\Big) \\
&+ 2\kappa \Big(T_\lambda \Big(-15 g_2^2 + 15 \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) + 15 \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) + 20 |\kappa|^2 - 3 g_1^2 + 5 \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big)\Big) \\
&+ \lambda \Big(15 g_2^2 M_2 + 15 \mathrm{Tr}\Big(Y_d^\dagger T_d\Big) + 15 \mathrm{Tr}\Big(Y_u^\dagger T_u\Big) + 3 g_1^2 M_1 + 5 \mathrm{Tr}\Big(Y_e^\dagger T_e\Big)\Big)\Big)\Big)\Big) \qquad (52)
\end{aligned}
$$

$$
\begin{aligned}
\beta_{T_u}^{(1)} = &+ 2 Y_u Y_d^\dagger T_d + 4 Y_u Y_u^\dagger T_u + T_u Y_d^\dagger Y_d + 5 T_u Y_u^\dagger Y_u - \frac{13}{15} g_1^2 T_u - 3 g_2^2 T_u - \frac{16}{3} g_3^2 T_u \\
&+ |\lambda|^2 T_u + 3 T_u \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) + Y_u \Big(2\lambda^* T_\lambda + 6 g_2^2 M_2 + 6 \mathrm{Tr}\Big(Y_u^\dagger T_u\Big) + \frac{26}{15} g_1^2 M_1 + \frac{32}{3} g_3^2 M_3\Big) \qquad (53)
\end{aligned}
$$

$$
\begin{aligned}
\beta^{(2)}_{T_u} =& +\frac{4}{5}g_1^2 Y_u Y_d^\dagger T_d - 2|\lambda|^2 Y_u Y_d^\dagger T_d - \frac{4}{5}g_1^2 M_1 Y_u Y_u^\dagger Y_u - 12 g_2^2 M_2 Y_u Y_u^\dagger Y_u \\
&+ \frac{6}{5}g_1^2 Y_u Y_u^\dagger T_u + 6 g_2^2 Y_u Y_u^\dagger T_u - 4|\lambda|^2 Y_u Y_u^\dagger T_u + \frac{2}{5}g_1^2 T_u Y_d^\dagger Y_d \\
&- |\lambda|^2 T_u Y_d^\dagger Y_d + 12 g_2^2 T_u Y_u^\dagger Y_u - 5|\lambda|^2 T_u Y_u^\dagger Y_u - 4 Y_u Y_d^\dagger Y_d Y_d^\dagger T_d \\
&- 2 Y_u Y_d^\dagger Y_d Y_u^\dagger T_u - 4 Y_u Y_d^\dagger T_d Y_d^\dagger Y_d - 4 Y_u Y_d^\dagger T_d Y_u^\dagger Y_u - 6 Y_u Y_u^\dagger Y_u Y_u^\dagger T_u \\
&- 8 Y_u Y_u^\dagger T_u Y_u^\dagger Y_u - 2 T_u Y_d^\dagger Y_d Y_d^\dagger Y_d - 4 T_u Y_d^\dagger Y_d Y_u^\dagger Y_u - 6 T_u Y_u^\dagger Y_u Y_u^\dagger Y_u + \frac{2743}{450}g_1^4 T_u \\
&+ g_1^2 g_2^2 T_u + \frac{15}{2}g_2^4 T_u + \frac{136}{45}g_1^2 g_3^2 T_u + 8 g_2^2 g_3^2 T_u - \frac{16}{9}g_3^4 T_u - 2\lambda|\kappa|^2 \lambda^* T_u - 3\lambda^2 \lambda^{*,2} T_u \\
&- 6\lambda^* Y_u Y_u^\dagger Y_u T_\lambda - 6 Y_u Y_d^\dagger T_d \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) - 3 T_u Y_d^\dagger Y_d \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) \\
&- 3|\lambda|^2 T_u \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) - 2 Y_u Y_d^\dagger T_d \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) - T_u Y_d^\dagger Y_d \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) \\
&- |\lambda|^2 T_u \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) - 12 Y_u Y_u^\dagger T_u \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) - 15 T_u Y_u^\dagger Y_u \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) \\
&+ \frac{4}{5}g_1^2 T_u \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) + 16 g_3^2 T_u \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) \\
&- \frac{2}{5}Y_u Y_d^\dagger Y_d \Big(15 \mathrm{Tr}\Big(Y_d^\dagger T_d\Big) + 2 g_1^2 M_1 + 5\lambda^* T_\lambda + 5 \mathrm{Tr}\Big(Y_e^\dagger T_e\Big)\Big) - 18 Y_u Y_u^\dagger Y_u \mathrm{Tr}\Big(Y_u^\dagger T_u\Big) \\
&- 3 T_u \mathrm{Tr}\Big(Y_d Y_u^\dagger Y_u Y_d^\dagger\Big) - 9 T_u \mathrm{Tr}\Big(Y_u Y_u^\dagger Y_u Y_u^\dagger\Big) \\
&- \frac{2}{225}Y_u \Big(2743 g_1^4 M_1 + 225 g_1^2 g_2^2 M_1 + 680 g_1^2 g_3^2 M_1 + 680 g_1^2 g_3^2 M_3 + 1800 g_2^2 g_3^2 M_3 - 800 g_3^4 M_3 \\
&+ 225 g_1^2 g_2^2 M_2 + 3375 g_2^4 M_2 + 1800 g_2^2 g_3^2 M_2 + 1350 \lambda \lambda^{*,2} T_\lambda + 450 \kappa^* \lambda^* \Big(\kappa T_\lambda + \lambda T_\kappa\Big) \\
&+ 180 g_1^2 M_1 \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) + 3600 g_3^2 M_3 \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) \\
&+ 225 \lambda^* \Big(\lambda \Big(3 \mathrm{Tr}\Big(Y_d^\dagger T_d\Big) + \mathrm{Tr}\Big(Y_e^\dagger T_e\Big)\Big) + T_\lambda \Big(3 \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) + \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big)\Big)\Big) - 180 g_1^2 \mathrm{Tr}\Big(Y_u^\dagger T_u\Big) \\
&- 3600 g_3^2 \mathrm{Tr}\Big(Y_u^\dagger T_u\Big) + 675 \mathrm{Tr}\Big(Y_d Y_u^\dagger T_u Y_d^\dagger\Big) + 675 \mathrm{Tr}\Big(Y_u Y_d^\dagger T_d Y_u^\dagger\Big) + 4050 \mathrm{Tr}\Big(Y_u Y_u^\dagger T_u Y_u^\dagger\Big)\Big)
\end{aligned} \tag{54}
$$

3.6 Soft-Breaking Scalar Masses

$$
\sigma_{1,1} = \sqrt{\frac{3}{5}}g_1 \Big(-2 \mathrm{Tr}\Big(m_u^2\Big) - \mathrm{Tr}\Big(m_l^2\Big) - m_{H_d}^2 + m_{H_u}^2 + \mathrm{Tr}\Big(m_d^2\Big) + \mathrm{Tr}\Big(m_e^2\Big) + \mathrm{Tr}\Big(m_q^2\Big)\Big) \tag{55}
$$

$$
\sigma_{2,11} = \frac{1}{10}g_1^2 \Big(2 \mathrm{Tr}\Big(m_d^2\Big) + 3 \mathrm{Tr}\Big(m_l^2\Big) + 3 m_{H_d}^2 + 3 m_{H_u}^2 + 6 \mathrm{Tr}\Big(m_e^2\Big) + 8 \mathrm{Tr}\Big(m_u^2\Big) + \mathrm{Tr}\Big(m_q^2\Big)\Big) \tag{56}
$$

$$
\begin{aligned}
\sigma_{3,1} =& \frac{1}{20}\frac{1}{\sqrt{15}}g_1 \Big(-9 g_1^2 m_{H_d}^2 - 45 g_2^2 m_{H_d}^2 + 9 g_1^2 m_{H_u}^2 + 45 g_2^2 m_{H_u}^2 + 30 \Big(-m_{H_u}^2 + m_{H_d}^2\Big)|\lambda|^2 + 4 \Big(20 g_3^2 + g_1^2\Big) \mathrm{Tr}\Big(m_d^2\Big) \\
&+ 36 g_1^2 \mathrm{Tr}\Big(m_e^2\Big) - 9 g_1^2 \mathrm{Tr}\Big(m_l^2\Big) - 45 g_2^2 \mathrm{Tr}\Big(m_l^2\Big) + g_1^2 \mathrm{Tr}\Big(m_q^2\Big) + 45 g_2^2 \mathrm{Tr}\Big(m_q^2\Big) + 80 g_3^2 \mathrm{Tr}\Big(m_q^2\Big) \\
&- 32 g_1^2 \mathrm{Tr}\Big(m_u^2\Big) - 160 g_3^2 \mathrm{Tr}\Big(m_u^2\Big) + 90 m_{H_d}^2 \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) + 30 m_{H_d}^2 \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) - 90 m_{H_u}^2 \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big)
\end{aligned}
$$

$$
\begin{aligned}
&- 60\mathrm{Tr}\Big(Y_d Y_d^\dagger m_d^{2*}\Big) - 30\mathrm{Tr}\Big(Y_d m_q^{2*} Y_d^\dagger\Big) - 60\mathrm{Tr}\Big(Y_e Y_e^\dagger m_e^{2*}\Big) + 30\mathrm{Tr}\Big(Y_e m_l^{2*} Y_e^\dagger\Big) \\
&+ 120\mathrm{Tr}\Big(Y_u Y_u^\dagger m_u^{2*}\Big) - 30\mathrm{Tr}\Big(Y_u m_q^{2*} Y_u^\dagger\Big)\Big) \qquad (57)
\end{aligned}
$$

$$
\sigma_{2,2} = \frac{1}{2}\Big(3\mathrm{Tr}\Big(m_q^2\Big) + m_{H_d}^2 + m_{H_u}^2 + \mathrm{Tr}\Big(m_l^2\Big)\Big) \qquad (58)
$$

$$
\sigma_{2,3} = \frac{1}{2}\Big(2\mathrm{Tr}\Big(m_q^2\Big) + \mathrm{Tr}\Big(m_d^2\Big) + \mathrm{Tr}\Big(m_u^2\Big)\Big) \qquad (59)
$$

$$
\begin{aligned}
\beta_{m_q^2}^{(1)} &= -\frac{2}{15}g_1^2\mathbf{1}|M_1|^2 - \frac{32}{3}g_3^2\mathbf{1}|M_3|^2 - 6g_2^2\mathbf{1}|M_2|^2 + 2m_{H_d}^2 Y_d^\dagger Y_d + 2m_{H_u}^2 Y_u^\dagger Y_u + 2T_d^\dagger T_d \\
&+ 2T_u^\dagger T_u + m_q^2 Y_d^\dagger Y_d + m_q^2 Y_u^\dagger Y_u + 2Y_d^\dagger m_d^2 Y_d + Y_d^\dagger Y_d m_q^2 + 2Y_u^\dagger m_u^2 Y_u \\
&+ Y_u^\dagger Y_u m_q^2 + \frac{1}{\sqrt{15}}g_1\mathbf{1}\sigma_{1,1} \qquad (60) \\
\beta_{m_q^2}^{(2)} &= +\frac{2}{5}g_1^2 g_2^2\mathbf{1}|M_2|^2 + 33g_2^4\mathbf{1}|M_2|^2 + 32g_2^2 g_3^2\mathbf{1}|M_2|^2 \\
&+ \frac{16}{45}g_3^2\Big(15\Big(3g_2^2\Big(2M_3 + M_2\Big) - 8g_3^2 M_3\Big) + g_1^2\Big(2M_3 + M_1\Big)\Big)\mathbf{1}M_3^* + \frac{1}{5}g_1^2 g_2^2 M_1\mathbf{1}M_2^* + 16g_2^2 g_3^2 M_3\mathbf{1}M_2^* \\
&+ \frac{4}{5}g_1^2 m_{H_d}^2 Y_d^\dagger Y_d - 4m_{H_d}^2|\lambda|^2 Y_d^\dagger Y_d - 2m_{H_u}^2|\lambda|^2 Y_d^\dagger Y_d \\
&- 2m_S^2|\lambda|^2 Y_d^\dagger Y_d - 2|T_\lambda|^2 Y_d^\dagger Y_d - 2\lambda T_\lambda^* Y_d^\dagger T_d + \frac{8}{5}g_1^2 m_{H_u}^2 Y_u^\dagger Y_u \\
&- 2m_{H_d}^2|\lambda|^2 Y_u^\dagger Y_u - 4m_{H_u}^2|\lambda|^2 Y_u^\dagger Y_u - 2m_S^2|\lambda|^2 Y_u^\dagger Y_u - 2|T_\lambda|^2 Y_u^\dagger Y_u \\
&+ \frac{1}{225}g_1^2 M_1^*\Big(\Big(5\Big(16g_3^2\Big(2M_1 + M_3\Big) + 9g_2^2\Big(2M_1 + M_2\Big)\Big) + 597g_1^2 M_1\Big)\mathbf{1} \\
&+ 180\Big(2M_1 Y_d^\dagger Y_d - 2Y_u^\dagger T_u + 4M_1 Y_u^\dagger Y_u - Y_d^\dagger T_d\Big)\Big) \\
&- 2\lambda T_\lambda^* Y_u^\dagger T_u - \frac{4}{5}g_1^2 M_1 T_d^\dagger Y_d + \frac{4}{5}g_1^2 T_d^\dagger T_d - 2|\lambda|^2 T_d^\dagger T_d \\
&- \frac{8}{5}g_1^2 M_1 T_u^\dagger Y_u + \frac{8}{5}g_1^2 T_u^\dagger T_u - 2|\lambda|^2 T_u^\dagger T_u + \frac{2}{5}g_1^2 m_q^2 Y_d^\dagger Y_d \\
&- |\lambda|^2 m_q^2 Y_d^\dagger Y_d + \frac{4}{5}g_1^2 m_q^2 Y_u^\dagger Y_u - |\lambda|^2 m_q^2 Y_u^\dagger Y_u + \frac{4}{5}g_1^2 Y_d^\dagger m_d^2 Y_d \\
&- 2|\lambda|^2 Y_d^\dagger m_d^2 Y_d + \frac{2}{5}g_1^2 Y_d^\dagger Y_d m_q^2 - |\lambda|^2 Y_d^\dagger Y_d m_q^2 + \frac{8}{5}g_1^2 Y_u^\dagger m_u^2 Y_u \\
&- 2|\lambda|^2 Y_u^\dagger m_u^2 Y_u + \frac{4}{5}g_1^2 Y_u^\dagger Y_u m_q^2 - |\lambda|^2 Y_u^\dagger Y_u m_q^2 - 8m_{H_d}^2 Y_d^\dagger Y_d Y_d^\dagger Y_d \\
&- 4Y_d^\dagger Y_d T_d^\dagger T_d - 4Y_d^\dagger T_d T_d^\dagger Y_d - 8m_{H_u}^2 Y_u^\dagger Y_u Y_u^\dagger Y_u - 4Y_u^\dagger Y_u T_u^\dagger T_u \\
&- 4Y_u^\dagger T_u T_u^\dagger Y_u - 4T_d^\dagger Y_d Y_d^\dagger T_d - 4T_d^\dagger T_d Y_d^\dagger Y_d - 4T_u^\dagger Y_u Y_u^\dagger T_u \\
&- 4T_u^\dagger T_u Y_u^\dagger Y_u - 2m_q^2 Y_d^\dagger Y_d Y_d^\dagger Y_d - 2m_q^2 Y_u^\dagger Y_u Y_u^\dagger Y_u - 4Y_d^\dagger m_d^2 Y_d Y_d^\dagger Y_d \\
&- 4Y_d^\dagger Y_d m_q^2 Y_d^\dagger Y_d - 4Y_d^\dagger Y_d Y_d^\dagger m_d^2 Y_d - 2Y_d^\dagger Y_d Y_d^\dagger Y_d m_q^2 - 4Y_u^\dagger m_u^2 Y_u Y_u^\dagger Y_u \\
&- 4Y_u^\dagger Y_u m_q^2 Y_u^\dagger Y_u - 4Y_u^\dagger Y_u Y_u^\dagger m_u^2 Y_u - 2Y_u^\dagger Y_u Y_u^\dagger Y_u m_q^2 - 2\lambda^* T_d^\dagger Y_d T_\lambda
\end{aligned}
$$

$$
\begin{aligned}
&- 2\lambda^* T_u^\dagger Y_u T_\lambda + 6 g_2^4 \mathbf{1}\sigma_{2,2} + \frac{32}{3} g_3^4 \mathbf{1}\sigma_{2,3} + \frac{2}{15} g_1^2 \mathbf{1}\sigma_{2,11} + 4\frac{1}{\sqrt{15}} g_1 \mathbf{1}\sigma_{3,1} \\
&- 12 m_{H_d}^2 Y_d^\dagger Y_d \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) - 6 T_d^\dagger T_d \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) - 3 m_q^2 Y_d^\dagger Y_d \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) \\
&- 6 Y_d^\dagger m_d^2 Y_d \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) - 3 Y_d^\dagger Y_d m_q^2 \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) - 4 m_{H_d}^2 Y_d^\dagger Y_d \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) \\
&- 2 T_d^\dagger T_d \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) - m_q^2 Y_d^\dagger Y_d \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) - 2 Y_d^\dagger m_d^2 Y_d \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) \\
&- Y_d^\dagger Y_d m_q^2 \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) - 12 m_{H_u}^2 Y_u^\dagger Y_u \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) - 6 T_u^\dagger T_u \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) \\
&- 3 m_q^2 Y_u^\dagger Y_u \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) - 6 Y_u^\dagger m_u^2 Y_u \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) - 3 Y_u^\dagger Y_u m_q^2 \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) \\
&- 6 T_d^\dagger Y_d \mathrm{Tr}\Big(Y_d^\dagger T_d\Big) - 2 T_d^\dagger Y_d \mathrm{Tr}\Big(Y_e^\dagger T_e\Big) - 6 T_u^\dagger Y_u \mathrm{Tr}\Big(Y_u^\dagger T_u\Big) \\
&- 6 Y_d^\dagger T_d \mathrm{Tr}\Big(T_d^* Y_d^T\Big) - 6 Y_d^\dagger Y_d \mathrm{Tr}\Big(T_d^* T_d^T\Big) - 2 Y_d^\dagger T_d \mathrm{Tr}\Big(T_e^* Y_e^T\Big) \\
&- 2 Y_d^\dagger Y_d \mathrm{Tr}\Big(T_e^* T_e^T\Big) - 6 Y_u^\dagger T_u \mathrm{Tr}\Big(T_u^* Y_u^T\Big) - 6 Y_u^\dagger Y_u \mathrm{Tr}\Big(T_u^* T_u^T\Big) \\
&- 6 Y_d^\dagger Y_d \mathrm{Tr}\Big(m_d^2 Y_d Y_d^\dagger\Big) - 2 Y_d^\dagger Y_d \mathrm{Tr}\Big(m_e^2 Y_e Y_e^\dagger\Big) - 2 Y_d^\dagger Y_d \mathrm{Tr}\Big(m_l^2 Y_e^\dagger Y_e\Big) \\
&- 6 Y_d^\dagger Y_d \mathrm{Tr}\Big(m_q^2 Y_d^\dagger Y_d\Big) - 6 Y_u^\dagger Y_u \mathrm{Tr}\Big(m_q^2 Y_u^\dagger Y_u\Big) - 6 Y_u^\dagger Y_u \mathrm{Tr}\Big(m_u^2 Y_u Y_u^\dagger\Big)
\end{aligned} \tag{61}
$$

$$
\begin{aligned}
\beta_{m_l^2}^{(1)} = &- \frac{6}{5} g_1^2 \mathbf{1} |M_1|^2 - 6 g_2^2 \mathbf{1} |M_2|^2 + 2 m_{H_d}^2 Y_e^\dagger Y_e + 2 T_e^\dagger T_e + m_l^2 Y_e^\dagger Y_e + 2 Y_e^\dagger m_e^2 Y_e \\
&+ Y_e^\dagger Y_e m_l^2 - \sqrt{\frac{3}{5}} g_1 \mathbf{1}\sigma_{1,1}
\end{aligned} \tag{62}
$$

$$
\begin{aligned}
\beta_{m_l^2}^{(2)} = &+ \frac{3}{5} g_2^2 \Big(3 g_1^2 \Big(2M_2 + M_1\Big) + 55 g_2^2 M_2\Big) \mathbf{1} M_2^* + \frac{12}{5} g_1^2 m_{H_d}^2 Y_e^\dagger Y_e - 4 m_{H_d}^2 |\lambda|^2 Y_e^\dagger Y_e \\
&- 2 m_{H_u}^2 |\lambda|^2 Y_e^\dagger Y_e - 2 m_S^2 |\lambda|^2 Y_e^\dagger Y_e - 2 |T_\lambda|^2 Y_e^\dagger Y_e \\
&+ \frac{3}{25} g_1^2 M_1^* \Big(- 20 Y_e^\dagger T_e + 3 \Big(5 g_2^2 \Big(2M_1 + M_2\Big) + 69 g_1^2 M_1\Big) \mathbf{1} + 40 M_1 Y_e^\dagger Y_e\Big) - 2\lambda T_\lambda^* Y_e^\dagger T_e \\
&- \frac{12}{5} g_1^2 M_1 T_e^\dagger Y_e + \frac{12}{5} g_1^2 T_e^\dagger T_e - 2 |\lambda|^2 T_e^\dagger T_e + \frac{6}{5} g_1^2 m_l^2 Y_e^\dagger Y_e \\
&- |\lambda|^2 m_l^2 Y_e^\dagger Y_e + \frac{12}{5} g_1^2 Y_e^\dagger m_e^2 Y_e - 2 |\lambda|^2 Y_e^\dagger m_e^2 Y_e + \frac{6}{5} g_1^2 Y_e^\dagger Y_e m_l^2 \\
&- |\lambda|^2 Y_e^\dagger Y_e m_l^2 - 8 m_{H_d}^2 Y_e^\dagger Y_e Y_e^\dagger Y_e - 4 Y_e^\dagger Y_e T_e^\dagger T_e - 4 Y_e^\dagger T_e T_e^\dagger Y_e \\
&- 4 T_e^\dagger Y_e Y_e^\dagger T_e - 4 T_e^\dagger T_e Y_e^\dagger Y_e - 2 m_l^2 Y_e^\dagger Y_e Y_e^\dagger Y_e - 4 Y_e^\dagger m_e^2 Y_e Y_e^\dagger Y_e \\
&- 4 Y_e^\dagger Y_e m_l^2 Y_e^\dagger Y_e - 4 Y_e^\dagger Y_e Y_e^\dagger m_e^2 Y_e - 2 Y_e^\dagger Y_e Y_e^\dagger Y_e m_l^2 - 2\lambda^* T_e^\dagger Y_e T_\lambda + 6 g_2^4 \mathbf{1}\sigma_{2,2} \\
&+ \frac{6}{5} g_1^2 \mathbf{1}\sigma_{2,11} - 4\sqrt{\frac{3}{5}} g_1 \mathbf{1}\sigma_{3,1} - 12 m_{H_d}^2 Y_e^\dagger Y_e \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) - 6 T_e^\dagger T_e \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) \\
&- 3 m_l^2 Y_e^\dagger Y_e \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) - 6 Y_e^\dagger m_e^2 Y_e \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) - 3 Y_e^\dagger Y_e m_l^2 \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) \\
&- 4 m_{H_d}^2 Y_e^\dagger Y_e \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) - 2 T_e^\dagger T_e \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) - m_l^2 Y_e^\dagger Y_e \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) \\
&- 2 Y_e^\dagger m_e^2 Y_e \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) - Y_e^\dagger Y_e m_l^2 \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) - 6 T_e^\dagger Y_e \mathrm{Tr}\Big(Y_d^\dagger T_d\Big)
\end{aligned}
$$

$$
\begin{aligned}
&- 2T_e^\dagger Y_e \mathrm{Tr}\Big(Y_e^\dagger T_e\Big) - 6Y_e^\dagger T_e \mathrm{Tr}\Big(T_d^* Y_d^T\Big) - 6Y_e^\dagger Y_e \mathrm{Tr}\Big(T_d^* T_d^T\Big) \\
&- 2Y_e^\dagger T_e \mathrm{Tr}\Big(T_e^* Y_e^T\Big) - 2Y_e^\dagger Y_e \mathrm{Tr}\Big(T_e^* T_e^T\Big) - 6Y_e^\dagger Y_e \mathrm{Tr}\Big(m_d^2 Y_d Y_d^\dagger\Big) \\
&- 2Y_e^\dagger Y_e \mathrm{Tr}\Big(m_e^2 Y_e Y_e^\dagger\Big) - 2Y_e^\dagger Y_e \mathrm{Tr}\Big(m_l^2 Y_e^\dagger Y_e\Big) - 6Y_e^\dagger Y_e \mathrm{Tr}\Big(m_q^2 Y_d^\dagger Y_d\Big) \qquad (63)
\end{aligned}
$$

$$
\begin{aligned}
\beta^{(1)}_{m^2_{H_d}} &= -\frac{6}{5} g_1^2 |M_1|^2 - 6 g_2^2 |M_2|^2 + 2 m_{H_d}^2 |\lambda|^2 + 2 m_{H_u}^2 |\lambda|^2 + 2 m_S^2 |\lambda|^2 + 2|T_\lambda|^2 - \sqrt{\frac{3}{5}} g_1 \sigma_{1,1} \\
&+ 6 m_{H_d}^2 \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) + 2 m_{H_d}^2 \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) + 6 \mathrm{Tr}\Big(T_d^* T_d^T\Big) + 2 \mathrm{Tr}\Big(T_e^* T_e^T\Big) + 6 \mathrm{Tr}\Big(m_d^2 Y_d Y_d^\dagger\Big) \\
&+ 2 \mathrm{Tr}\Big(m_e^2 Y_e Y_e^\dagger\Big) + 2 \mathrm{Tr}\Big(m_l^2 Y_e^\dagger Y_e\Big) + 6 \mathrm{Tr}\Big(m_q^2 Y_d^\dagger Y_d\Big) \qquad (64)
\end{aligned}
$$

$$
\begin{aligned}
\beta^{(2)}_{m^2_{H_d}} &= \frac{1}{25} \Big(g_1^2 M_1^* \Big(621 g_1^2 M_1 + 90 g_2^2 M_1 + 45 g_2^2 M_2 - 40 M_1 \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) + 120 M_1 \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) + 20 \mathrm{Tr}\Big(Y_d^\dagger T_d\Big) \\
&- 60 \mathrm{Tr}\Big(Y_e^\dagger T_e\Big)\Big) \\
&+ 5 \Big(3 g_2^2 \Big(3 g_1^2 \Big(2 M_2 + M_1 \Big) + 55 g_2^2 M_2 \Big) M_2^* \\
&- 2 \Big(30 \Big(m_{H_d}^2 + m_{H_u}^2 + m_S^2 \Big) \lambda^2 \lambda^{*,2} + 10 \kappa^* \Big(\Big(4 m_S^2 + m_{H_d}^2 + m_{H_u}^2 \Big) \kappa |\lambda|^2 + T_\lambda^* \Big(\kappa T_\lambda + \lambda T_\kappa \Big) \Big) - 15 g_2^4 \sigma_{2,2} - 3 g_1^2 \sigma_{2,11} \\
&+ 2\sqrt{15} g_1 \sigma_{3,1} + 2 g_1^2 m_{H_d}^2 \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) - 80 g_3^2 m_{H_d}^2 \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) - 160 g_3^2 |M_3|^2 \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) \\
&- 6 g_1^2 m_{H_d}^2 \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) + 15 |T_\lambda|^2 \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) + 80 g_3^2 M_3^* \mathrm{Tr}\Big(Y_d^\dagger T_d\Big) + 15 \lambda T_\lambda^* \mathrm{Tr}\Big(Y_u^\dagger T_u\Big) \\
&- 2 g_1^2 M_1 \mathrm{Tr}\Big(T_d^* Y_d^T\Big) + 80 g_3^2 M_3 \mathrm{Tr}\Big(T_d^* Y_d^T\Big) + 2 g_1^2 \mathrm{Tr}\Big(T_d^* T_d^T\Big) - 80 g_3^2 \mathrm{Tr}\Big(T_d^* T_d^T\Big) \\
&+ 6 g_1^2 M_1 \mathrm{Tr}\Big(T_e^* Y_e^T\Big) - 6 g_1^2 \mathrm{Tr}\Big(T_e^* T_e^T\Big) + 2 g_1^2 \mathrm{Tr}\Big(m_d^2 Y_d Y_d^\dagger\Big) - 80 g_3^2 \mathrm{Tr}\Big(m_d^2 Y_d Y_d^\dagger\Big) \\
&- 6 g_1^2 \mathrm{Tr}\Big(m_e^2 Y_e Y_e^\dagger\Big) - 6 g_1^2 \mathrm{Tr}\Big(m_l^2 Y_e^\dagger Y_e\Big) + 2 g_1^2 \mathrm{Tr}\Big(m_q^2 Y_d^\dagger Y_d\Big) - 80 g_3^2 \mathrm{Tr}\Big(m_q^2 Y_d^\dagger Y_d\Big) \\
&+ 5 \lambda^* \Big(2 T_\kappa^* \Big(\kappa T_\lambda + \lambda T_\kappa \Big) \\
&+ 3 \Big(4 \lambda |T_\lambda|^2 + \Big(2 m_{H_u}^2 + m_{H_d}^2 + m_S^2 \Big) \lambda \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) + T_\lambda \mathrm{Tr}\Big(T_u^* Y_u^T\Big) + \lambda \mathrm{Tr}\Big(T_u^* T_u^T\Big) + \lambda \mathrm{Tr}\Big(m_q^2 Y_u^\dagger Y_u\Big) \\
&+ \lambda \mathrm{Tr}\Big(m_u^2 Y_u Y_u^\dagger\Big)\Big)\Big) \\
&+ 90 m_{H_d}^2 \mathrm{Tr}\Big(Y_d Y_d^\dagger Y_d Y_d^\dagger\Big) + 90 \mathrm{Tr}\Big(Y_d Y_d^\dagger T_d T_d^\dagger\Big) + 15 m_{H_d}^2 \mathrm{Tr}\Big(Y_d Y_u^\dagger Y_u Y_d^\dagger\Big) \\
&+ 15 m_{H_u}^2 \mathrm{Tr}\Big(Y_d Y_u^\dagger Y_u Y_d^\dagger\Big) + 15 \mathrm{Tr}\Big(Y_d Y_u^\dagger T_u T_d^\dagger\Big) + 90 \mathrm{Tr}\Big(Y_d T_d^\dagger T_d Y_d^\dagger\Big) \\
&+ 15 \mathrm{Tr}\Big(Y_d T_u^\dagger T_u Y_d^\dagger\Big) + 30 m_{H_d}^2 \mathrm{Tr}\Big(Y_e Y_e^\dagger Y_e Y_e^\dagger\Big) + 30 \mathrm{Tr}\Big(Y_e Y_e^\dagger T_e T_e^\dagger\Big) + 30 \mathrm{Tr}\Big(Y_e T_e^\dagger T_e Y_e^\dagger\Big) \\
&+ 15 \mathrm{Tr}\Big(Y_u Y_d^\dagger T_d T_u^\dagger\Big) + 15 \mathrm{Tr}\Big(Y_u T_d^\dagger T_d Y_u^\dagger\Big) + 90 \mathrm{Tr}\Big(m_d^2 Y_d Y_d^\dagger Y_d Y_d^\dagger\Big) + 15 \mathrm{Tr}\Big(m_d^2 Y_d Y_u^\dagger Y_u Y_d^\dagger\Big) \\
&+ 30 \mathrm{Tr}\Big(m_e^2 Y_e Y_e^\dagger Y_e Y_e^\dagger\Big) + 30 \mathrm{Tr}\Big(m_l^2 Y_e^\dagger Y_e Y_e^\dagger Y_e\Big) + 90 \mathrm{Tr}\Big(m_q^2 Y_d^\dagger Y_d Y_d^\dagger Y_d\Big) + 15 \mathrm{Tr}\Big(m_q^2 Y_d^\dagger Y_d Y_u^\dagger Y_u\Big) \\
&+ 15 \mathrm{Tr}\Big(m_q^2 Y_u^\dagger Y_u Y_d^\dagger Y_d\Big) + 15 \mathrm{Tr}\Big(m_u^2 Y_u Y_d^\dagger Y_d Y_u^\dagger\Big)\Big)\Big)\Big) \qquad (65)
\end{aligned}
$$

$$\beta^{(1)}_{m^2_{H_u}} = -\frac{6}{5}g_1^2|M_1|^2 - 6g_2^2|M_2|^2 + 2m^2_{H_d}|\lambda|^2 + 2m^2_{H_u}|\lambda|^2 + 2m_S^2|\lambda|^2 + 2|T_\lambda|^2 + \sqrt{\frac{3}{5}}g_1\sigma_{1,1}$$
$$+ 6m^2_{H_u}\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) + 6\mathrm{Tr}\Big(T_u^*T_u^T\Big) + 6\mathrm{Tr}\Big(m_q^2Y_u^\dagger Y_u\Big) + 6\mathrm{Tr}\Big(m_u^2Y_uY_u^\dagger\Big) \tag{66}$$

$$\beta^{(2)}_{m^2_{H_u}} = \frac{1}{25}\Big(g_1^2M_1^*\Big(- 40\mathrm{Tr}\Big(Y_u^\dagger T_u\Big) + 45g_2^2M_2 + 621g_1^2M_1 + 80M_1\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) + 90g_2^2M_1\Big)$$
$$+ 5\Big(3g_2^2\Big(3g_1^2\Big(2M_2 + M_1\Big) + 55g_2^2M_2\Big)M_2^*$$
$$- 2\Big(30\Big(m^2_{H_d} + m^2_{H_u} + m_S^2\Big)\lambda^2\lambda^{*,2} + 10\kappa^*\Big(\Big(4m_S^2 + m^2_{H_d} + m^2_{H_u}\Big)\kappa|\lambda|^2 + T_\lambda^*\Big(\kappa T_\lambda + \lambda T_\kappa\Big)\Big) - 15g_2^4\sigma_{2,2} - 3g_1^2\sigma_{2,11}$$
$$- 2\sqrt{15}g_1\sigma_{3,1} + 15|T_\lambda|^2\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) + 5|T_\lambda|^2\mathrm{Tr}\Big(Y_eY_e^\dagger\Big) - 4g_1^2m^2_{H_u}\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)$$
$$- 80g_3^2m^2_{H_u}\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) - 160g_3^2|M_3|^2\mathrm{Tr}\Big(Y_uY_u^\dagger\Big) + 15\lambda T_\lambda^*\mathrm{Tr}\Big(Y_d^\dagger T_d\Big) + 5\lambda T_\lambda^*\mathrm{Tr}\Big(Y_e^\dagger T_e\Big)$$
$$+ 80g_3^2M_3^*\mathrm{Tr}\Big(Y_u^\dagger T_u\Big) + 4g_1^2M_1\mathrm{Tr}\Big(T_u^*Y_u^T\Big) + 80g_3^2M_3\mathrm{Tr}\Big(T_u^*Y_u^T\Big) - 4g_1^2\mathrm{Tr}\Big(T_u^*T_u^T\Big)$$
$$- 80g_3^2\mathrm{Tr}\Big(T_u^*T_u^T\Big)$$
$$+ 5\lambda^*\Big(12\lambda|T_\lambda|^2 + 2T_\kappa^*\Big(\kappa T_\lambda + \lambda T_\kappa\Big) + 6m^2_{H_d}\lambda\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) + 3m^2_{H_u}\lambda\mathrm{Tr}\Big(Y_dY_d^\dagger\Big) + 3m_S^2\lambda\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)$$
$$+ 2m^2_{H_d}\lambda\mathrm{Tr}\Big(Y_eY_e^\dagger\Big) + m^2_{H_u}\lambda\mathrm{Tr}\Big(Y_eY_e^\dagger\Big) + m_S^2\lambda\mathrm{Tr}\Big(Y_eY_e^\dagger\Big) + 3T_\lambda\mathrm{Tr}\Big(T_d^*Y_d^T\Big) + 3\lambda\mathrm{Tr}\Big(T_d^*T_d^T\Big)$$
$$+ T_\lambda\mathrm{Tr}\Big(T_e^*Y_e^T\Big) + \lambda\mathrm{Tr}\Big(T_e^*T_e^T\Big) + 3\lambda\mathrm{Tr}\Big(m_d^2Y_dY_d^\dagger\Big) + \lambda\mathrm{Tr}\Big(m_e^2Y_eY_e^\dagger\Big) + \lambda\mathrm{Tr}\Big(m_l^2Y_e^\dagger Y_e\Big)$$
$$+ 3\lambda\mathrm{Tr}\Big(m_q^2Y_d^\dagger Y_d\Big)\Big)$$
$$- 4g_1^2\mathrm{Tr}\Big(m_q^2Y_u^\dagger Y_u\Big) - 80g_3^2\mathrm{Tr}\Big(m_q^2Y_u^\dagger Y_u\Big) - 4g_1^2\mathrm{Tr}\Big(m_u^2Y_uY_u^\dagger\Big) - 80g_3^2\mathrm{Tr}\Big(m_u^2Y_uY_u^\dagger\Big)$$
$$+ 15m^2_{H_d}\mathrm{Tr}\Big(Y_dY_u^\dagger Y_uY_d^\dagger\Big) + 15m^2_{H_u}\mathrm{Tr}\Big(Y_dY_u^\dagger Y_uY_d^\dagger\Big) + 15\mathrm{Tr}\Big(Y_dY_u^\dagger T_uT_d^\dagger\Big)$$
$$+ 15\mathrm{Tr}\Big(Y_dT_u^\dagger T_uY_d^\dagger\Big) + 15\mathrm{Tr}\Big(Y_uY_d^\dagger T_dT_u^\dagger\Big) + 90m^2_{H_u}\mathrm{Tr}\Big(Y_uY_u^\dagger Y_uY_u^\dagger\Big) + 90\mathrm{Tr}\Big(Y_uY_u^\dagger T_uT_u^\dagger\Big)$$
$$+ 15\mathrm{Tr}\Big(Y_uT_d^\dagger T_dY_u^\dagger\Big) + 90\mathrm{Tr}\Big(Y_uT_u^\dagger T_uY_u^\dagger\Big) + 15\mathrm{Tr}\Big(m_d^2Y_dY_u^\dagger Y_uY_d^\dagger\Big)$$
$$+ 15\mathrm{Tr}\Big(m_q^2Y_d^\dagger Y_dY_u^\dagger Y_u\Big) + 15\mathrm{Tr}\Big(m_q^2Y_u^\dagger Y_uY_d^\dagger Y_d\Big) + 90\mathrm{Tr}\Big(m_q^2Y_u^\dagger Y_uY_u^\dagger Y_u\Big)$$
$$+ 15\mathrm{Tr}\Big(m_u^2Y_uY_d^\dagger Y_dY_u^\dagger\Big) + 90\mathrm{Tr}\Big(m_u^2Y_uY_u^\dagger Y_uY_u^\dagger\Big)\Big)\Big)\Big) \tag{67}$$

$$\beta^{(1)}_{m_d^2} = -\frac{8}{15}g_1^2\mathbf{1}|M_1|^2 - \frac{32}{3}g_3^2\mathbf{1}|M_3|^2 + 4m^2_{H_d}Y_dY_d^\dagger + 4T_dT_d^\dagger + 2m_d^2Y_dY_d^\dagger + 4Y_dm_q^2Y_d^\dagger$$
$$+ 2Y_dY_d^\dagger m_d^2 + 2\frac{1}{\sqrt{15}}g_1\mathbf{1}\sigma_{1,1} \tag{68}$$

$$\beta^{(2)}_{m_d^2} = +\frac{64}{45}g_3^2\Big(- 30g_3^2M_3 + g_1^2\Big(2M_3 + M_1\Big)\Big)\mathbf{1}M_3^* + \frac{4}{5}g_1^2m^2_{H_d}Y_dY_d^\dagger + 12g_2^2m^2_{H_d}Y_dY_d^\dagger$$
$$+ 24g_2^2|M_2|^2Y_dY_d^\dagger - 8m^2_{H_d}|\lambda|^2Y_dY_d^\dagger - 4m^2_{H_u}|\lambda|^2Y_dY_d^\dagger$$
$$- 4m_S^2|\lambda|^2Y_dY_d^\dagger - 4|T_\lambda|^2Y_dY_d^\dagger - \frac{4}{5}g_1^2M_1Y_dT_d^\dagger - 12g_2^2M_2Y_dT_d^\dagger$$

$$
\begin{aligned}
&+\frac{4}{225}g_1^2M_1^*\Big(2\Big(303g_1^2M_1+40g_3^2\Big(2M_1+M_3\Big)\Big)\mathbf{1}-45T_dY_d^\dagger+90M_1Y_dY_d^\dagger\Big)-12g_2^2M_2^*T_dY_d^\dagger\\
&-4\lambda T_\lambda^*T_dY_d^\dagger+\frac{4}{5}g_1^2T_dT_d^\dagger+12g_2^2T_dT_d^\dagger-4|\lambda|^2T_dT_d^\dagger\\
&+\frac{2}{5}g_1^2m_d^2Y_dY_d^\dagger+6g_2^2m_d^2Y_dY_d^\dagger-2|\lambda|^2m_d^2Y_dY_d^\dagger+\frac{4}{5}g_1^2Y_dm_q^2Y_d^\dagger\\
&+12g_2^2Y_dm_q^2Y_d^\dagger-4|\lambda|^2Y_dm_q^2Y_d^\dagger+\frac{2}{5}g_1^2Y_dY_d^\dagger m_d^2+6g_2^2Y_dY_d^\dagger m_d^2\\
&-2|\lambda|^2Y_dY_d^\dagger m_d^2-8m_{H_d}^2Y_dY_d^\dagger Y_dY_d^\dagger-4Y_dY_d^\dagger T_dT_d^\dagger-4m_{H_d}^2Y_dY_u^\dagger Y_uY_d^\dagger\\
&-4m_{H_u}^2Y_dY_u^\dagger Y_uY_d^\dagger-4Y_dY_u^\dagger T_uT_d^\dagger-4Y_dT_d^\dagger T_dY_d^\dagger-4Y_dT_u^\dagger T_uY_d^\dagger\\
&-4T_dY_d^\dagger Y_dT_d^\dagger-4T_dY_u^\dagger Y_uT_d^\dagger-4T_dT_d^\dagger Y_dY_d^\dagger-4T_dT_u^\dagger Y_uY_d^\dagger\\
&-2m_d^2Y_dY_d^\dagger Y_dY_d^\dagger-2m_d^2Y_dY_u^\dagger Y_uY_d^\dagger-4Y_dm_q^2Y_d^\dagger Y_dY_d^\dagger-4Y_dm_q^2Y_u^\dagger Y_uY_d^\dagger\\
&-4Y_dY_d^\dagger m_d^2Y_dY_d^\dagger-4Y_dY_d^\dagger Y_dm_q^2Y_d^\dagger-2Y_dY_d^\dagger Y_dY_d^\dagger m_d^2-4Y_dY_u^\dagger m_u^2Y_uY_d^\dagger\\
&-4Y_dY_u^\dagger Y_um_q^2Y_d^\dagger-2Y_dY_u^\dagger Y_uY_d^\dagger m_d^2-4\lambda^*Y_dT_d^\dagger T_\lambda+\frac{32}{3}g_3^4\mathbf{1}\sigma_{2,3}+\frac{8}{15}g_1^2\mathbf{1}\sigma_{2,11}\\
&+8\frac{1}{\sqrt{15}}g_1\mathbf{1}\sigma_{3,1}-24m_{H_d}^2Y_dY_d^\dagger\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)-12T_dT_d^\dagger\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)\\
&-6m_d^2Y_dY_d^\dagger\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)-12Y_dm_q^2Y_d^\dagger\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)-6Y_dY_d^\dagger m_d^2\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)\\
&-8m_{H_d}^2Y_dY_d^\dagger\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)-4T_dT_d^\dagger\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)-2m_d^2Y_dY_d^\dagger\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)\\
&-4Y_dm_q^2Y_d^\dagger\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)-2Y_dY_d^\dagger m_d^2\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)-12Y_dT_d^\dagger\mathrm{Tr}\Big(Y_d^\dagger T_d\Big)\\
&-4Y_dT_d^\dagger\mathrm{Tr}\Big(Y_e^\dagger T_e\Big)-12T_dY_d^\dagger\mathrm{Tr}\Big(T_d^*Y_d^T\Big)-12Y_dY_d^\dagger\mathrm{Tr}\Big(T_d^*T_d^T\Big)\\
&-4T_dY_d^\dagger\mathrm{Tr}\Big(T_e^*Y_e^T\Big)-4Y_dY_d^\dagger\mathrm{Tr}\Big(T_e^*T_e^T\Big)-12Y_dY_d^\dagger\mathrm{Tr}\Big(m_d^2Y_dY_d^\dagger\Big)\\
&-4Y_dY_d^\dagger\mathrm{Tr}\Big(m_e^2Y_eY_e^\dagger\Big)-4Y_dY_d^\dagger\mathrm{Tr}\Big(m_l^2Y_e^\dagger Y_e\Big)-12Y_dY_d^\dagger\mathrm{Tr}\Big(m_q^2Y_d^\dagger Y_d\Big)
\end{aligned}
\tag{69}
$$

$$
\begin{aligned}
\beta_{m_u^2}^{(1)}=&-\frac{32}{15}g_1^2\mathbf{1}|M_1|^2-\frac{32}{3}g_3^2\mathbf{1}|M_3|^2+4m_{H_u}^2Y_uY_u^\dagger+4T_uT_u^\dagger+2m_u^2Y_uY_u^\dagger+4Y_um_q^2Y_u^\dagger\\
&+2Y_uY_u^\dagger m_u^2-4\frac{1}{\sqrt{15}}g_1\mathbf{1}\sigma_{1,1}
\end{aligned}
\tag{70}
$$

$$
\begin{aligned}
\beta_{m_u^2}^{(2)}=&-\frac{128}{45}g_3^2\Big(15g_3^2M_3-2g_1^2\Big(2M_3+M_1\Big)\Big)\mathbf{1}M_3^*-\frac{4}{5}g_1^2m_{H_u}^2Y_uY_u^\dagger+12g_2^2m_{H_u}^2Y_uY_u^\dagger\\
&+24g_2^2|M_2|^2Y_uY_u^\dagger-4m_{H_d}^2|\lambda|^2Y_uY_u^\dagger-8m_{H_u}^2|\lambda|^2Y_uY_u^\dagger\\
&-4m_S^2|\lambda|^2Y_uY_u^\dagger-4|T_\lambda|^2Y_uY_u^\dagger+\frac{4}{5}g_1^2M_1Y_uT_u^\dagger-12g_2^2M_2Y_uT_u^\dagger\\
&-12g_2^2M_2^*T_uY_u^\dagger-4\lambda T_\lambda^*T_uY_u^\dagger\\
&+\frac{4}{225}g_1^2M_1^*\Big(45\Big(-2M_1Y_uY_u^\dagger+T_uY_u^\dagger\Big)+8\Big(321g_1^2M_1+40g_3^2\Big(2M_1+M_3\Big)\Big)\mathbf{1}\Big)-\frac{4}{5}g_1^2T_uT_u^\dagger\\
&+12g_2^2T_uT_u^\dagger-4|\lambda|^2T_uT_u^\dagger-\frac{2}{5}g_1^2m_u^2Y_uY_u^\dagger+6g_2^2m_u^2Y_uY_u^\dagger
\end{aligned}
$$

$$
\begin{aligned}
&- 2|\lambda|^2 m_u^2 Y_u Y_u^\dagger - \frac{4}{5} g_1^2 Y_u m_q^2 Y_u^\dagger + 12 g_2^2 Y_u m_q^2 Y_u^\dagger - 4|\lambda|^2 Y_u m_q^2 Y_u^\dagger \\
&- \frac{2}{5} g_1^2 Y_u Y_u^\dagger m_u^2 + 6 g_2^2 Y_u Y_u^\dagger m_u^2 - 2|\lambda|^2 Y_u Y_u^\dagger m_u^2 - 4 m_{H_d}^2 Y_u Y_d^\dagger Y_d Y_u^\dagger \\
&- 4 m_{H_u}^2 Y_u Y_d^\dagger Y_d Y_u^\dagger - 4 Y_u Y_d^\dagger T_d T_u^\dagger - 8 m_{H_u}^2 Y_u Y_u^\dagger Y_u Y_u^\dagger - 4 Y_u Y_u^\dagger T_u T_u^\dagger \\
&- 4 Y_u T_d^\dagger T_d Y_u^\dagger - 4 Y_u T_u^\dagger T_u Y_u^\dagger - 4 T_u Y_d^\dagger Y_d T_u^\dagger - 4 T_u Y_u^\dagger Y_u T_u^\dagger \\
&- 4 T_u T_d^\dagger Y_d Y_u^\dagger - 4 T_u T_u^\dagger Y_u Y_u^\dagger - 2 m_u^2 Y_u Y_d^\dagger Y_d Y_u^\dagger - 2 m_u^2 Y_u Y_u^\dagger Y_u Y_u^\dagger \\
&- 4 Y_u m_q^2 Y_d^\dagger Y_d Y_u^\dagger - 4 Y_u m_q^2 Y_u^\dagger Y_u Y_u^\dagger - 4 Y_u Y_d^\dagger m_d^2 Y_d Y_u^\dagger \\
&- 4 Y_u Y_d^\dagger Y_d m_q^2 Y_u^\dagger - 2 Y_u Y_d^\dagger Y_d Y_u^\dagger m_u^2 - 4 Y_u Y_u^\dagger m_u^2 Y_u Y_u^\dagger - 4 Y_u Y_u^\dagger Y_u m_q^2 Y_u^\dagger \\
&- 2 Y_u Y_u^\dagger Y_u Y_u^\dagger m_u^2 - 4 \lambda^* Y_u T_u^\dagger T_\lambda + \frac{32}{3} g_3^4 \mathbf{1} \sigma_{2,3} + \frac{32}{15} g_1^2 \mathbf{1} \sigma_{2,11} - 16 \frac{1}{\sqrt{15}} g_1 \mathbf{1} \sigma_{3,1} \\
&- 24 m_{H_u}^2 Y_u Y_u^\dagger \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) - 12 T_u T_u^\dagger \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) - 6 m_u^2 Y_u Y_u^\dagger \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) \\
&- 12 Y_u m_q^2 Y_u^\dagger \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) - 6 Y_u Y_u^\dagger m_u^2 \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) - 12 Y_u T_u^\dagger \mathrm{Tr}\Big(Y_u^\dagger T_u\Big) \\
&- 12 T_u Y_u^\dagger \mathrm{Tr}\Big(T_u^* Y_u^T\Big) - 12 Y_u Y_u^\dagger \mathrm{Tr}\Big(T_u^* T_u^T\Big) - 12 Y_u Y_u^\dagger \mathrm{Tr}\Big(m_q^2 Y_u^\dagger Y_u\Big) \\
&- 12 Y_u Y_u^\dagger \mathrm{Tr}\Big(m_u^2 Y_u Y_u^\dagger\Big)
\end{aligned} \tag{71}
$$

$$
\begin{aligned}
\beta_{m_e^2}^{(1)} = &- \frac{24}{5} g_1^2 \mathbf{1} |M_1|^2 + 2\Big(2 m_{H_d}^2 Y_e Y_e^\dagger + 2 T_e T_e^\dagger + 2 Y_e m_l^2 Y_e^\dagger + m_e^2 Y_e Y_e^\dagger + Y_e Y_e^\dagger m_e^2\Big) \\
&+ 2\sqrt{\frac{3}{5}} g_1 \mathbf{1} \sigma_{1,1}
\end{aligned} \tag{72}
$$

$$
\begin{aligned}
\beta_{m_e^2}^{(2)} = &\frac{2}{25}\Big(6 g_1^2 M_1^* \Big(234 g_1^2 M_1 \mathbf{1} + 5\Big(-2 M_1 Y_e Y_e^\dagger + T_e Y_e^\dagger\Big)\Big) + 20 g_1 \mathbf{1}\Big(3 g_1 \sigma_{2,11} + \sqrt{15}\sigma_{3,1}\Big) \\
&- 5\Big(30 g_2^2 M_2^* T_e Y_e^\dagger + 10 \lambda T_\lambda^* T_e Y_e^\dagger + 6 g_1^2 T_e T_e^\dagger - 30 g_2^2 T_e T_e^\dagger \\
&+ 10|\lambda|^2 T_e T_e^\dagger + 3 g_1^2 m_e^2 Y_e Y_e^\dagger - 15 g_2^2 m_e^2 Y_e Y_e^\dagger + 5|\lambda|^2 m_e^2 Y_e Y_e^\dagger \\
&+ 6 g_1^2 Y_e m_l^2 Y_e^\dagger - 30 g_2^2 Y_e m_l^2 Y_e^\dagger + 10|\lambda|^2 Y_e m_l^2 Y_e^\dagger + 3 g_1^2 Y_e Y_e^\dagger m_e^2 \\
&- 15 g_2^2 Y_e Y_e^\dagger m_e^2 + 5|\lambda|^2 Y_e Y_e^\dagger m_e^2 + 20 m_{H_d}^2 Y_e Y_e^\dagger Y_e Y_e^\dagger + 10 Y_e Y_e^\dagger T_e T_e^\dagger \\
&+ 10 Y_e T_e^\dagger T_e Y_e^\dagger + 10 T_e Y_e^\dagger Y_e T_e^\dagger + 10 T_e T_e^\dagger Y_e Y_e^\dagger + 5 m_e^2 Y_e Y_e^\dagger Y_e Y_e^\dagger \\
&+ 10 Y_e m_l^2 Y_e^\dagger Y_e Y_e^\dagger + 10 Y_e Y_e^\dagger m_e^2 Y_e Y_e^\dagger + 10 Y_e Y_e^\dagger Y_e m_l^2 Y_e^\dagger + 5 Y_e Y_e^\dagger Y_e Y_e^\dagger m_e^2 \\
&+ 30 T_e T_e^\dagger \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) + 15 m_e^2 Y_e Y_e^\dagger \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) + 30 Y_e m_l^2 Y_e^\dagger \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) \\
&+ 15 Y_e Y_e^\dagger m_e^2 \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) + 10 T_e T_e^\dagger \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) + 5 m_e^2 Y_e Y_e^\dagger \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) \\
&+ 10 Y_e m_l^2 Y_e^\dagger \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) + 5 Y_e Y_e^\dagger m_e^2 \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) \\
&+ Y_e T_e^\dagger \Big(10 \lambda^* T_\lambda + 10 \mathrm{Tr}\Big(Y_e^\dagger T_e\Big) + 30 g_2^2 M_2 + 30 \mathrm{Tr}\Big(Y_d^\dagger T_d\Big) - 6 g_1^2 M_1\Big) + 30 T_e Y_e^\dagger \mathrm{Tr}\Big(T_d^* Y_d^T\Big) \\
&+ 10 T_e Y_e^\dagger \mathrm{Tr}\Big(T_e^* Y_e^T\Big)
\end{aligned}
$$

$$+ 2Y_e Y_e^\dagger \Big(3g_1^2 m_{H_d}^2 - 15 g_2^2 m_{H_d}^2 - 30 g_2^2 |M_2|^2 + 5\Big(2m_{H_d}^2 + m_{H_u}^2 + m_S^2\Big)|\lambda|^2 + 5|T_\lambda|^2 + 30 m_{H_d}^2 \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big)$$
$$+ 10 m_{H_d}^2 \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) + 15\mathrm{Tr}\Big(T_d^* T_d^T\Big) + 5\mathrm{Tr}\Big(T_e^* T_e^T\Big) + 15\mathrm{Tr}\Big(m_d^2 Y_d Y_d^\dagger\Big) + 5\mathrm{Tr}\Big(m_e^2 Y_e Y_e^\dagger\Big)$$
$$+ 5\mathrm{Tr}\Big(m_l^2 Y_e^\dagger Y_e\Big) + 15\mathrm{Tr}\Big(m_q^2 Y_d^\dagger Y_d\Big)\Big)\Big)\Big) \tag{73}$$

$$\beta^{(1)}_{m_S^2} = 4\Big(3m_S^2|\kappa|^2 + \Big(m_{H_d}^2 + m_{H_u}^2 + m_S^2\Big)|\lambda|^2 + |T_\kappa|^2 + |T_\lambda|^2\Big) \tag{74}$$

$$\beta^{(2)}_{m_S^2} = -\frac{4}{5}\Big(120 m_S^2 \kappa^2 \kappa^{*,2} + 20\Big(m_{H_d}^2 + m_{H_u}^2 + m_S^2\Big)\lambda^2\lambda^{*,2}$$
$$+ 20\kappa^*\Big(4\kappa|T_\kappa|^2 + \Big(4m_S^2 + m_{H_d}^2 + m_{H_u}^2\Big)\kappa|\lambda|^2 + T_\lambda^*\Big(\kappa T_\lambda + \lambda T_\kappa\Big)\Big)$$
$$+ T_\lambda^*\Big(T_\lambda\Big(15\mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) - 3\Big(5g_2^2 - 5\mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) + g_1^2\Big) + 5\mathrm{Tr}\Big(Y_e Y_e^\dagger\Big)\Big)$$
$$+ \lambda\Big(15\mathrm{Tr}\Big(Y_d^\dagger T_d\Big) + 3\Big(5g_2^2 M_2 + 5\mathrm{Tr}\Big(Y_u^\dagger T_u\Big) + g_1^2 M_1\Big) + 5\mathrm{Tr}\Big(Y_e^\dagger T_e\Big)\Big)\Big)$$
$$+ \lambda^*\Big(- 3g_1^2 m_{H_d}^2\lambda - 15 g_2^2 m_{H_d}^2\lambda - 3g_1^2 m_{H_u}^2\lambda - 15 g_2^2 m_{H_u}^2\lambda - 3g_1^2 m_S^2\lambda - 15 g_2^2 m_S^2\lambda$$
$$+ 20\lambda|T_\kappa|^2 + 40\lambda|T_\lambda|^2 + 20\kappa T_\kappa^* T_\lambda + 3g_1^2 M_1^*\Big(- 2M_1\lambda + T_\lambda\Big) + 15 g_2^2 M_2^*\Big(- 2M_2\lambda + T_\lambda\Big)$$
$$+ 30 m_{H_d}^2\lambda\mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) + 15 m_{H_u}^2\lambda\mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) + 15 m_S^2\lambda\mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) + 10 m_{H_d}^2\lambda\mathrm{Tr}\Big(Y_e Y_e^\dagger\Big)$$
$$+ 5 m_{H_u}^2\lambda\mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) + 5 m_S^2\lambda\mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) + 15 m_{H_d}^2\lambda\mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) + 30 m_{H_u}^2\lambda\mathrm{Tr}\Big(Y_u Y_u^\dagger\Big)$$
$$+ 15 m_S^2\lambda\mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) + 15 T_\lambda\mathrm{Tr}\Big(T_d^* Y_d^T\Big) + 15\lambda\mathrm{Tr}\Big(T_d^* T_d^T\Big) + 5 T_\lambda\mathrm{Tr}\Big(T_e^* Y_e^T\Big) + 5\lambda\mathrm{Tr}\Big(T_e^* T_e^T\Big)$$
$$+ 15 T_\lambda\mathrm{Tr}\Big(T_u^* Y_u^T\Big) + 15\lambda\mathrm{Tr}\Big(T_u^* T_u^T\Big) + 15\lambda\mathrm{Tr}\Big(m_d^2 Y_d Y_d^\dagger\Big) + 5\lambda\mathrm{Tr}\Big(m_e^2 Y_e Y_e^\dagger\Big) + 5\lambda\mathrm{Tr}\Big(m_l^2 Y_e^\dagger Y_e\Big)$$
$$+ 15\lambda\mathrm{Tr}\Big(m_q^2 Y_d^\dagger Y_d\Big) + 15\lambda\mathrm{Tr}\Big(m_q^2 Y_u^\dagger Y_u\Big) + 15\lambda\mathrm{Tr}\Big(m_u^2 Y_u Y_u^\dagger\Big)\Big)\Big) \tag{75}$$

3.7 Vacuum expectation values

$$\beta^{(1)}_{v_d} = \frac{1}{20} v_d\Big(15g_2^2 + 15g_2^2\mathrm{Xi} - 20|\lambda|^2 - 20\mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) + 3g_1^2 + 3g_1^2\mathrm{Xi} - 60\mathrm{Tr}\Big(Y_d Y_d^\dagger\Big)\Big) \tag{76}$$

$$\beta^{(2)}_{v_d} = \frac{1}{400} v_d\Big(- 414 g_1^4 - 180 g_1^2 g_2^2 - 1200 g_2^4 - 9 g_1^4\mathrm{Xi} - 90 g_1^2 g_2^2\mathrm{Xi} + 875 g_2^4\mathrm{Xi} + 9 g_1^4\mathrm{Xi}^2 + 90 g_1^2 g_2^2\mathrm{Xi}^2$$
$$- 225 g_2^4\mathrm{Xi}^2 + 1200\lambda^2\lambda^{*,2} - 40\Big(5\Big(32g_3^2 + 9g_2^2\mathrm{Xi}\Big) + g_1^2\Big(9\mathrm{Xi} - 4\Big)\Big)\mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) - 480 g_1^2\mathrm{Tr}\Big(Y_e Y_e^\dagger\Big)$$
$$- 120 g_1^2\mathrm{Xi}\mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) - 600 g_2^2\mathrm{Xi}\mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) - 40|\lambda|^2\Big(15g_2^2\mathrm{Xi} - 20\kappa\kappa^* - 30\mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) + 3g_1^2\mathrm{Xi}\Big)$$
$$+ 3600\mathrm{Tr}\Big(Y_d Y_d^\dagger Y_d Y_d^\dagger\Big) + 1200\mathrm{Tr}\Big(Y_d Y_u^\dagger Y_u Y_d^\dagger\Big) + 1200\mathrm{Tr}\Big(Y_e Y_e^\dagger Y_e Y_e^\dagger\Big)\Big) \tag{77}$$

$$\beta^{(1)}_{v_u} = \frac{1}{20} v_u\Big(- 20|\lambda|^2 + 3\Big(- 20\mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) + \Big(5g_2^2 + g_1^2\Big)\Big(1 + \mathrm{Xi}\Big)\Big)\Big) \tag{78}$$

$$\beta^{(2)}_{v_u} = \frac{1}{400} v_u\Big(- 414 g_1^4 - 180 g_1^2 g_2^2 - 1200 g_2^4 - 9 g_1^4\mathrm{Xi} - 90 g_1^2 g_2^2\mathrm{Xi} + 875 g_2^4\mathrm{Xi} + 9 g_1^4\mathrm{Xi}^2 + 90 g_1^2 g_2^2\mathrm{Xi}^2$$

$$- 225 g_2^4 \mathrm{Xi}^2 + 1200 \lambda^2 \lambda^{*,2} - 40 |\lambda|^2 \Big(- 10 \mathrm{Tr}\Big(Y_e Y_e^\dagger \Big) + 15 g_2^2 \mathrm{Xi} - 20 \kappa \kappa^* - 30 \mathrm{Tr}\Big(Y_d Y_d^\dagger \Big) + 3 g_1^2 \mathrm{Xi} \Big)$$
$$- 40 \Big(5 \Big(32 g_3^2 + 9 g_2^2 \mathrm{Xi} \Big) + g_1^2 \Big(9 \mathrm{Xi} + 8 \Big) \Big) \mathrm{Tr}\Big(Y_u Y_u^\dagger \Big) + 1200 \mathrm{Tr}\Big(Y_d Y_u^\dagger Y_u Y_d^\dagger \Big) + 3600 \mathrm{Tr}\Big(Y_u Y_u^\dagger Y_u Y_u^\dagger \Big) \Big) \tag{79}$$

$$\beta_{v_s}^{(1)} = -2 v_s \Big(|\kappa|^2 + |\lambda|^2 \Big) \tag{80}$$

$$\beta_{v_s}^{(2)} = +8 v_s \kappa^2 \kappa^{*,2} + 8 v_s \lambda |\kappa|^2 \lambda^*$$
$$+ \frac{2}{5} v_s |\lambda|^2 \Big(10 \lambda \lambda^* - 15 g_2^2 + 15 \mathrm{Tr}\Big(Y_d Y_d^\dagger \Big) + 15 \mathrm{Tr}\Big(Y_u Y_u^\dagger \Big) - 3 g_1^2 + 5 \mathrm{Tr}\Big(Y_e Y_e^\dagger \Big) \Big) \tag{81}$$

4 Field Rotations

4.1 Rotations in gauge sector for eigenstates 'EWSB'

$$\begin{pmatrix} B_\rho \\ W_{3\rho} \end{pmatrix} = Z^{\gamma Z} \begin{pmatrix} \gamma_\rho \\ Z_\rho \end{pmatrix} \tag{82}$$

$$\begin{pmatrix} W_{1\rho} \\ W_{2\rho} \end{pmatrix} = Z^{W} \begin{pmatrix} W_\rho^- \\ W_\rho^- \end{pmatrix} \tag{83}$$

$$\begin{pmatrix} \lambda_{\tilde{W},1} \\ \lambda_{\tilde{W},2} \\ \lambda_{\tilde{W},3} \end{pmatrix} = Z^{\tilde{W}} \begin{pmatrix} \tilde{W}^- \\ \tilde{W}^+ \\ \tilde{W}^0 \end{pmatrix} \tag{84}$$

$$\tag{85}$$

The mixing matrices are parametrized by

$$Z^{\gamma Z} = \begin{pmatrix} \cos\Theta_W & -\sin\Theta_W \\ \sin\Theta_W & \cos\Theta_W \end{pmatrix} \tag{86}$$

$$Z^{W} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -i\frac{1}{\sqrt{2}} & i\frac{1}{\sqrt{2}} \end{pmatrix} \tag{87}$$

$$Z^{\tilde{W}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -i\frac{1}{\sqrt{2}} & i\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{88}$$

$$\tag{89}$$

4.2 Rotations in Mass sector for eigenstates 'EWSB'

4.2.1 Mass Matrices for Scalars

- **Mass matrix for Higgs**, Basis: $(\phi_d, \phi_u, \phi_s), (\phi_d, \phi_u, \phi_s)$

$$m_h^2 = \begin{pmatrix} m_{\phi_d\phi_d} & m_{\phi_u\phi_d} & m_{\phi_s\phi_d} \\ m_{\phi_d\phi_u} & m_{\phi_u\phi_u} & m_{\phi_s\phi_u} \\ m_{\phi_d\phi_s} & m_{\phi_u\phi_s} & m_{\phi_s\phi_s} \end{pmatrix} \tag{90}$$

$$m_{\phi_d\phi_d} = \frac{1}{2}\Big(v_s^2 + v_u^2\Big)|\lambda|^2 + \frac{1}{8}\Big(g_1^2 + g_2^2\Big)\Big(3v_d^2 - v_u^2\Big) + m_{H_d}^2 \tag{91}$$

$$m_{\phi_d\phi_u} = \frac{1}{4}\Big(- 2\sqrt{2}v_s\Re\Big(T_\lambda\Big) + \Big(4v_dv_u\lambda - v_s^2\kappa\Big)\lambda^* - v_s^2\lambda\kappa^*\Big) - \frac{1}{4}\Big(g_1^2 + g_2^2\Big)v_dv_u \tag{92}$$

$$m_{\phi_u\phi_u} = \frac{1}{2}\Big(v_d^2 + v_s^2\Big)|\lambda|^2 - \frac{1}{8}\Big(g_1^2 + g_2^2\Big)\Big(- 3v_u^2 + v_d^2\Big) + m_{H_u}^2 \tag{93}$$

$$m_{\phi_d\phi_s} = -\frac{1}{\sqrt{2}}v_u\Re\Big(T_\lambda\Big) + v_s\Big(\Big(- \frac{1}{2}v_u\kappa + v_d\lambda\Big)\lambda^* - \frac{1}{2}v_u\lambda\kappa^*\Big) \tag{94}$$

$$m_{\phi_u\phi_s} = \frac{1}{2}\Big(- v_d\Big(\sqrt{2}\Re\Big(T_\lambda\Big) + v_s\lambda\kappa^*\Big) - v_s\Big(- 2v_u\lambda + v_d\kappa\Big)\lambda^*\Big) \tag{95}$$

$$m_{\phi_s\phi_s} = \frac{1}{2}\Big(2\sqrt{2}v_s\Re\Big(T_\kappa\Big) + \Big(6v_s^2\kappa - v_dv_u\lambda\Big)\kappa^* + \Big(\Big(v_d^2 + v_u^2\Big)\lambda - v_dv_u\kappa\Big)\lambda^*\Big) + m_S^2 \tag{96}$$

This matrix is diagonalized by Z^H:

$$Z^H m_h^2 Z^{H,\dagger} = m_{2,h}^{dia} \tag{97}$$

with

$$\phi_d = \sum_j Z_{j1}^H h_j\,, \qquad \phi_u = \sum_j Z_{j2}^H h_j\,, \qquad \phi_s = \sum_j Z_{j3}^H h_j \tag{98}$$

- **Mass matrix for Pseudo-Scalar Higgs**, Basis: $(\sigma_d, \sigma_u, \sigma_s)\,, (\sigma_d, \sigma_u, \sigma_s)$

$$m_{A^0}^2 = \begin{pmatrix} m_{\sigma_d\sigma_d} & \frac{1}{4}v_s\Big(2\sqrt{2}\Re\Big(T_\lambda\Big) + 2v_s\Re\Big(\lambda\kappa^*\Big)\Big) & m_{\sigma_s\sigma_d} \\ \frac{1}{4}v_s\Big(2\sqrt{2}\Re\Big(T_\lambda\Big) + 2v_s\Re\Big(\lambda\kappa^*\Big)\Big) & m_{\sigma_u\sigma_u} & m_{\sigma_s\sigma_u} \\ m_{\sigma_d\sigma_s} & m_{\sigma_u\sigma_s} & m_{\sigma_s\sigma_s} \end{pmatrix} + \xi_Z m^2(Z) \tag{99}$$

$$m_{\sigma_d\sigma_d} = \frac{1}{2}\Big(v_s^2 + v_u^2\Big)|\lambda|^2 + \frac{1}{8}\Big(g_1^2 + g_2^2\Big)\Big(- v_u^2 + v_d^2\Big) + m_{H_d}^2 \tag{100}$$

$$m_{\sigma_u\sigma_u} = \frac{1}{2}\Big(v_d^2 + v_s^2\Big)|\lambda|^2 - \frac{1}{8}\Big(g_1^2 + g_2^2\Big)\Big(- v_u^2 + v_d^2\Big) + m_{H_u}^2 \tag{101}$$

$$m_{\sigma_d\sigma_s} = -\frac{1}{2}v_u\Big(2v_s\Re\Big(\lambda\kappa^*\Big) - \sqrt{2}\Re\Big(T_\lambda\Big)\Big) \tag{102}$$

$$m_{\sigma_u\sigma_s} = -\frac{1}{2}v_d\Big(2v_s\Re\Big(\lambda\kappa^*\Big) - \sqrt{2}\Re\Big(T_\lambda\Big)\Big) \tag{103}$$

$$m_{\sigma_s\sigma_s} = \frac{1}{2}\Big(- 2\sqrt{2}v_s\Re\Big(T_\kappa\Big) + \Big(2v_s^2\kappa + v_dv_u\lambda\Big)\kappa^* + \Big(\Big(v_d^2 + v_u^2\Big)\lambda + v_dv_u\kappa\Big)\lambda^*\Big) + m_S^2 \tag{104}$$

Gauge fixing contributions:

$$m^2(\xi_Z) = \begin{pmatrix} m_{\sigma_d\sigma_d} & m_{\sigma_u\sigma_d} & 0 \\ m_{\sigma_d\sigma_u} & m_{\sigma_u\sigma_u} & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{105}$$

$$m_{\sigma_d\sigma_d} = \frac{1}{4}v_d^2\Big(g_1\sin\Theta_W + g_2\cos\Theta_W\Big)^2 \tag{106}$$

$$m_{\sigma_d\sigma_u} = -\frac{1}{4}v_d v_u\Big(g_1\sin\Theta_W + g_2\cos\Theta_W\Big)^2 \tag{107}$$

$$m_{\sigma_u\sigma_u} = \frac{1}{4}v_u^2\Big(g_1\sin\Theta_W + g_2\cos\Theta_W\Big)^2 \tag{108}$$

This matrix is diagonalized by Z^A:

$$Z^A m^2_{A^0} Z^{A,\dagger} = m^{dia}_{2,A^0} \tag{109}$$

with

$$\sigma_d = \sum_j Z^A_{j1}A^0_j\,, \qquad \sigma_u = \sum_j Z^A_{j2}A^0_j\,, \qquad \sigma_s = \sum_j Z^A_{j3}A^0_j \tag{110}$$

- **Mass matrix for Charged Higgs**, Basis: $\left(H_d^-, H_u^{+,*}\right), \left(H_d^{-,*}, H_u^+\right)$

$$m^2_{H^-} = \begin{pmatrix} m_{H_d^- H_d^{-,*}} & m^*_{H_u^{+,*}H_d^{-,*}} \\ m_{H_d^- H_u^+} & m_{H_u^{+,*}H_u^+} \end{pmatrix} + \xi_{W^-} m^2(W^-) \tag{111}$$

$$m_{H_d^- H_d^{-,*}} = \frac{1}{2}v_s^2|\lambda|^2 + \frac{1}{8}\Big(g_1^2\Big(-v_u^2+v_d^2\Big) + g_2^2\Big(v_d^2+v_u^2\Big)\Big) + m^2_{H_d} \tag{112}$$

$$m_{H_d^- H_u^+} = \frac{1}{2}\Big(\lambda\Big(-v_d v_u\lambda^* + v_s^2\kappa^*\Big) + \sqrt{2}v_s T_\lambda\Big) + \frac{1}{4}g_2^2 v_d v_u \tag{113}$$

$$m_{H_u^{+,*}H_u^+} = \frac{1}{2}v_s^2|\lambda|^2 + \frac{1}{8}\Big(g_1^2\Big(-v_d^2+v_u^2\Big) + g_2^2\Big(v_d^2+v_u^2\Big)\Big) + m^2_{H_u} \tag{114}$$

Gauge fixing contributions:

$$m^2(\xi_{W^-}) = \begin{pmatrix} \frac{1}{4}g_2^2 v_d^2 & -\frac{1}{4}g_2^2 v_d v_u \\ -\frac{1}{4}g_2^2 v_d v_u & \frac{1}{4}g_2^2 v_u^2 \end{pmatrix} \tag{115}$$

This matrix is diagonalized by Z^+:

$$Z^+ m^2_{H^-} Z^{+,\dagger} = m^{dia}_{2,H^-} \tag{116}$$

with

$$H_d^- = \sum_j Z^+_{j1}H^-_j\,, \qquad H_u^+ = \sum_j Z^+_{j2}H^+_j \tag{117}$$

- **Mass matrix for Down-Squarks**, Basis: $\left(\tilde{d}_{L,\alpha_1}, \tilde{d}_{R,\alpha_2}\right), \left(\tilde{d}^*_{L,\beta_1}, \tilde{d}^*_{R,\beta_2}\right)$

$$m^2_{\tilde{d}} = \begin{pmatrix} m_{\tilde{d}_L \tilde{d}^*_L} & \frac{1}{2}\text{Delta}\Big(\sqrt{2} v_d T^\dagger_d - v_s v_u \lambda Y^\dagger_d\Big)\delta_{\alpha_1\beta_2} \\ \frac{1}{2}\text{Delta}\delta_{\alpha_2\beta_1}\Big(\sqrt{2} v_d T_d - v_s v_u Y_d \lambda^*\Big) & m_{\tilde{d}_R \tilde{d}^*_R} \end{pmatrix} \tag{118}$$

$$m_{\tilde{d}_L \tilde{d}^*_L} = \frac{1}{2}\Big(2m^2_q + \mathbf{1} v^2_d Y_d Y^*_{d,o_1 o_1}\Big)\delta_{\alpha_1\beta_1} - \frac{1}{24}\Big(3g^2_2 + g^2_1\Big)\mathbf{1}\Big(-v^2_u + v^2_d\Big)\delta_{\alpha_1\beta_1} \tag{119}$$

$$m_{\tilde{d}_R \tilde{d}^*_R} = \frac{1}{12} g^2_1 \mathbf{1}\Big(-v^2_d + v^2_u\Big)\delta_{\alpha_2\beta_2} + \frac{1}{2}\delta_{\alpha_2\beta_2}\Big(2m^2_d + \mathbf{1} v^2_d Y^\dagger_d Y_{d,o_2 o_2}\Big) \tag{120}$$

This matrix is diagonalized by Z^D:

$$Z^D m^2_{\tilde{d}} Z^{D,\dagger} = m^{dia}_{2,\tilde{d}} \tag{121}$$

with

$$\tilde{d}_{L,i\alpha} = \sum_j Z^{D,*}_{ji} \tilde{d}_{j\text{ft}1\alpha}\,, \qquad \tilde{d}_{R,i\alpha} = \sum_j Z^{D,*}_{ji} \tilde{d}_{j\text{ft}1\alpha} \tag{122}$$

- **Mass matrix for Up-Squarks**, Basis: $\left(\tilde{u}_{L,\alpha_1}, \tilde{u}_{R,\alpha_2}\right), \left(\tilde{u}^*_{L,\beta_1}, \tilde{u}^*_{R,\beta_2}\right)$

$$m^2_{\tilde{u}} = \begin{pmatrix} m_{\tilde{u}_L \tilde{u}^*_L} & \frac{1}{2}\text{Delta}\Big(\sqrt{2} v_u T^\dagger_u - v_d v_s \lambda Y^\dagger_u\Big)\delta_{\alpha_1\beta_2} \\ \frac{1}{2}\text{Delta}\delta_{\alpha_2\beta_1}\Big(\sqrt{2} v_u T_u - v_d v_s Y_u \lambda^*\Big) & m_{\tilde{u}_R \tilde{u}^*_R} \end{pmatrix} \tag{123}$$

$$m_{\tilde{u}_L \tilde{u}^*_L} = \frac{1}{2}\Big(2m^2_q + \mathbf{1} v^2_u Y_u Y^*_{u,o_1 o_1}\Big)\delta_{\alpha_1\beta_1} - \frac{1}{24}\Big(-3g^2_2 + g^2_1\Big)\mathbf{1}\Big(-v^2_u + v^2_d\Big)\delta_{\alpha_1\beta_1} \tag{124}$$

$$m_{\tilde{u}_R \tilde{u}^*_R} = \frac{1}{2}\delta_{\alpha_2\beta_2}\Big(2m^2_u + \mathbf{1} v^2_u Y^\dagger_u Y_{u,o_2 o_2}\Big) + \frac{1}{6} g^2_1 \mathbf{1}\Big(-v^2_u + v^2_d\Big)\delta_{\alpha_2\beta_2} \tag{125}$$

This matrix is diagonalized by Z^U:

$$Z^U m^2_{\tilde{u}} Z^{U,\dagger} = m^{dia}_{2,\tilde{u}} \tag{126}$$

with

$$\tilde{u}_{L,i\alpha} = \sum_j Z^{U,*}_{ji} \tilde{u}_{j\text{ft}1\alpha}\,, \qquad \tilde{u}_{R,i\alpha} = \sum_j Z^{U,*}_{ji} \tilde{u}_{j\text{ft}1\alpha} \tag{127}$$

- **Mass matrix for Sleptons**, Basis: $(\tilde{e}_L, \tilde{e}_R), (\tilde{e}^*_L, \tilde{e}^*_R)$

$$m^2_{\tilde{e}} = \begin{pmatrix} m_{\tilde{e}_L \tilde{e}^*_L} & \frac{1}{2}\text{Delta}\Big(\sqrt{2} v_d T^\dagger_e - v_s v_u \lambda Y^\dagger_e\Big) \\ \frac{1}{2}\text{Delta}\Big(\sqrt{2} v_d T_e - v_s v_u Y_e \lambda^*\Big) & m_{\tilde{e}_R \tilde{e}^*_R} \end{pmatrix} \tag{128}$$

$$m_{\tilde{e}_L \tilde{e}^*_L} = \frac{1}{2}\mathbf{1} v^2_d Y_e Y^*_{e,o_1 o_1} + \frac{1}{8}\Big(-g^2_2 + g^2_1\Big)\mathbf{1}\Big(-v^2_u + v^2_d\Big) + m^2_l \tag{129}$$

$$m_{\tilde{e}_R \tilde{e}_R^*} = \frac{1}{2}\mathbf{1} v_d^2 Y_e^\dagger Y_{e,o_2 o_2} + \frac{1}{4} g_1^2 \mathbf{1}\Big(- v_d^2 + v_u^2\Big) + m_e^2 \tag{130}$$

This matrix is diagonalized by Z^E:

$$Z^E m_{\tilde{e}}^2 Z^{E,\dagger} = m_{2,\tilde{e}}^{dia} \tag{131}$$

with

$$\tilde{e}_{L,i} = \sum_j Z_{ji}^{E,*} \tilde{e}_{j\text{ft}1}\,, \qquad \tilde{e}_{R,i} = \sum_j Z_{ji}^{E,*} \tilde{e}_{j\text{ft}1} \tag{132}$$

4.2.2 Mass Matrices for Fermions

- **Mass matrix for Neutralinos**, Basis: $\left(\lambda_{\tilde{B}}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{S}\right), \left(\lambda_{\tilde{B}}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{S}\right)$

$$m_{\tilde{\chi}^0} = \begin{pmatrix} M_1 & 0 & -\frac{1}{2} g_1 v_d & \frac{1}{2} g_1 v_u & 0 \\ 0 & M_2 & \frac{1}{2} g_2 v_d & -\frac{1}{2} g_2 v_u & 0 \\ -\frac{1}{2} g_1 v_d & \frac{1}{2} g_2 v_d & 0 & -\frac{1}{\sqrt{2}} v_s \lambda & -\frac{1}{\sqrt{2}} v_u \lambda \\ \frac{1}{2} g_1 v_u & -\frac{1}{2} g_2 v_u & -\frac{1}{\sqrt{2}} v_s \lambda & 0 & -\frac{1}{\sqrt{2}} v_d \lambda \\ 0 & 0 & -\frac{1}{\sqrt{2}} v_u \lambda & -\frac{1}{\sqrt{2}} v_d \lambda & \sqrt{2} v_s \kappa \end{pmatrix} \tag{133}$$

This matrix is diagonalized by N:

$$N^* m_{\tilde{\chi}^0} N^\dagger = m_{\tilde{\chi}^0}^{dia} \tag{134}$$

with

$$\lambda_{\tilde{B}} = \sum_j N_{j1}^* \lambda_j^0\,, \qquad \tilde{W}^0 = \sum_j N_{j2}^* \lambda_j^0\,, \qquad \tilde{H}_d^0 = \sum_j N_{j3}^* \lambda_j^0 \tag{135}$$

$$\tilde{H}_u^0 = \sum_j N_{j4}^* \lambda_j^0\,, \qquad \tilde{S} = \sum_j N_{j5}^* \lambda_j^0 \tag{136}$$

- **Mass matrix for Charginos**, Basis: $\left(\tilde{W}^-, \tilde{H}_d^-\right), \left(\tilde{W}^+, \tilde{H}_u^+\right)$

$$m_{\tilde{\chi}^-} = \begin{pmatrix} M_2 & \frac{1}{\sqrt{2}} g_2 v_u \\ \frac{1}{\sqrt{2}} g_2 v_d & \frac{1}{\sqrt{2}} v_s \lambda \end{pmatrix} \tag{137}$$

This matrix is diagonalized by U and V

$$U^* m_{\tilde{\chi}^-} V^\dagger = m_{\tilde{\chi}^-}^{dia} \tag{138}$$

with

$$\tilde{W}^- = \sum_{t_2} U_{j1}^* \lambda_j^-\,, \qquad \tilde{H}_d^- = \sum_{t_2} U_{j2}^* \lambda_j^- \tag{139}$$

$$\tilde{W}^+ = \sum_{t_2} V_{1j}^* \lambda_j^+\,, \qquad \tilde{H}_u^+ = \sum_{t_2} V_{2j}^* \lambda_j^+ \tag{140}$$

5 Vacuum Expectation Values

$$H_d^0 = \frac{1}{\sqrt{2}}\phi_d + \frac{1}{\sqrt{2}}v_d + i\frac{1}{\sqrt{2}}\sigma_d \tag{141}$$

$$H_u^0 = \frac{1}{\sqrt{2}}\phi_u + \frac{1}{\sqrt{2}}v_u + i\frac{1}{\sqrt{2}}\sigma_u \tag{142}$$

$$S = \frac{1}{\sqrt{2}}\phi_s + \frac{1}{\sqrt{2}}v_s + i\frac{1}{\sqrt{2}}\sigma_s \tag{143}$$

6 Tadpole Equations

$$\frac{\partial V}{\partial \phi_d} = \frac{1}{4}\Big(\Big(2v_d\Big(v_s^2 + v_u^2\Big)\lambda - v_s^2 v_u \kappa\Big)\lambda^* + 4m_{H_d}^2 v_d - v_s v_u\Big(2\sqrt{2}\Re\Big(T_\lambda\Big) + v_s\lambda\kappa^*\Big)\Big) + \frac{1}{8}\Big(g_1^2 + g_2^2\Big)v_d\Big(-v_u + v_d\Big)\Big(v_d + v_u\Big) \tag{144}$$

$$\begin{aligned}\frac{\partial V}{\partial \phi_u} = &+\frac{1}{8}\Big(g_1^2 + g_2^2\Big)v_u\Big(-v_d^2 + v_u^2\Big) \\ &+ \frac{1}{4}\Big(\Big(2\Big(v_d^2 + v_s^2\Big)v_u\lambda - v_d v_s^2\kappa\Big)\lambda^* + 4m_{H_u}^2 v_u - v_d v_s\Big(2\sqrt{2}\Re\Big(T_\lambda\Big) + v_s\lambda\kappa^*\Big)\Big)\end{aligned} \tag{145}$$

$$\begin{aligned}\frac{\partial V}{\partial \phi_s} = &\frac{1}{4}\Big(\Big(-2v_d v_s v_u\lambda + 4v_s^3\kappa\Big)\kappa^* + v_s\Big(2\Big(\Big(v_d^2 + v_u^2\Big)\lambda - v_d v_u\kappa\Big)\lambda^* + 4m_S^2\Big) \\ &+ \sqrt{2}\Big(-v_d v_u\Big(T_\lambda^* + T_\lambda\Big) + v_s^2\Big(T_\kappa^* + T_\kappa\Big)\Big)\Big)\end{aligned} \tag{146}$$

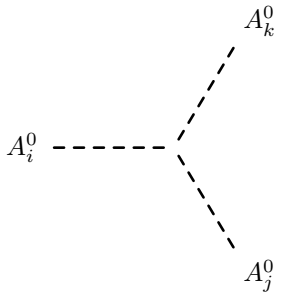
7 Particle content for eigenstates 'EWSB'

Name	Type	complex/real	Generations	Indices
$\tilde{\nu}_L$	Scalar	complex	3	generation, 3
h	Scalar	real	3	generation, 3
A^0	Scalar	real	3	generation, 3
H^-	Scalar	complex	2	generation, 2
$\tilde{d}$	Scalar	complex	3	generation, 3, flavor, 2, color, 3
$\tilde{u}$	Scalar	complex	3	generation, 3, flavor, 2, color, 3
$\tilde{e}$	Scalar	complex	3	generation, 3, flavor, 2
$\tilde{g}$	Fermion	Majorana	1	color, 8
d	Fermion	Dirac	3	generation, 3, color, 3
u	Fermion	Dirac	3	generation, 3, color, 3
e	Fermion	Dirac	3	generation, 3
ν	Fermion	Dirac	3	generation, 3

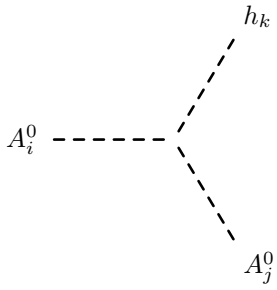
$\tilde{\chi}^0$	Fermion	Majorana	5	generation, 5
$\tilde{\chi}^-$	Fermion	Dirac	2	generation, 2
g	Vector	real	1	color, 8, lorentz, 4
γ	Vector	real	1	lorentz, 4
Z	Vector	real	1	lorentz, 4
W^-	Vector	complex	1	lorentz, 4
η^G	Ghost	real	1	color, 8
η^γ	Ghost	real	1	
η^Z	Ghost	real	1	
η^-	Ghost	complex	1	
η^+	Ghost	complex	1	

8 Interactions for eigenstates 'EWSB'

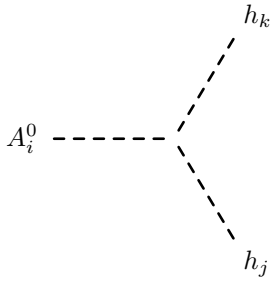
8.1 Three Scalar-Interaction



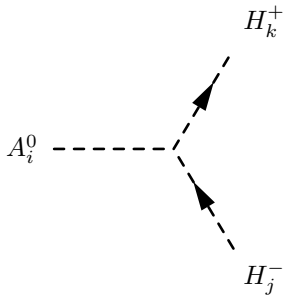
$$\frac{1}{4}\Big(- 2\lambda\kappa^*\Big(Z_{i2}^A\Big(v_s Z_{j1}^A Z_{k3}^A + Z_{j3}^A\Big(- v_d Z_{k3}^A + v_s Z_{k1}^A\Big)\Big) + Z_{i1}^A\Big(v_s Z_{j2}^A Z_{k3}^A + Z_{j3}^A\Big(v_s Z_{k2}^A - v_u Z_{k3}^A\Big)\Big)$$
$$- Z_{i3}^A\Big(Z_{j1}^A\Big(- v_s Z_{k2}^A + v_u Z_{k3}^A\Big) + Z_{j2}^A\Big(v_d Z_{k3}^A - v_s Z_{k1}^A\Big) + Z_{j3}^A\Big(v_d Z_{k2}^A + v_u Z_{k1}^A\Big)\Big)\Big)$$
$$+ 2\kappa\lambda^*\Big(Z_{i2}^A\Big(v_s Z_{j1}^A Z_{k3}^A + Z_{j3}^A\Big(- v_d Z_{k3}^A + v_s Z_{k1}^A\Big)\Big) + Z_{i1}^A\Big(v_s Z_{j2}^A Z_{k3}^A + Z_{j3}^A\Big(v_s Z_{k2}^A - v_u Z_{k3}^A\Big)\Big)$$
$$- Z_{i3}^A\Big(Z_{j1}^A\Big(- v_s Z_{k2}^A + v_u Z_{k3}^A\Big) + Z_{j2}^A\Big(v_d Z_{k3}^A - v_s Z_{k1}^A\Big) + Z_{j3}^A\Big(v_d Z_{k2}^A + v_u Z_{k1}^A\Big)\Big)\Big)$$
$$- \sqrt{2}\Big(2\Big(- T_\kappa^* + T_\kappa\Big)Z_{i3}^A Z_{j3}^A Z_{k3}^A$$
$$+ T_\lambda^*\Big(Z_{i1}^A\Big(Z_{j2}^A Z_{k3}^A + Z_{j3}^A Z_{k2}^A\Big) + Z_{i2}^A\Big(Z_{j1}^A Z_{k3}^A + Z_{j3}^A Z_{k1}^A\Big) + Z_{i3}^A\Big(Z_{j1}^A Z_{k2}^A + Z_{j2}^A Z_{k1}^A\Big)\Big)$$
$$- T_\lambda\Big(Z_{i1}^A\Big(Z_{j2}^A Z_{k3}^A + Z_{j3}^A Z_{k2}^A\Big) + Z_{i2}^A\Big(Z_{j1}^A Z_{k3}^A + Z_{j3}^A Z_{k1}^A\Big) + Z_{i3}^A\Big(Z_{j1}^A Z_{k2}^A + Z_{j2}^A Z_{k1}^A\Big)\Big)\Big)\Big) \tag{147}$$



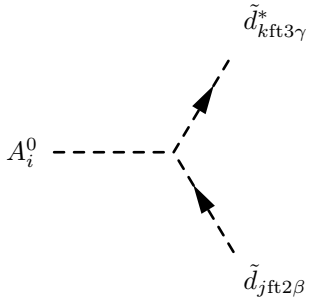
$$
\begin{aligned}
&\frac{i}{4}\Big(- Z_{i1}^A\Big(- 2v_s\lambda\kappa^* Z_{j3}^A Z_{k2}^H - 2v_s\kappa\lambda^* Z_{j3}^A Z_{k2}^H + \sqrt{2}T_\lambda^* Z_{j3}^A Z_{k2}^H + \sqrt{2}T_\lambda Z_{j3}^A Z_{k2}^H \\
&+ 2v_s\lambda\kappa^* Z_{j2}^A Z_{k3}^H + 2v_s\kappa\lambda^* Z_{j2}^A Z_{k3}^H + \sqrt{2}T_\lambda^* Z_{j2}^A Z_{k3}^H + \sqrt{2}T_\lambda Z_{j2}^A Z_{k3}^H \\
&- 2v_u\lambda\kappa^* Z_{j3}^A Z_{k3}^H - 2v_u\kappa\lambda^* Z_{j3}^A Z_{k3}^H \\
&+ Z_{j1}^A\Big(4v_s|\lambda|^2 Z_{k3}^H + \Big(g_1^2 + g_2^2\Big)v_d Z_{k1}^H - v_u\Big(- 4|\lambda|^2 + g_1^2 + g_2^2\Big)Z_{k2}^H\Big)\Big) \\
&+ Z_{i2}^A\Big(2v_s\kappa\lambda^* Z_{j3}^A Z_{k1}^H - \sqrt{2}T_\lambda^* Z_{j3}^A Z_{k1}^H - \sqrt{2}T_\lambda Z_{j3}^A Z_{k1}^H - 2v_s\kappa\lambda^* Z_{j1}^A Z_{k3}^H \\
&- \sqrt{2}T_\lambda^* Z_{j1}^A Z_{k3}^H - \sqrt{2}T_\lambda Z_{j1}^A Z_{k3}^H + 2v_d\kappa\lambda^* Z_{j3}^A Z_{k3}^H \\
&+ Z_{j2}^A\Big(- 4v_s|\lambda|^2 Z_{k3}^H - \Big(g_1^2 + g_2^2\Big)v_u Z_{k2}^H + v_d\Big(- 4|\lambda|^2 + g_1^2 + g_2^2\Big)Z_{k1}^H\Big) \\
&+ 2\lambda\kappa^*\Big(- v_s Z_{j1}^A Z_{k3}^H + Z_{j3}^A\Big(v_d Z_{k3}^H + v_s Z_{k1}^H\Big)\Big)\Big) \\
&+ Z_{i3}^A\Big(- \sqrt{2}\Big(- 2\Big(T_\kappa^* + T_\kappa\Big)Z_{j3}^A Z_{k3}^H + T_\lambda^*\Big(Z_{j1}^A Z_{k2}^H + Z_{j2}^A Z_{k1}^H\Big) + T_\lambda\Big(Z_{j1}^A Z_{k2}^H + Z_{j2}^A Z_{k1}^H\Big)\Big) \\
&+ 2\lambda^*\Big(- Z_{j3}^A\Big(\Big(2v_d\lambda + v_u\kappa\Big)Z_{k1}^H + \Big(2v_u\lambda + v_d\kappa\Big)Z_{k2}^H\Big) + \kappa Z_{j2}^A\Big(v_d Z_{k3}^H + v_s Z_{k1}^H\Big) \\
&+ \kappa Z_{j1}^A\Big(v_s Z_{k2}^H + v_u Z_{k3}^H\Big)\Big) \\
&+ 2\kappa^*\Big(\lambda Z_{j2}^A\Big(v_d Z_{k3}^H + v_s Z_{k1}^H\Big) + \lambda Z_{j1}^A\Big(v_s Z_{k2}^H + v_u Z_{k3}^H\Big) \\
&- Z_{j3}^A\Big(4v_s\kappa Z_{k3}^H + v_d\lambda Z_{k2}^H + v_u\lambda Z_{k1}^H\Big)\Big)\Big)\Big)
\end{aligned}
\tag{148}
$$



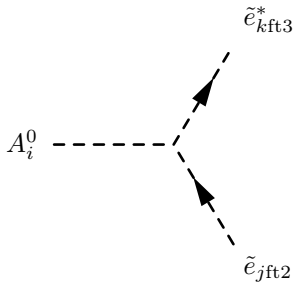
$$
\begin{aligned}
&\frac{1}{4}\Big(2\lambda\kappa^*\Big(-Z^A_{i2}\Big(v_sZ^H_{j1}Z^H_{k3}+Z^H_{j3}\Big(v_dZ^H_{k3}+v_sZ^H_{k1}\Big)\Big)\\
&+Z^A_{i3}\Big(Z^H_{j1}\Big(v_sZ^H_{k2}+v_uZ^H_{k3}\Big)+Z^H_{j2}\Big(v_dZ^H_{k3}+v_sZ^H_{k1}\Big)+Z^H_{j3}\Big(v_dZ^H_{k2}+v_uZ^H_{k1}\Big)\Big)\\
&-Z^A_{i1}\Big(v_sZ^H_{j2}Z^H_{k3}+Z^H_{j3}\Big(v_sZ^H_{k2}+v_uZ^H_{k3}\Big)\Big)\Big)\\
&-2\kappa\lambda^*\Big(-Z^A_{i2}\Big(v_sZ^H_{j1}Z^H_{k3}+Z^H_{j3}\Big(v_dZ^H_{k3}+v_sZ^H_{k1}\Big)\Big)\\
&+Z^A_{i3}\Big(Z^H_{j1}\Big(v_sZ^H_{k2}+v_uZ^H_{k3}\Big)+Z^H_{j2}\Big(v_dZ^H_{k3}+v_sZ^H_{k1}\Big)+Z^H_{j3}\Big(v_dZ^H_{k2}+v_uZ^H_{k1}\Big)\Big)\\
&-Z^A_{i1}\Big(v_sZ^H_{j2}Z^H_{k3}+Z^H_{j3}\Big(v_sZ^H_{k2}+v_uZ^H_{k3}\Big)\Big)\Big)\\
&-\sqrt{2}\Big(2\Big(-T_\kappa+T^*_\kappa\Big)Z^A_{i3}Z^H_{j3}Z^H_{k3}\\
&-T^*_\lambda\Big(Z^A_{i1}\Big(Z^H_{j2}Z^H_{k3}+Z^H_{j3}Z^H_{k2}\Big)+Z^A_{i2}\Big(Z^H_{j1}Z^H_{k3}+Z^H_{j3}Z^H_{k1}\Big)+Z^A_{i3}\Big(Z^H_{j1}Z^H_{k2}+Z^H_{j2}Z^H_{k1}\Big)\Big)\\
&+T_\lambda\Big(Z^A_{i1}\Big(Z^H_{j2}Z^H_{k3}+Z^H_{j3}Z^H_{k2}\Big)+Z^A_{i2}\Big(Z^H_{j1}Z^H_{k3}+Z^H_{j3}Z^H_{k1}\Big)+Z^A_{i3}\Big(Z^H_{j1}Z^H_{k2}+Z^H_{j2}Z^H_{k1}\Big)\Big)\Big)\Big)
\end{aligned}
\tag{149}
$$



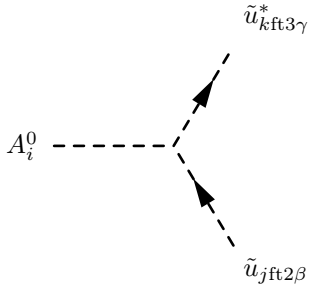
$$
\begin{aligned}
&\frac{1}{4}\Big(v_u\Big(-2|\lambda|^2+g_2^2\Big)Z^A_{i1}\Big(-Z^+_{j1}Z^+_{k2}+Z^+_{j2}Z^+_{k1}\Big)\\
&+v_d\Big(-2|\lambda|^2+g_2^2\Big)Z^A_{i2}\Big(-Z^+_{j1}Z^+_{k2}+Z^+_{j2}Z^+_{k1}\Big)\\
&+2Z^A_{i3}\Big(2v_s\kappa\lambda^*Z^+_{j2}Z^+_{k1}+\Big(-2v_s\lambda\kappa^*+\sqrt{2}T_\lambda\Big)Z^+_{j1}Z^+_{k2}-\sqrt{2}T^*_\lambda Z^+_{j2}Z^+_{k1}\Big)\Big)
\end{aligned}
\tag{150}
$$



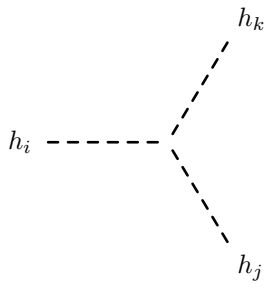
$$-\frac{1}{2}\delta_{\beta\gamma}\delta_{jk}\Big(\sqrt{2}T^{*}_{d,jk}\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)Z^{A}_{i1}\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)+\lambda Y^{*}_{d,jk}\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(v_{s}Z^{A}_{i2}+v_{u}Z^{A}_{i3}\Big)\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)$$
$$-\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big)\Big(\lambda^{*}Y_{d,kj}\Big(v_{s}Z^{A}_{i2}+v_{u}Z^{A}_{i3}\Big)+\sqrt{2}Z^{A}_{i1}T_{d,kj}\Big)\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big) \tag{151}$$



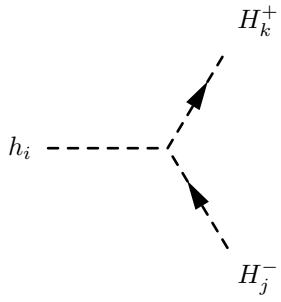
$$-\frac{1}{2}\delta_{jk}\Big(\sqrt{2}T^{*}_{e,jk}\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)Z^{A}_{i1}\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)+\lambda Y^{*}_{e,jk}\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(v_{s}Z^{A}_{i2}+v_{u}Z^{A}_{i3}\Big)\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)$$
$$-\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big)\Big(\lambda^{*}Y_{e,kj}\Big(v_{s}Z^{A}_{i2}+v_{u}Z^{A}_{i3}\Big)+\sqrt{2}Z^{A}_{i1}T_{e,kj}\Big)\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big) \tag{152}$$



$$-\frac{1}{2}\delta_{\beta\gamma}\delta_{jk}\Big(\sqrt{2}T^{*}_{u,jk}\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)Z^{A}_{i2}\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)+\lambda Y^{*}_{u,jk}\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(v_{d}Z^{A}_{i3}+v_{s}Z^{A}_{i1}\Big)\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)$$
$$-\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big)\Big(\lambda^{*}Y_{u,kj}\Big(v_{d}Z^{A}_{i3}+v_{s}Z^{A}_{i1}\Big)+\sqrt{2}Z^{A}_{i2}T_{u,kj}\Big)\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big) \tag{153}$$

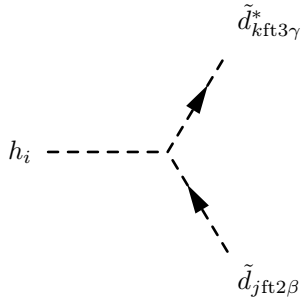


$$
\begin{aligned}
&\frac{i}{4}\Big(Z_{i1}^H\Big(Z_{j1}^H\Big(-3\Big(g_1^2+g_2^2\Big)v_dZ_{k1}^H-4v_s|\lambda|^2Z_{k3}^H+v_u\Big(-4|\lambda|^2+g_1^2+g_2^2\Big)Z_{k2}^H\Big)\\
&+Z_{j2}^H\Big(v_u\Big(-4|\lambda|^2+g_1^2+g_2^2\Big)Z_{k1}^H+v_d\Big(-4|\lambda|^2+g_1^2+g_2^2\Big)Z_{k2}^H\\
&+\Big(2v_s\kappa\lambda^*+2v_s\lambda\kappa^*+\sqrt{2}\Big(T_\lambda^*+T_\lambda\Big)\Big)Z_{k3}^H\Big)\\
&+Z_{j3}^H\Big(\sqrt{2}\Big(T_\lambda^*+T_\lambda\Big)Z_{k2}^H+2\lambda\kappa^*\Big(v_sZ_{k2}^H+v_uZ_{k3}^H\Big)\\
&+2\lambda^*\Big(\Big(-2v_d\lambda+v_u\kappa\Big)Z_{k3}^H-2v_s\lambda Z_{k1}^H+v_s\kappa Z_{k2}^H\Big)\Big)\Big)\\
&+Z_{i2}^H\Big(Z_{j2}^H\Big(-3\Big(g_1^2+g_2^2\Big)v_uZ_{k2}^H-4v_s|\lambda|^2Z_{k3}^H+v_d\Big(-4|\lambda|^2+g_1^2+g_2^2\Big)Z_{k1}^H\Big)\\
&+Z_{j1}^H\Big(v_u\Big(-4|\lambda|^2+g_1^2+g_2^2\Big)Z_{k1}^H+v_d\Big(-4|\lambda|^2+g_1^2+g_2^2\Big)Z_{k2}^H\\
&+\Big(2v_s\kappa\lambda^*+2v_s\lambda\kappa^*+\sqrt{2}\Big(T_\lambda^*+T_\lambda\Big)\Big)Z_{k3}^H\Big)\\
&+Z_{j3}^H\Big(\sqrt{2}\Big(T_\lambda^*+T_\lambda\Big)Z_{k1}^H+2\lambda\kappa^*\Big(v_dZ_{k3}^H+v_sZ_{k1}^H\Big)\\
&+2\lambda^*\Big(-2v_s\lambda Z_{k2}^H+\Big(-2v_u\lambda+v_d\kappa\Big)Z_{k3}^H+v_s\kappa Z_{k1}^H\Big)\Big)\Big)\\
&+Z_{i3}^H\Big(\sqrt{2}\Big(-2\Big(T_\kappa^*+T_\kappa\Big)Z_{j3}^HZ_{k3}^H+T_\lambda^*\Big(Z_{j1}^HZ_{k2}^H+Z_{j2}^HZ_{k1}^H\Big)+T_\lambda\Big(Z_{j1}^HZ_{k2}^H+Z_{j2}^HZ_{k1}^H\Big)\Big)\\
&+2\kappa^*\Big(\lambda Z_{j2}^H\Big(v_dZ_{k3}^H+v_sZ_{k1}^H\Big)+\lambda Z_{j1}^H\Big(v_sZ_{k2}^H+v_uZ_{k3}^H\Big)\\
&+Z_{j3}^H\Big(-12v_s\kappa Z_{k3}^H+v_d\lambda Z_{k2}^H+v_u\lambda Z_{k1}^H\Big)\Big)\\
&+2\lambda^*\Big(Z_{j3}^H\Big(\Big(-2v_d\lambda+v_u\kappa\Big)Z_{k1}^H+\Big(-2v_u\lambda+v_d\kappa\Big)Z_{k2}^H\Big)\\
&+Z_{j1}^H\Big(\Big(-2v_d\lambda+v_u\kappa\Big)Z_{k3}^H-2v_s\lambda Z_{k1}^H+v_s\kappa Z_{k2}^H\Big)\\
&+Z_{j2}^H\Big(-2v_s\lambda Z_{k2}^H+\Big(-2v_u\lambda+v_d\kappa\Big)Z_{k3}^H+v_s\kappa Z_{k1}^H\Big)\Big)\Big)\Big)
\end{aligned}
\tag{154}
$$

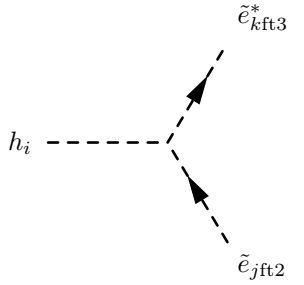


$$
\begin{aligned}
&\frac{i}{4}\Big(Z_{i2}^H\Big(-Z_{j2}^+\Big(\Big(g_1^2+g_2^2\Big)v_uZ_{k2}^++v_d\Big(-2|\lambda|^2+g_2^2\Big)Z_{k1}^+\Big)\\
&+Z_{j1}^+\Big(\Big(-g_2^2+g_1^2\Big)v_uZ_{k1}^+-v_d\Big(-2|\lambda|^2+g_2^2\Big)Z_{k2}^+\Big)\Big)
\end{aligned}
$$

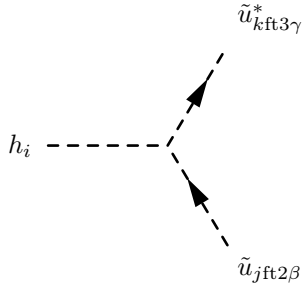
$$
\begin{aligned}
&- Z_{i1}^{H}\Big(Z_{j2}^{+}\Big(\Big(-g_1^2+g_2^2\Big)v_d Z_{k2}^{+}+v_u\Big(-2|\lambda|^2+g_2^2\Big)Z_{k1}^{+}\Big)\\
&+ Z_{j1}^{+}\Big(\Big(g_1^2+g_2^2\Big)v_d Z_{k1}^{+}+v_u\Big(-2|\lambda|^2+g_2^2\Big)Z_{k2}^{+}\Big)\Big)\\
&- 2Z_{i3}^{H}\Big(\Big(2v_s\lambda\kappa^{*}+\sqrt{2}T_{\lambda}\Big)Z_{j1}^{+}Z_{k2}^{+}+2v_s\lambda^{*}\Big(\lambda Z_{j1}^{+}Z_{k1}^{+}+Z_{j2}^{+}\Big(\kappa Z_{k1}^{+}+\lambda Z_{k2}^{+}\Big)\Big)+\sqrt{2}T_{\lambda}^{*}Z_{j2}^{+}Z_{k1}^{+}\Big)\Big)
\end{aligned}
\tag{155}
$$



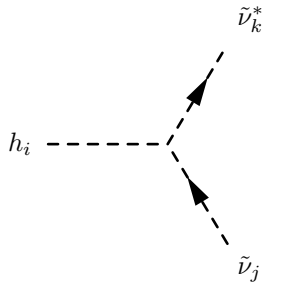
$$
\begin{aligned}
&\frac{i}{12}\delta_{\beta\gamma}\delta_{jk}\Big(2\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(-3\sqrt{2}T_{d,jk}^{*}Z_{i1}^{H}\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)+3\lambda Y_{d,jk}^{*}\Big(v_s Z_{i2}^{H}+v_u Z_{i3}^{H}\Big)\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\\
&+\Big(-g_1^2 v_u Z_{i2}^{H}+v_d\Big(-6Y_{d,jj}^{*}Y_{d,kj}+g_1^2\Big)Z_{i1}^{H}\Big)\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big)\\
&+\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big)\Big(6v_u\lambda^{*}Y_{d,kj}Z_{i3}^{H}\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)-Z_{i2}^{H}\Big(\Big(3g_2^2+g_1^2\Big)v_u\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)-6v_s\lambda^{*}Y_{d,kj}\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big)\\
&+Z_{i1}^{H}\Big(-6\sqrt{2}T_{d,kj}\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)+v_d\Big(-12Y_{d,jk}^{*}Y_{d,jj}+3g_2^2+g_1^2\Big)\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)\Big)\Big)
\end{aligned}
\tag{156}
$$



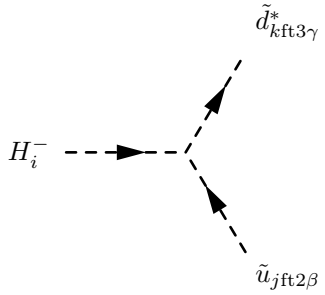
$$
\begin{aligned}
&\frac{i}{4}\delta_{jk}\Big(2\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(-\sqrt{2}T_{e,jk}^{*}Z_{i1}^{H}\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)+\lambda Y_{e,jk}^{*}\Big(v_s Z_{i2}^{H}+v_u Z_{i3}^{H}\Big)\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\\
&+\Big(-g_1^2 v_u Z_{i2}^{H}+v_d\Big(-2Y_{e,jj}^{*}Y_{e,kj}+g_1^2\Big)Z_{i1}^{H}\Big)\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big)\\
&+\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big)\Big(2v_u\lambda^{*}Y_{e,kj}Z_{i3}^{H}\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)+Z_{i2}^{H}\Big(2v_s\lambda^{*}Y_{e,kj}\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)+\Big(-g_2^2+g_1^2\Big)v_u\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)\\
&-Z_{i1}^{H}\Big(2\sqrt{2}T_{e,kj}\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)+v_d\Big(4Y_{e,jk}^{*}Y_{e,jj}-g_2^2+g_1^2\Big)\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)\Big)\Big)
\end{aligned}
\tag{157}
$$



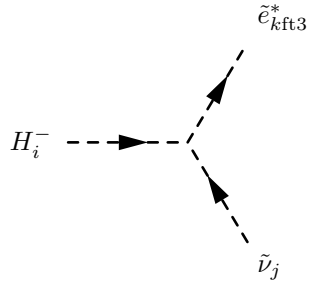
$$\frac{i}{12}\delta_{\beta\gamma}\delta_{jk}\Big(2\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(-3\sqrt{2}T^*_{u,jk}Z^H_{i2}\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)+3\lambda Y^*_{u,jk}\Big(v_dZ^H_{i3}+v_sZ^H_{i1}\Big)\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)$$
$$-2\Big(g_1^2v_dZ^H_{i1}-v_u\Big(-3Y^*_{u,jj}Y_{u,kj}+g_1^2\Big)Z^H_{i2}\Big)\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big)$$
$$+\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big)\Big(6v_d\lambda^*Y_{u,kj}Z^H_{i3}\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)+Z^H_{i1}\Big(\Big(-3g_2^2+g_1^2\Big)v_d\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)+6v_s\lambda^*Y_{u,kj}\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big)$$
$$-Z^H_{i2}\Big(6\sqrt{2}T_{u,kj}\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)+v_u\Big(12Y^*_{u,jk}Y_{u,jj}-3g_2^2+g_1^2\Big)\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)\Big)\Big) \tag{158}$$



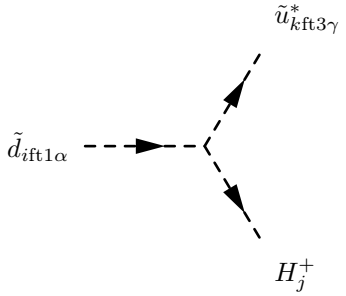
$$-\frac{i}{4}\Big(g_1^2+g_2^2\Big)\delta_{jk}\Big(v_dZ^H_{i1}-v_uZ^H_{i2}\Big) \tag{159}$$



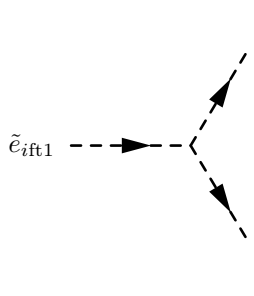
$$-\frac{i}{4}\delta_{\beta\gamma}\delta_{jk}\Big(-2\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(2T^*_{u,jk}Z^+_{i2}\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)+\sqrt{2}v_s\lambda Y^*_{u,jk}Z^+_{i1}\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)+\sqrt{2}Y^*_{u,jj}Y_{d,kj}\Big(v_dZ^+_{i2}+v_uZ_i$$
$$+\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big)\Big(\sqrt{2}Z^+_{i2}\Big(-2v_s\lambda^*Y_{d,kj}\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)+v_u\Big(-2Y^*_{u,jk}Y_{u,jj}+g_2^2\Big)\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)$$
$$+Z^+_{i1}\Big(-4T_{d,kj}\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)+\sqrt{2}v_d\Big(-2Y^*_{d,jk}Y_{d,jj}+g_2^2\Big)\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)\Big)\Big) \tag{160}$$



$$-\frac{i}{4}\delta_{jk}\Big(\sqrt{2}Z^+_{i2}\Big(-2v_s\lambda^*Y_{e,kj}\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)+g_2^2v_u\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)+Z^+_{i1}\Big(-4T_{e,kj}\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)+\sqrt{2}v_d\Big(-2Y^*_{e,jk}Y_{e,jj}+g$$
(161)

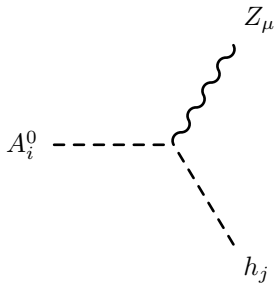


$$-\frac{i}{4}\delta_{\alpha\gamma}\delta_{ik}\Big(-2\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},2\Big)\Big)\Big(2T^*_{d,ik}Z^+_{j1}\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)+\sqrt{2}\Big(v_s\lambda Y^*_{d,ik}Z^+_{j2}\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)+Y^*_{d,ii}Y_{u,ki}\Big(v_dZ^+_{j2}+v_uZ^+_{j1}\Big)$$
$$+\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\Big(\sqrt{2}Z^+_{j1}\Big(-2v_s\lambda^*Y_{u,ki}\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)+v_d\Big(-2Y^*_{d,ik}Y_{d,ii}+g_2^2\Big)\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)$$
$$+Z^+_{j2}\Big(-4T_{u,ki}\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)+\sqrt{2}v_u\Big(-2Y^*_{u,ik}Y_{u,ii}+g_2^2\Big)\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)\Big)\Big) \tag{162}$$

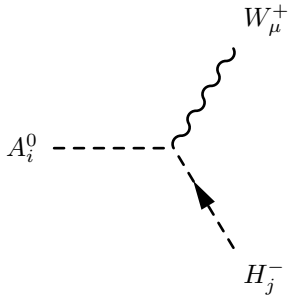


$$\frac{i}{4}\delta_{ik}\Big(2\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},2\Big)\Big)\Big(2T^*_{e,ik}Z^+_{j1}+\sqrt{2}v_s\lambda Y^*_{e,ik}Z^+_{j2}\Big)-\sqrt{2}\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\Big(g_2^2v_uZ^+_{j2}+v_d\Big(-2Y^*_{e,ik}Y_{e,ii}+g_2^2\Big)Z^+_{j1}\Big)\Big) \tag{163}$$

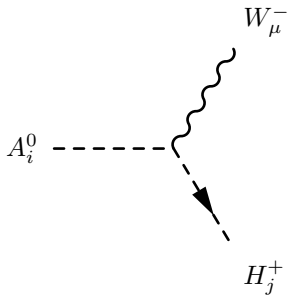
8.2 Two Scalar-One Vector Boson-Interaction



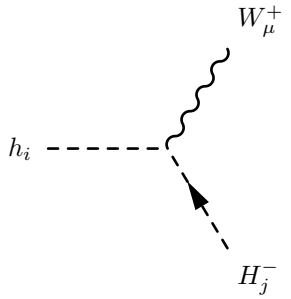
$$\frac{1}{2}\Big(g_1\sin\Theta_W+g_2\cos\Theta_W\Big)\Big(Z^A_{i1}Z^H_{j1}-Z^A_{i2}Z^H_{j2}\Big)\Big(-p_\mu^{h_j}+p_\mu^{A_i^0}\Big) \tag{164}$$



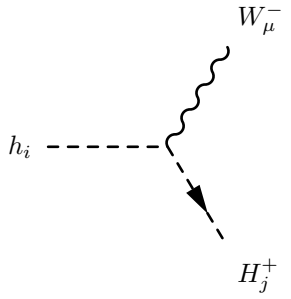
$$\frac{1}{2}g_2\Big(Z^A_{i1}Z^+_{j1}+Z^A_{i2}Z^+_{j2}\Big)\Big(-p_\mu^{H_j^-}+p_\mu^{A_i^0}\Big) \tag{165}$$



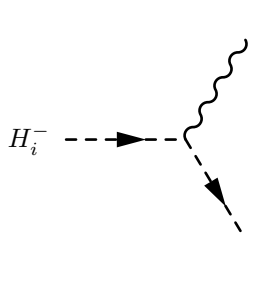
$$\frac{1}{2}g_2\Big(Z_{i1}^A Z_{j1}^+ + Z_{i2}^A Z_{j2}^+\Big)\Big(-p_\mu^{H_j^+} + p_\mu^{A_i^0}\Big) \tag{166}$$



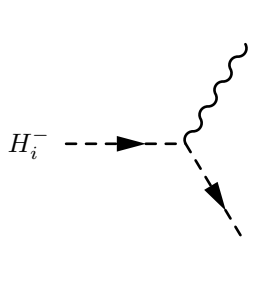
$$\frac{i}{2}g_2\Big(Z_{i1}^H Z_{j1}^+ - Z_{i2}^H Z_{j2}^+\Big)\Big(-p_\mu^{H_j^-} + p_\mu^{h_i}\Big) \tag{167}$$



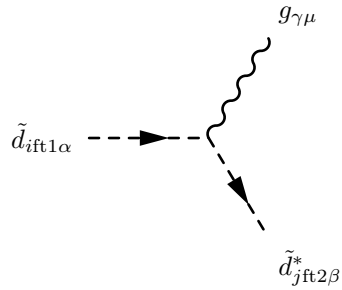
$$-\frac{i}{2}g_2\Big(Z_{i1}^H Z_{j1}^+ - Z_{i2}^H Z_{j2}^+\Big)\Big(-p_\mu^{H_j^+} + p_\mu^{h_i}\Big) \tag{168}$$



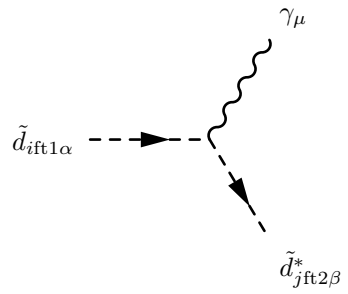
$$\frac{i}{2}\delta_{ij}\Big(g_1\cos\Theta_W + g_2\sin\Theta_W\Big)\Big(-p_\mu^{H_j^+} + p_\mu^{H_i^-}\Big) \tag{169}$$



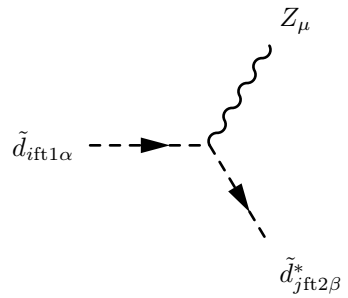
$$\frac{i}{2}\delta_{ij}\Big(-g_1\sin\Theta_W + g_2\cos\Theta_W\Big)\Big(-p_\mu^{H_j^+} + p_\mu^{H_i^-}\Big) \tag{170}$$



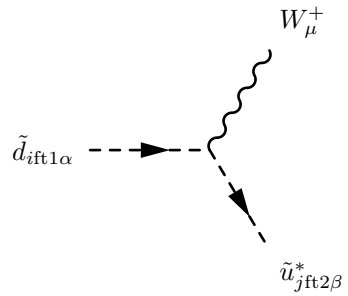
$$-\frac{i}{2}g_3\delta_{\mathrm{ft}1\mathrm{ft}2}\delta_{ij}\lambda^{\gamma}_{\beta,\alpha}\Big(-p_\mu^{\tilde{d}^*_{j\mathrm{ft}2\beta}} + p_\mu^{\tilde{d}_{i\mathrm{ft}1\alpha}}\Big) \tag{171}$$



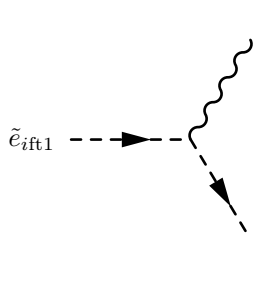
$$-\frac{i}{6}\delta_{\alpha\beta}\delta_{ij}\Big(-2g_1\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt}1\Big)\Big(\mathrm{ft}1,2\Big)\Big)\cos\Theta_W\mathrm{ZD}\Big(\mathrm{gt}2\Big)\Big(\mathrm{ft}2,2\Big)+\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt}1\Big)\Big(\mathrm{ft}1,1\Big)\Big)\Big(-3g_2\sin\Theta_W+g_1\cos\Theta_W\Big)\mathrm{ZD}\Big(\mathrm{gt}2\Big)\Big($$
(172)



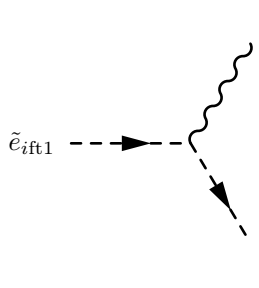
$$\frac{i}{6}\delta_{\alpha\beta}\delta_{ij}\Big(-2g_1\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt}1\Big)\Big(\mathrm{ft}1,2\Big)\Big)\sin\Theta_W\mathrm{ZD}\Big(\mathrm{gt}2\Big)\Big(\mathrm{ft}2,2\Big)+\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt}1\Big)\Big(\mathrm{ft}1,1\Big)\Big)\Big(3g_2\cos\Theta_W+g_1\sin\Theta_W\Big)\mathrm{ZD}\Big(\mathrm{gt}2\Big)\Big(\mathrm{ft}2,1$$
(173)



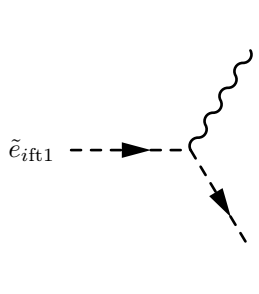
$$-i\frac{1}{\sqrt{2}}g_2\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt}1\Big)\Big(\mathrm{ft}1,1\Big)\Big)\delta_{\alpha\beta}\delta_{ij}\mathrm{ZU}\Big(\mathrm{gt}2\Big)\Big(\mathrm{ft}2,1\Big)\Big(-p_\mu^{\tilde{u}^*_{j\mathrm{ft}2\beta}}+p_\mu^{\tilde{d}_{i\mathrm{ft}1\alpha}}\Big)\tag{174}$$



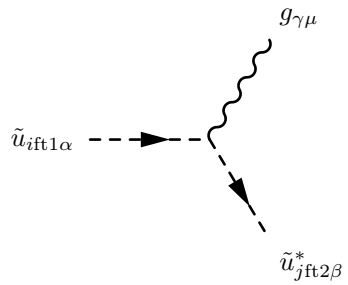
$$\frac{i}{2}\delta_{ij}\Big(2g_1\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},2\Big)\Big)\cos\Theta_W\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)+\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\Big(g_1\cos\Theta_W+g_2\sin\Theta_W\Big)\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big)\Big(-p$$
(175)



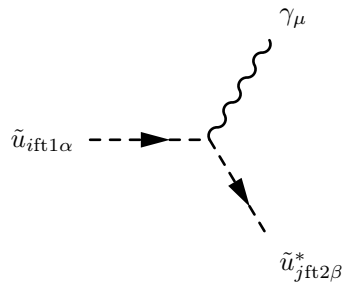
$$\frac{i}{2}\delta_{ij}\Big(-2g_1\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},2\Big)\Big)\sin\Theta_W\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)+\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\Big(-g_1\sin\Theta_W+g_2\cos\Theta_W\Big)\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big)$$
(176)



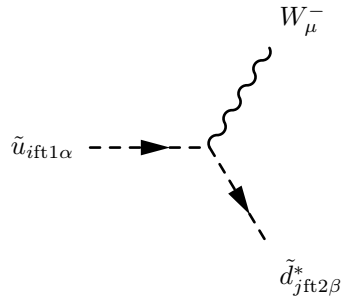
$$-i\frac{1}{\sqrt{2}}g_2\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\delta_{ij}\Big(-p_\mu^{\tilde{\nu}^*_j}+p_\mu^{\tilde{e}_{i\mathrm{ft1}}}\Big)$$
(177)



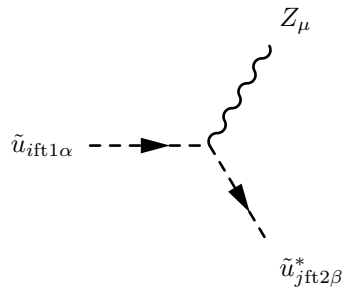
$$-\frac{i}{2}g_3\delta_{\mathrm{ft}1\mathrm{ft}2}\delta_{ij}\lambda^{\gamma}_{\beta,\alpha}\Big(-p_{\mu}^{\tilde{u}^*_{j\mathrm{ft}2\beta}}+p_{\mu}^{\tilde{u}_{i\mathrm{ft}1\alpha}}\Big) \tag{178}$$



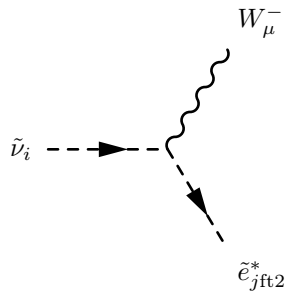
$$-\frac{i}{6}\delta_{\alpha\beta}\delta_{ij}\Big(4g_1\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt}1\Big)\Big(\mathrm{ft}1,2\Big)\Big)\cos\Theta_W\mathrm{ZU}\Big(\mathrm{gt}2\Big)\Big(\mathrm{ft}2,2\Big)+\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt}1\Big)\Big(\mathrm{ft}1,1\Big)\Big)\Big(3g_2\sin\Theta_W+g_1\cos\Theta_W\Big)\mathrm{ZU}\Big(\mathrm{gt}2\Big)\Big(\mathrm{ft}2,1 \tag{179}$$



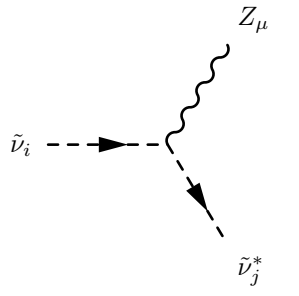
$$-i\frac{1}{\sqrt{2}}g_2\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt}1\Big)\Big(\mathrm{ft}1,1\Big)\Big)\delta_{\alpha\beta}\delta_{ij}\mathrm{ZD}\Big(\mathrm{gt}2\Big)\Big(\mathrm{ft}2,1\Big)\Big(-p_{\mu}^{\tilde{d}^*_{j\mathrm{ft}2\beta}}+p_{\mu}^{\tilde{u}_{i\mathrm{ft}1\alpha}}\Big) \tag{180}$$



$$-\frac{i}{6}\delta_{\alpha\beta}\delta_{ij}\Big(-4g_1\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt}1\Big)\Big(\mathrm{ft}1,2\Big)\Big)\sin\Theta_W\mathrm{ZU}\Big(\mathrm{gt}2\Big)\Big(\mathrm{ft}2,2\Big)+\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt}1\Big)\Big(\mathrm{ft}1,1\Big)\Big)\Big(3g_2\cos\Theta_W-g_1\sin\Theta_W\Big)\mathrm{ZU}\Big(\mathrm{gt}2\Big)\Big(\mathrm{ft}2 \tag{181}$$

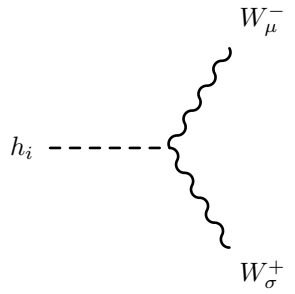


$$-i\frac{1}{\sqrt{2}}g_2\delta_{ij}\mathrm{ZE}\Big(\mathrm{gt}2\Big)\Big(\mathrm{ft}2,1\Big)\Big(-p_\mu^{\tilde{e}^*_{j\mathrm{ft}2}}+p_\mu^{\tilde{\nu}_i}\Big) \tag{182}$$

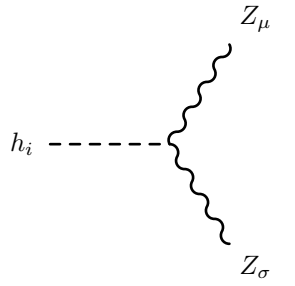


$$-\frac{i}{2}\delta_{ij}\Big(g_1\sin\Theta_W+g_2\cos\Theta_W\Big)\Big(-p_\mu^{\tilde{\nu}^*_j}+p_\mu^{\tilde{\nu}_i}\Big) \tag{183}$$

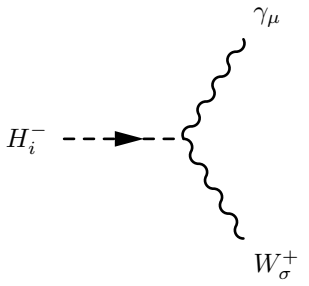
8.3 One Scalar-Two Vector Boson-Interaction



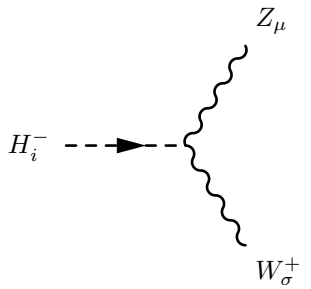
$$\frac{i}{2}g_2^2\Big(v_d Z_{i1}^H + v_u Z_{i2}^H\Big)\Big(g_{\sigma\mu}\Big) \tag{184}$$



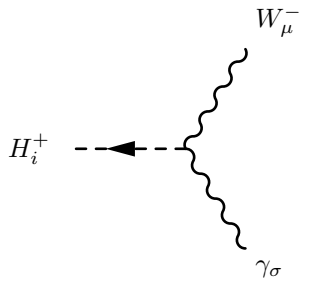
$$\frac{i}{2}\Big(g_1\sin\Theta_W + g_2\cos\Theta_W\Big)^2\Big(v_d Z_{i1}^H + v_u Z_{i2}^H\Big)\Big(g_{\sigma\mu}\Big) \tag{185}$$



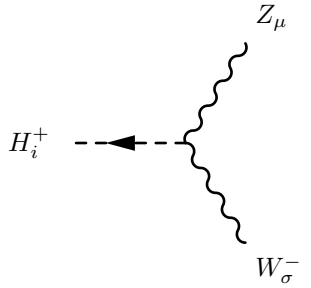
$$-\frac{i}{2}g_1 g_2\cos\Theta_W\Big(v_d Z_{i1}^+ - v_u Z_{i2}^+\Big)\Big(g_{\sigma\mu}\Big) \tag{186}$$



$$\frac{i}{2}g_1 g_2 \sin\Theta_W \Big(v_d Z_{i1}^+ - v_u Z_{i2}^+\Big)\Big(g_{\sigma\mu}\Big) \tag{187}$$

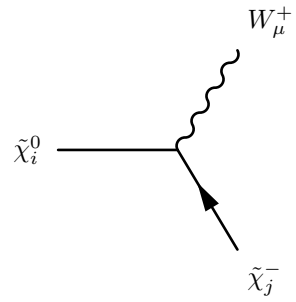


$$-\frac{i}{2}g_1 g_2 \cos\Theta_W \Big(v_d Z_{i1}^+ - v_u Z_{i2}^+\Big)\Big(g_{\sigma\mu}\Big) \tag{188}$$



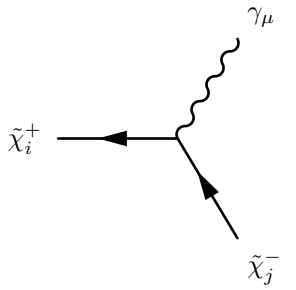
$$\frac{i}{2}g_1 g_2 \sin\Theta_W \Big(v_d Z_{i1}^+ - v_u Z_{i2}^+\Big)\Big(g_{\sigma\mu}\Big) \tag{189}$$

8.4 Two Fermion-One Vector Boson-Interaction



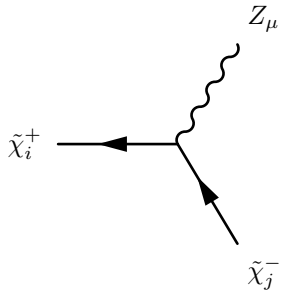
$$-\frac{i}{2}g_2\Big(2U_{j1}^*N_{i2}+\sqrt{2}U_{j2}^*N_{i3}\Big)\Big(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\Big) \tag{190}$$

$$+\ -\frac{i}{2}g_2\Big(2N_{i2}^*V_{j1}-\sqrt{2}N_{i4}^*V_{j2}\Big)\Big(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\Big) \tag{191}$$



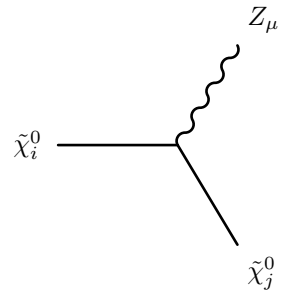
$$\frac{i}{2}\Big(2g_2U_{j1}^*\sin\Theta_W U_{i1}+U_{j2}^*\Big(g_1\cos\Theta_W+g_2\sin\Theta_W\Big)U_{i2}\Big)\Big(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\Big) \tag{192}$$

$$+\ \frac{i}{2}\Big(2g_2V_{i1}^*\sin\Theta_W V_{j1}+V_{i2}^*\Big(g_1\cos\Theta_W+g_2\sin\Theta_W\Big)V_{j2}\Big)\Big(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\Big) \tag{193}$$



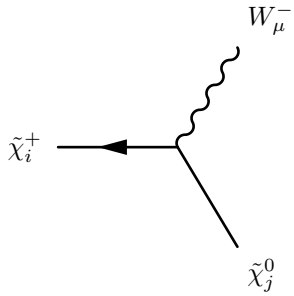
$$\frac{i}{2}\Big(2g_2U^*_{j1}\cos\Theta_W U_{i1}+U^*_{j2}\Big(-g_1\sin\Theta_W+g_2\cos\Theta_W\Big)U_{i2}\Big)\Big(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\Big) \tag{194}$$

$$+\ \frac{i}{2}\Big(2g_2V^*_{i1}\cos\Theta_W V_{j1}+V^*_{i2}\Big(-g_1\sin\Theta_W+g_2\cos\Theta_W\Big)V_{j2}\Big)\Big(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\Big) \tag{195}$$



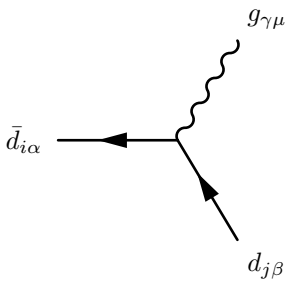
$$-\frac{i}{2}\Big(g_1\sin\Theta_W+g_2\cos\Theta_W\Big)\Big(N^*_{j3}N_{i3}-N^*_{j4}N_{i4}\Big)\Big(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\Big) \tag{196}$$

$$+\ \frac{i}{2}\Big(g_1\sin\Theta_W+g_2\cos\Theta_W\Big)\Big(N^*_{i3}N_{j3}-N^*_{i4}N_{j4}\Big)\Big(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\Big) \tag{197}$$



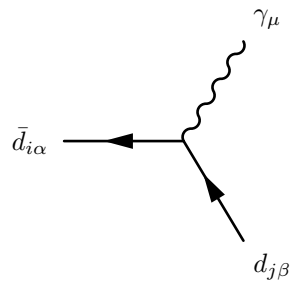
$$-\frac{i}{2}g_2\Big(2N^*_{j2}U_{i1}+\sqrt{2}N^*_{j3}U_{i2}\Big)\Big(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\Big) \tag{198}$$

$$+\ -\frac{i}{2}g_2\Big(2V^*_{i1}N_{j2}-\sqrt{2}V^*_{i2}N_{j4}\Big)\Big(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\Big) \tag{199}$$



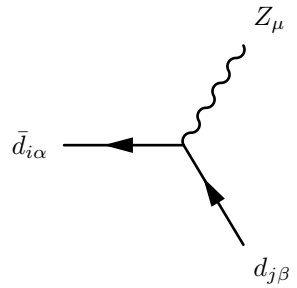
$$- \frac{i}{2} g_3 \delta_{ij} \lambda^{\gamma}_{\alpha,\beta} \Big(\gamma_\mu \cdot \frac{1-\gamma_5}{2} \Big) \tag{200}$$

$$+ \; - \frac{i}{2} g_3 \delta_{ij} \lambda^{\gamma}_{\alpha,\beta} \Big(\gamma_\mu \cdot \frac{1+\gamma_5}{2} \Big) \tag{201}$$



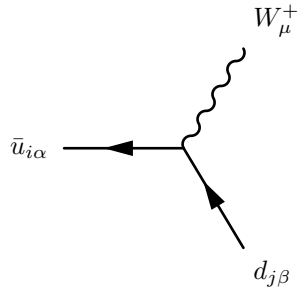
$$- \frac{i}{6} \delta_{\alpha\beta} \delta_{ij} \Big(-3 g_2 \sin\Theta_W + g_1 \cos\Theta_W \Big) \Big(\gamma_\mu \cdot \frac{1-\gamma_5}{2} \Big) \tag{202}$$

$$+ \; \frac{i}{3} g_1 \cos\Theta_W \delta_{\alpha\beta} \delta_{ij} \Big(\gamma_\mu \cdot \frac{1+\gamma_5}{2} \Big) \tag{203}$$

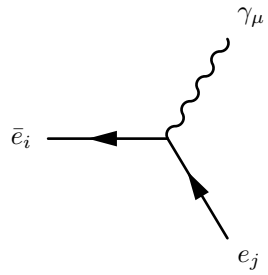


$$\frac{i}{6} \delta_{\alpha\beta} \delta_{ij} \Big(3 g_2 \cos\Theta_W + g_1 \sin\Theta_W \Big) \Big(\gamma_\mu \cdot \frac{1-\gamma_5}{2} \Big) \tag{204}$$

$$+ \; - \frac{i}{3} g_1 \delta_{\alpha\beta} \delta_{ij} \sin\Theta_W \Big(\gamma_\mu \cdot \frac{1+\gamma_5}{2} \Big) \tag{205}$$

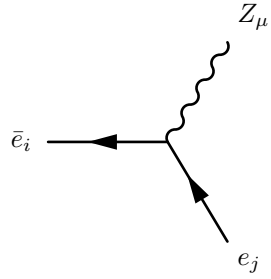


$$- i\frac{1}{\sqrt{2}}g_2\delta_{\alpha\beta}\delta_{ij}\Big(\gamma_\mu \cdot \frac{1-\gamma_5}{2}\Big) \tag{206}$$



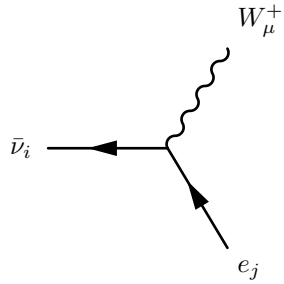
$$\frac{i}{2}\delta_{ij}\Big(g_1\cos\Theta_W + g_2\sin\Theta_W\Big)\Big(\gamma_\mu \cdot \frac{1-\gamma_5}{2}\Big) \tag{207}$$

$$+\ ig_1\cos\Theta_W\delta_{ij}\Big(\gamma_\mu \cdot \frac{1+\gamma_5}{2}\Big) \tag{208}$$

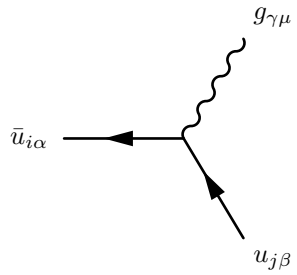


$$\frac{i}{2}\delta_{ij}\Big(-g_1\sin\Theta_W + g_2\cos\Theta_W\Big)\Big(\gamma_\mu \cdot \frac{1-\gamma_5}{2}\Big) \tag{209}$$

$$+\ -ig_1\delta_{ij}\sin\Theta_W\Big(\gamma_\mu \cdot \frac{1+\gamma_5}{2}\Big) \tag{210}$$

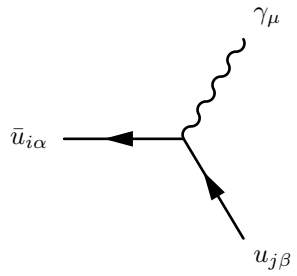


$$- i\frac{1}{\sqrt{2}}g_2\delta_{ij}\Big(\gamma_\mu \cdot \frac{1-\gamma_5}{2}\Big) \tag{211}$$



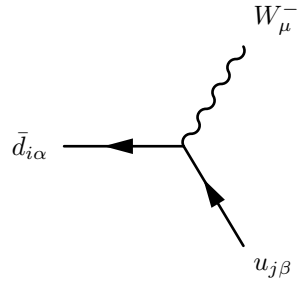
$$- \frac{i}{2}g_3\delta_{ij}\lambda^{\gamma}_{\alpha,\beta}\Big(\gamma_\mu \cdot \frac{1-\gamma_5}{2}\Big) \tag{212}$$

$$+ -\frac{i}{2}g_3\delta_{ij}\lambda^{\gamma}_{\alpha,\beta}\Big(\gamma_\mu \cdot \frac{1+\gamma_5}{2}\Big) \tag{213}$$

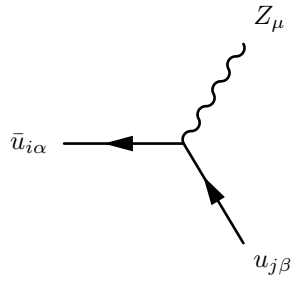


$$- \frac{i}{6}\delta_{\alpha\beta}\delta_{ij}\Big(3g_2\sin\Theta_W + g_1\cos\Theta_W\Big)\Big(\gamma_\mu \cdot \frac{1-\gamma_5}{2}\Big) \tag{214}$$

$$+ -\frac{2i}{3}g_1\cos\Theta_W\delta_{\alpha\beta}\delta_{ij}\Big(\gamma_\mu \cdot \frac{1+\gamma_5}{2}\Big) \tag{215}$$

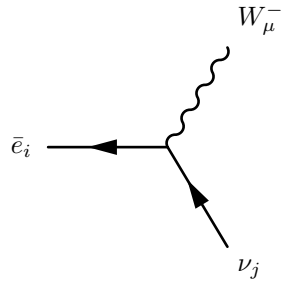


$$-i\frac{1}{\sqrt{2}}g_2\delta_{\alpha\beta}\delta_{ij}\Big(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\Big) \tag{216}$$

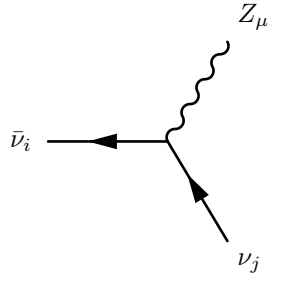


$$-\frac{i}{6}\delta_{\alpha\beta}\delta_{ij}\Big(3g_2\cos\Theta_W-g_1\sin\Theta_W\Big)\Big(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\Big) \tag{217}$$

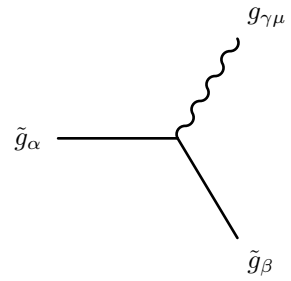
$$+\frac{2i}{3}g_1\delta_{\alpha\beta}\delta_{ij}\sin\Theta_W\Big(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\Big) \tag{218}$$



$$-i\frac{1}{\sqrt{2}}g_2\delta_{ij}\Big(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\Big) \tag{219}$$



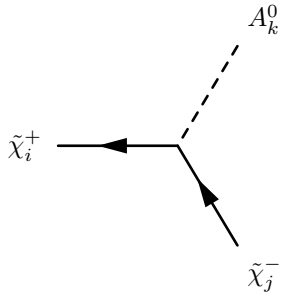
$$-\frac{i}{2}\delta_{ij}\Big(g_1\sin\Theta_W+g_2\cos\Theta_W\Big)\Big(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\Big) \tag{220}$$



$$-g_3|\phi_{\tilde{g}}|^2 f_{\alpha,\beta,\gamma}\Big(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\Big) \tag{221}$$

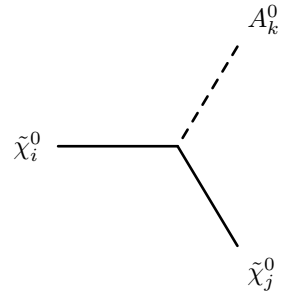
$$+\ -g_3|\phi_{\tilde{g}}|^2 f_{\alpha,\beta,\gamma}\Big(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\Big) \tag{222}$$

8.5 Two Fermion-One Scalar Boson-Interaction

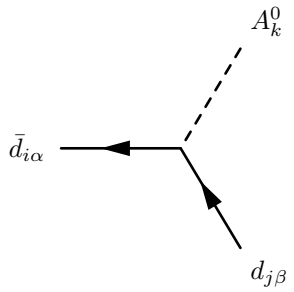


$$-\frac{1}{\sqrt{2}}\Big(g_2U_{j1}^*V_{i2}^*Z_{k2}^A+U_{j2}^*\Big(g_2V_{i1}^*Z_{k1}^A-\lambda V_{i2}^*Z_{k3}^A\Big)\Big)\Big(\frac{1-\gamma_5}{2}\Big) \tag{223}$$

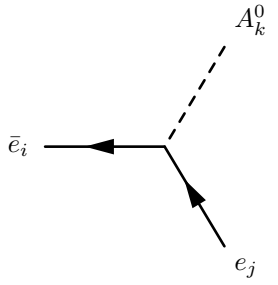
$$+\ \frac{1}{\sqrt{2}}\Big(g_2U_{i1}V_{j2}Z_{k2}^A+U_{i2}\Big(g_2V_{j1}Z_{k1}^A-\lambda^*V_{j2}Z_{k3}^A\Big)\Big)\Big(\frac{1+\gamma_5}{2}\Big) \tag{224}$$



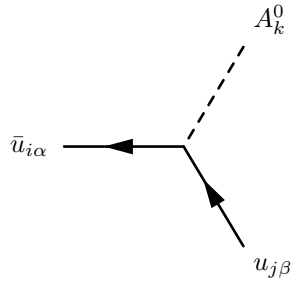
$$\frac{1}{2}\Big(- g_2 N_{i2}^* N_{j3}^* Z_{k1}^A - \sqrt{2}\lambda N_{i5}^* N_{j4}^* Z_{k1}^A - \sqrt{2}\lambda N_{i4}^* N_{j5}^* Z_{k1}^A - g_1 N_{i4}^* N_{j1}^* Z_{k2}^A$$
$$+ g_2 N_{i4}^* N_{j2}^* Z_{k2}^A - \sqrt{2}\lambda N_{i5}^* N_{j3}^* Z_{k2}^A + g_2 N_{i2}^* N_{j4}^* Z_{k2}^A$$
$$- N_{i1}^*\Big(- g_1 N_{j3}^* Z_{k1}^A + g_1 N_{j4}^* Z_{k2}^A\Big) - \sqrt{2}\lambda N_{i4}^* N_{j3}^* Z_{k3}^A + 2\sqrt{2}\kappa N_{i5}^* N_{j5}^* Z_{k3}^A$$
$$- N_{i3}^*\Big(- g_1 N_{j1}^* Z_{k1}^A + g_2 N_{j2}^* Z_{k1}^A + \sqrt{2}\lambda\Big(N_{j4}^* Z_{k3}^A + N_{j5}^* Z_{k2}^A\Big)\Big)\Big)\Big(\frac{1-\gamma_5}{2}\Big) \tag{225}$$
$$+ \frac{1}{2}\Big(- Z_{k1}^A\Big(g_1 N_{i1} N_{j3} - g_2 N_{i2} N_{j3} + N_{i3}\Big(g_1 N_{j1} - g_2 N_{j2}\Big) - \sqrt{2}\lambda^* N_{i4} N_{j5} - \sqrt{2}\lambda^* N_{i5} N_{j4}\Big)$$
$$+ \sqrt{2} Z_{k3}^A\Big(- 2\kappa^* N_{i5} N_{j5} + \lambda^*\Big(N_{i3} N_{j4} + N_{i4} N_{j3}\Big)\Big)$$
$$+ Z_{k2}^A\Big(\Big(g_1 N_{i1} - g_2 N_{i2}\Big) N_{j4} + N_{i4}\Big(g_1 N_{j1} - g_2 N_{j2}\Big) + \sqrt{2}\lambda^*\Big(N_{i3} N_{j5} + N_{i5} N_{j3}\Big)\Big)\Big)\Big(\frac{1+\gamma_5}{2}\Big) \tag{226}$$



$$\frac{1}{\sqrt{2}}\delta_{\alpha\beta}\delta_{ij} Y_{d,ij} Z_{k1}^A\Big(\frac{1-\gamma_5}{2}\Big) \tag{227}$$
$$+ \; -\frac{1}{\sqrt{2}} Y_{d,ji}^*\delta_{\alpha\beta}\delta_{ij} Z_{k1}^A\Big(\frac{1+\gamma_5}{2}\Big) \tag{228}$$

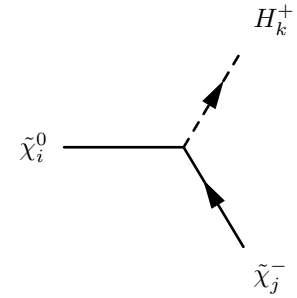


$$\frac{1}{\sqrt{2}}\delta_{ij} Y_{e,ij} Z_{k1}^A\Big(\frac{1-\gamma_5}{2}\Big) \tag{229}$$
$$+ \; -\frac{1}{\sqrt{2}} Y_{e,ji}^*\delta_{ij} Z_{k1}^A\Big(\frac{1+\gamma_5}{2}\Big) \tag{230}$$



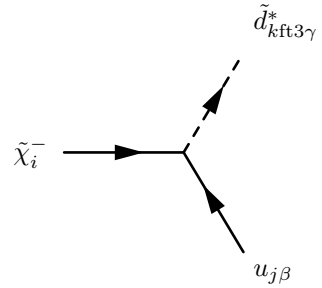
$$\frac{1}{\sqrt{2}}\delta_{\alpha\beta}\delta_{ij}Y_{u,ij}Z^A_{k2}\Big(\frac{1-\gamma_5}{2}\Big) \tag{231}$$

$$+\ -\frac{1}{\sqrt{2}}Y^*_{u,ji}\delta_{\alpha\beta}\delta_{ij}Z^A_{k2}\Big(\frac{1+\gamma_5}{2}\Big) \tag{232}$$



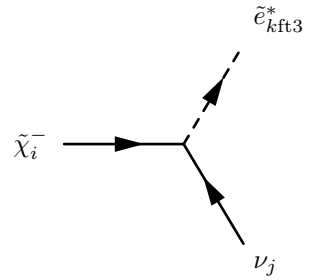
$$i\Big(-g_2U^*_{j1}N^*_{i3}Z^+_{k1}+U^*_{j2}\Big(\frac{1}{\sqrt{2}}g_1N^*_{i1}Z^+_{k1}+\frac{1}{\sqrt{2}}g_2N^*_{i2}Z^+_{k1}-\lambda N^*_{i5}Z^+_{k2}\Big)\Big)\Big(\frac{1-\gamma_5}{2}\Big) \tag{233}$$

$$+\ i\Big(-\frac{1}{2}\Big(2g_2V_{j1}N_{i4}+\sqrt{2}V_{j2}\Big(g_1N_{i1}+g_2N_{i2}\Big)\Big)Z^+_{k2}-\lambda^*V_{j2}N_{i5}Z^+_{k1}\Big)\Big(\frac{1+\gamma_5}{2}\Big) \tag{234}$$

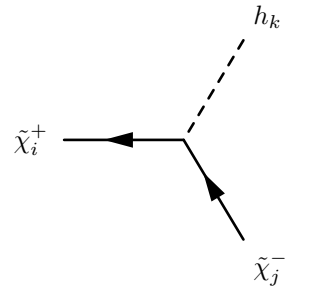


$$i\delta_{\beta\gamma}\delta_{jk}\Big(-g_2U^*_{i1}\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)+U^*_{i2}Y_{d,kj}\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big)\Big(\frac{1-\gamma_5}{2}\Big) \tag{235}$$

$$+\ iY^*_{u,jk}\delta_{\beta\gamma}\delta_{jk}V_{i2}\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big(\frac{1+\gamma_5}{2}\Big) \tag{236}$$

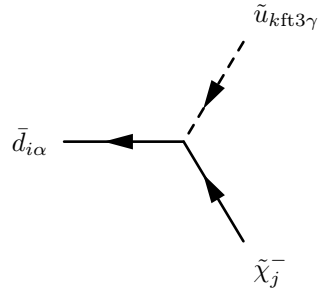


$$i\delta_{jk}\Big(-g_2U^*_{i1}\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)+U^*_{i2}Y_{e,kj}\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big)\Big(\frac{1-\gamma_5}{2}\Big) \tag{237}$$



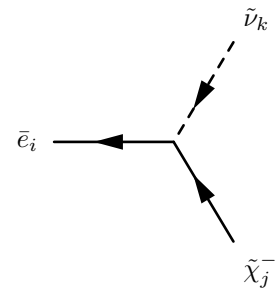
$$-\,i\frac{1}{\sqrt{2}}\Big(g_2U^*_{j1}V^*_{i2}Z^H_{k2}+U^*_{j2}\Big(g_2V^*_{i1}Z^H_{k1}+\lambda V^*_{i2}Z^H_{k3}\Big)\Big)\Big(\frac{1-\gamma_5}{2}\Big) \tag{238}$$

$$+\ -i\frac{1}{\sqrt{2}}\Big(g_2U_{i1}V_{j2}Z^H_{k2}+U_{i2}\Big(g_2V_{j1}Z^H_{k1}+\lambda^*V_{j2}Z^H_{k3}\Big)\Big)\Big(\frac{1+\gamma_5}{2}\Big) \tag{239}$$



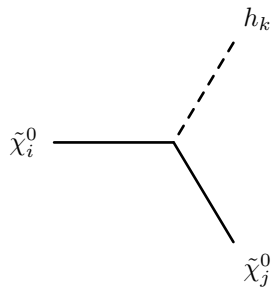
$$iU^*_{j2}\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)\delta_{\alpha\gamma}\delta_{ik}Y_{d,ik}\Big(\frac{1-\gamma_5}{2}\Big) \tag{240}$$

$$+\ i\delta_{\alpha\gamma}\delta_{ik}\Big(-g_2\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)V_{j1}+Y^*_{u,ki}\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big)V_{j2}\Big)\Big(\frac{1+\gamma_5}{2}\Big) \tag{241}$$



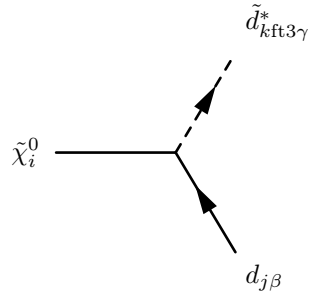
$$iU^*_{j2}\delta_{ik}Y_{e,ik}\Big(\frac{1-\gamma_5}{2}\Big) \tag{242}$$

$$+\ -ig_2\delta_{ik}V_{j1}\Big(\frac{1+\gamma_5}{2}\Big) \tag{243}$$



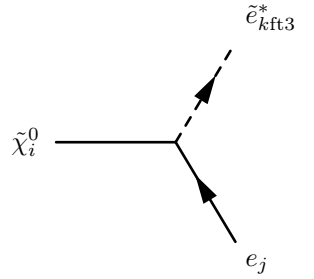
$$\begin{aligned}&\frac{i}{2}\Big(-g_2N^*_{i2}N^*_{j3}Z^H_{k1}+\sqrt{2}\lambda N^*_{i5}N^*_{j4}Z^H_{k1}+\sqrt{2}\lambda N^*_{i4}N^*_{j5}Z^H_{k1}-g_1N^*_{i4}N^*_{j1}Z^H_{k2}\\&+g_2N^*_{i4}N^*_{j2}Z^H_{k2}+\sqrt{2}\lambda N^*_{i5}N^*_{j3}Z^H_{k2}+g_2N^*_{i2}N^*_{j4}Z^H_{k2}\\&+g_1N^*_{i1}\Big(N^*_{j3}Z^H_{k1}-N^*_{j4}Z^H_{k2}\Big)+\sqrt{2}\lambda N^*_{i4}N^*_{j3}Z^H_{k3}-2\sqrt{2}\kappa N^*_{i5}N^*_{j5}Z^H_{k3}\\&+N^*_{i3}\Big(g_1N^*_{j1}Z^H_{k1}-g_2N^*_{j2}Z^H_{k1}+\sqrt{2}\lambda\Big(N^*_{j4}Z^H_{k3}+N^*_{j5}Z^H_{k2}\Big)\Big)\Big)\Big(\frac{1-\gamma_5}{2}\Big)\end{aligned} \tag{244}$$

$$\begin{aligned}&+\ \frac{i}{2}\Big(Z^H_{k1}\Big(g_1N_{i1}N_{j3}-g_2N_{i2}N_{j3}+N_{i3}\Big(g_1N_{j1}-g_2N_{j2}\Big)+\sqrt{2}\lambda^*N_{i4}N_{j5}+\sqrt{2}\lambda^*N_{i5}N_{j4}\Big)\\&+\sqrt{2}Z^H_{k3}\Big(-2\kappa^*N_{i5}N_{j5}+\lambda^*\Big(N_{i3}N_{j4}+N_{i4}N_{j3}\Big)\Big)\\&+Z^H_{k2}\Big(\Big(-g_1N_{i1}+g_2N_{i2}\Big)N_{j4}+N_{i4}\Big(-g_1N_{j1}+g_2N_{j2}\Big)+\sqrt{2}\lambda^*\Big(N_{i3}N_{j5}+N_{i5}N_{j3}\Big)\Big)\Big)\Big(\frac{1+\gamma_5}{2}\Big)\end{aligned} \tag{245}$$



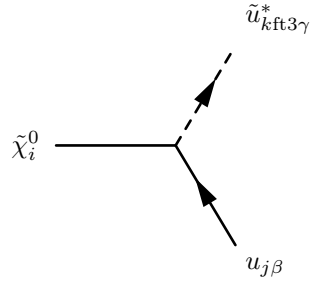
$$-\frac{i}{6}\delta_{\beta\gamma}\delta_{jk}\Big(-3\sqrt{2}g_2N^*_{i2}\mathrm{ZD}\Big(\mathrm{gt}3\Big)\Big(\mathrm{ft}3,1\Big)+6N^*_{i3}Y_{d,kj}\mathrm{ZD}\Big(\mathrm{gt}3\Big)\Big(\mathrm{ft}3,2\Big)+\sqrt{2}g_1N^*_{i1}\mathrm{ZD}\Big(\mathrm{gt}3\Big)\Big(\mathrm{ft}3,1\Big)\Big)\Big(\frac{1-\gamma_5}{2}\Big) \quad (246)$$

$$+\ -\frac{i}{3}\delta_{\beta\gamma}\delta_{jk}\Big(3Y^*_{d,jk}N_{i3}\mathrm{ZD}\Big(\mathrm{gt}3\Big)\Big(\mathrm{ft}3,1\Big)+\sqrt{2}g_1N_{i1}\mathrm{ZD}\Big(\mathrm{gt}3\Big)\Big(\mathrm{ft}3,2\Big)\Big)\Big(\frac{1+\gamma_5}{2}\Big) \quad (247)$$



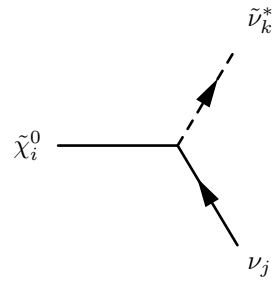
$$\frac{i}{2}\delta_{jk}\Big(-2N^*_{i3}Y_{e,kj}\mathrm{ZE}\Big(\mathrm{gt}3\Big)\Big(\mathrm{ft}3,2\Big)+\sqrt{2}g_1N^*_{i1}\mathrm{ZE}\Big(\mathrm{gt}3\Big)\Big(\mathrm{ft}3,1\Big)+\sqrt{2}g_2N^*_{i2}\mathrm{ZE}\Big(\mathrm{gt}3\Big)\Big(\mathrm{ft}3,1\Big)\Big)\Big(\frac{1-\gamma_5}{2}\Big) \quad (248)$$

$$+\ -i\delta_{jk}\Big(\sqrt{2}g_1N_{i1}\mathrm{ZE}\Big(\mathrm{gt}3\Big)\Big(\mathrm{ft}3,2\Big)+Y^*_{e,jk}N_{i3}\mathrm{ZE}\Big(\mathrm{gt}3\Big)\Big(\mathrm{ft}3,1\Big)\Big)\Big(\frac{1+\gamma_5}{2}\Big) \quad (249)$$

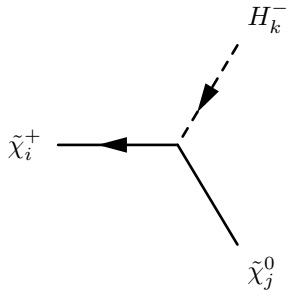


$$-\frac{i}{6}\delta_{\beta\gamma}\delta_{jk}\Big(3\sqrt{2}g_2N^*_{i2}\mathrm{ZU}\Big(\mathrm{gt}3\Big)\Big(\mathrm{ft}3,1\Big)+6N^*_{i4}Y_{u,kj}\mathrm{ZU}\Big(\mathrm{gt}3\Big)\Big(\mathrm{ft}3,2\Big)+\sqrt{2}g_1N^*_{i1}\mathrm{ZU}\Big(\mathrm{gt}3\Big)\Big(\mathrm{ft}3,1\Big)\Big)\Big(\frac{1-\gamma_5}{2}\Big) \quad (250)$$

$$+\ \frac{i}{3}\delta_{\beta\gamma}\delta_{jk}\Big(2\sqrt{2}g_1N_{i1}\mathrm{ZU}\Big(\mathrm{gt}3\Big)\Big(\mathrm{ft}3,2\Big)-3Y^*_{u,jk}N_{i4}\mathrm{ZU}\Big(\mathrm{gt}3\Big)\Big(\mathrm{ft}3,1\Big)\Big)\Big(\frac{1+\gamma_5}{2}\Big) \quad (251)$$

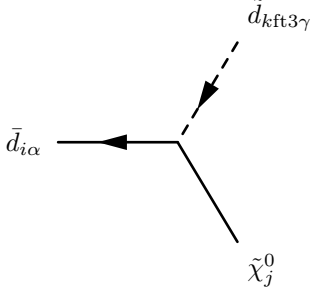


$$i\frac{1}{\sqrt{2}}\Big(g_1 N_{i1}^* - g_2 N_{i2}^*\Big)\delta_{jk}\Big(\frac{1-\gamma_5}{2}\Big) \tag{252}$$



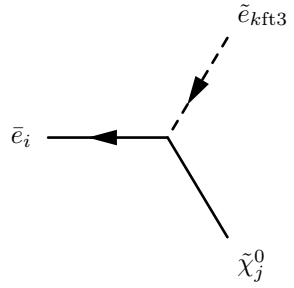
$$i\Big(-\frac{1}{2}V_{i2}^*\Big(2\lambda N_{j5}^* Z_{k1}^+ + \sqrt{2}\Big(g_1 N_{j1}^* + g_2 N_{j2}^*\Big)Z_{k2}^+\Big) - g_2 V_{i1}^* N_{j4}^* Z_{k2}^+\Big)\Big(\frac{1-\gamma_5}{2}\Big) \tag{253}$$

$$+\ i\Big(-g_2 U_{i1} N_{j3} Z_{k1}^+ + U_{i2}\Big(\frac{1}{\sqrt{2}}g_1 N_{j1} Z_{k1}^+ + \frac{1}{\sqrt{2}}g_2 N_{j2} Z_{k1}^+ - \lambda^* N_{j5} Z_{k2}^+\Big)\Big)\Big(\frac{1+\gamma_5}{2}\Big) \tag{254}$$



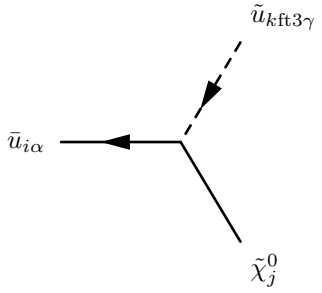
$$-\frac{i}{3}\delta_{\alpha\gamma}\delta_{ik}\Big(3N_{j3}^*\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)Y_{d,ik} + \sqrt{2}g_1 N_{j1}^*\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big)\Big)\Big(\frac{1-\gamma_5}{2}\Big) \tag{255}$$

$$+\ -\frac{i}{6}\delta_{\alpha\gamma}\delta_{ik}\Big(6Y_{d,ki}^*\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big)N_{j3} + \sqrt{2}\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)\Big(-3g_2 N_{j2} + g_1 N_{j1}\Big)\Big)\Big(\frac{1+\gamma_5}{2}\Big) \tag{256}$$



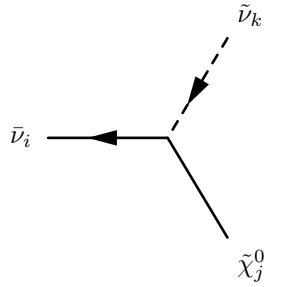
$$- i\delta_{ik}\Big(N_{j3}^{*}\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)Y_{e,ik} + \sqrt{2}g_1 N_{j1}^{*}\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big)\Big)\Big(\frac{1-\gamma_5}{2}\Big) \tag{257}$$

$$+ \frac{i}{2}\delta_{ik}\Big(-2Y_{e,ki}^{*}\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big)N_{j3} + \sqrt{2}\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)\Big(g_1 N_{j1} + g_2 N_{j2}\Big)\Big)\Big(\frac{1+\gamma_5}{2}\Big) \tag{258}$$



$$\frac{i}{3}\delta_{\alpha\gamma}\delta_{ik}\Big(2\sqrt{2}g_1 N_{j1}^{*}\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big) - 3N_{j4}^{*}\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)Y_{u,ik}\Big)\Big(\frac{1-\gamma_5}{2}\Big) \tag{259}$$

$$+ -\frac{i}{6}\delta_{\alpha\gamma}\delta_{ik}\Big(6Y_{u,ki}^{*}\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big)N_{j4} + \sqrt{2}\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)\Big(3g_2 N_{j2} + g_1 N_{j1}\Big)\Big)\Big(\frac{1+\gamma_5}{2}\Big) \tag{260}$$



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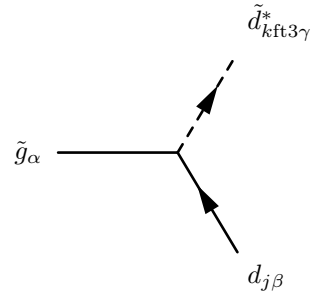
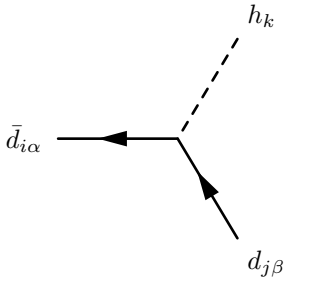
$$+ i\frac{1}{\sqrt{2}}\delta_{ik}\Big(g_1 N_{j1} - g_2 N_{j2}\Big)\Big(\frac{1+\gamma_5}{2}\Big) \tag{262}$$



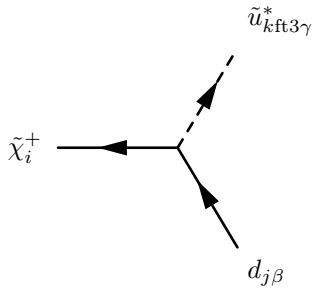
$$- i\frac{1}{\sqrt{2}}g_3\phi_{\tilde{g}}\delta_{jk}\lambda^{\alpha}_{\gamma,\beta}\mathrm{ZD}\Big(\mathrm{gt}3\Big)\Big(\mathrm{ft}3,1\Big)\Big(\frac{1-\gamma_5}{2}\Big) \tag{263}$$

$$+ \ i\frac{1}{\sqrt{2}}g_3\phi^*_{\tilde{g}}\delta_{jk}\lambda^{\alpha}_{\gamma,\beta}\mathrm{ZD}\Big(\mathrm{gt}3\Big)\Big(\mathrm{ft}3,2\Big)\Big(\frac{1+\gamma_5}{2}\Big) \tag{264}$$



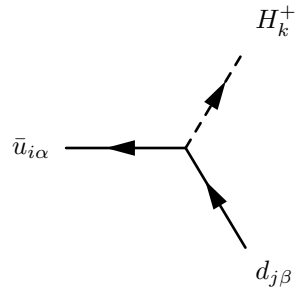
$$- i\frac{1}{\sqrt{2}}\delta_{\alpha\beta}\delta_{ij}Y_{d,ij}Z^H_{k1}\Big(\frac{1-\gamma_5}{2}\Big) \tag{265}$$

$$+ \ -i\frac{1}{\sqrt{2}}Y^*_{d,ji}\delta_{\alpha\beta}\delta_{ij}Z^H_{k1}\Big(\frac{1+\gamma_5}{2}\Big) \tag{266}$$



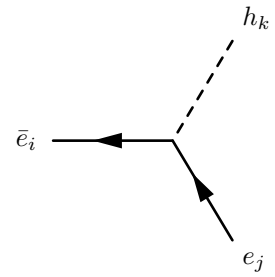
$$i\delta_{\beta\gamma}\delta_{jk}\Big(-g_2V^*_{i1}\mathrm{ZU}\Big(\mathrm{gt}3\Big)\Big(\mathrm{ft}3,1\Big) + V^*_{i2}Y_{u,kj}\mathrm{ZU}\Big(\mathrm{gt}3\Big)\Big(\mathrm{ft}3,2\Big)\Big)\Big(\frac{1-\gamma_5}{2}\Big) \tag{267}$$

$$+\ iY^*_{d,jk}\delta_{\beta\gamma}\delta_{jk}U_{i2}\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big(\frac{1+\gamma_5}{2}\Big) \tag{268}$$



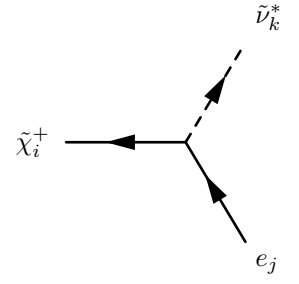
$$i\delta_{\alpha\beta}\delta_{ij}Y_{u,ij}Z^+_{k2}\Big(\frac{1-\gamma_5}{2}\Big) \tag{269}$$

$$+\ iY^*_{d,ji}\delta_{\alpha\beta}\delta_{ij}Z^+_{k1}\Big(\frac{1+\gamma_5}{2}\Big) \tag{270}$$



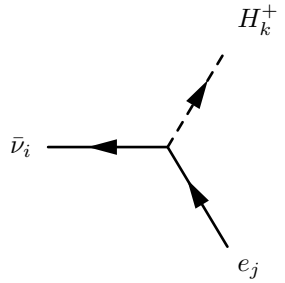
$$-\ i\frac{1}{\sqrt{2}}\delta_{ij}Y_{e,ij}Z^H_{k1}\Big(\frac{1-\gamma_5}{2}\Big) \tag{271}$$

$$+\ -i\frac{1}{\sqrt{2}}Y^*_{e,ji}\delta_{ij}Z^H_{k1}\Big(\frac{1+\gamma_5}{2}\Big) \tag{272}$$



$$- ig_2 V_{i1}^* \delta_{jk} \Big(\frac{1-\gamma_5}{2} \Big) \tag{273}$$

$$+ iY_{e,jk}^* \delta_{jk} U_{i2} \Big(\frac{1+\gamma_5}{2} \Big) \tag{274}$$



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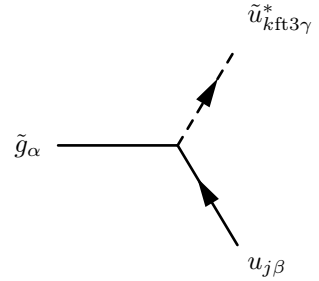
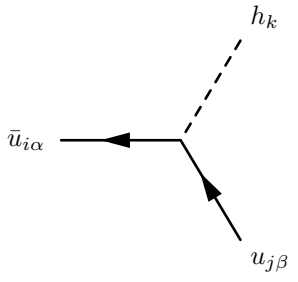
$$+ iY_{e,ji}^* \delta_{ij} Z_{k1}^+ \Big(\frac{1+\gamma_5}{2} \Big) \tag{276}$$



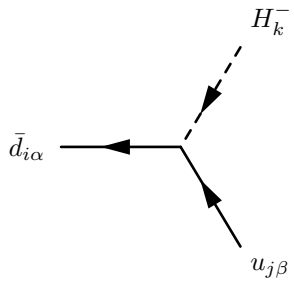
$$- i\frac{1}{\sqrt{2}} g_3 \phi_{\tilde{g}} \delta_{jk} \lambda_{\gamma,\beta}^{\alpha} \mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3}, 1\Big)\Big(\frac{1-\gamma_5}{2}\Big) \tag{277}$$

$$+ i\frac{1}{\sqrt{2}} g_3 \phi_{\tilde{g}}^* \delta_{jk} \lambda_{\gamma,\beta}^{\alpha} \mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3}, 2\Big)\Big(\frac{1+\gamma_5}{2}\Big) \tag{278}$$



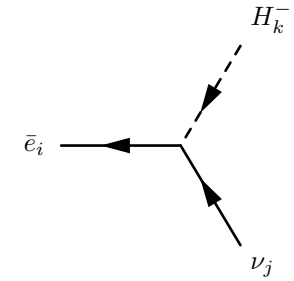
$$- i\frac{1}{\sqrt{2}}\delta_{\alpha\beta}\delta_{ij}Y_{u,ij}Z^H_{k2}\Big(\frac{1-\gamma_5}{2}\Big) \tag{279}$$

$$+ -i\frac{1}{\sqrt{2}}Y^*_{u,ji}\delta_{\alpha\beta}\delta_{ij}Z^H_{k2}\Big(\frac{1+\gamma_5}{2}\Big) \tag{280}$$

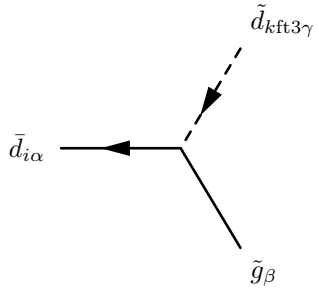


$$i\delta_{\alpha\beta}\delta_{ij}Y_{d,ij}Z^+_{k1}\Big(\frac{1-\gamma_5}{2}\Big) \tag{281}$$

$$+ iY^*_{u,ji}\delta_{\alpha\beta}\delta_{ij}Z^+_{k2}\Big(\frac{1+\gamma_5}{2}\Big) \tag{282}$$

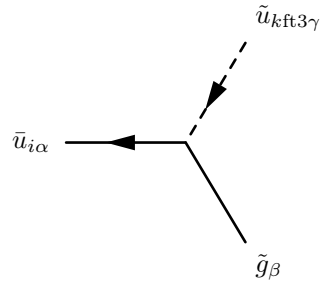


$$i\delta_{ij}Y_{e,ij}Z^+_{k1}\Big(\frac{1-\gamma_5}{2}\Big) \tag{283}$$



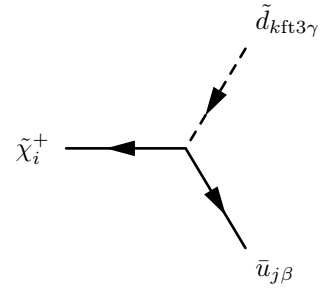
$$i\frac{1}{\sqrt{2}}g_3\phi_{\tilde{g}}\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big)\delta_{ik}\lambda^{\beta}_{\alpha,\gamma}\Big(\frac{1-\gamma_5}{2}\Big) \tag{284}$$

$$+\ -i\frac{1}{\sqrt{2}}g_3\phi^*_{\tilde{g}}\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)\delta_{ik}\lambda^{\beta}_{\alpha,\gamma}\Big(\frac{1+\gamma_5}{2}\Big) \tag{285}$$



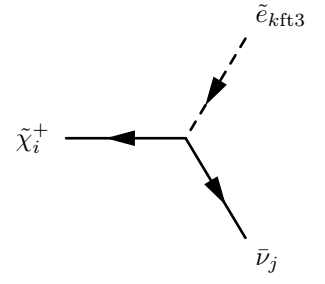
$$i\frac{1}{\sqrt{2}}g_3\phi_{\tilde{g}}\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big)\delta_{ik}\lambda^{\beta}_{\alpha,\gamma}\Big(\frac{1-\gamma_5}{2}\Big) \tag{286}$$

$$+\ -i\frac{1}{\sqrt{2}}g_3\phi^*_{\tilde{g}}\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)\delta_{ik}\lambda^{\beta}_{\alpha,\gamma}\Big(\frac{1+\gamma_5}{2}\Big) \tag{287}$$



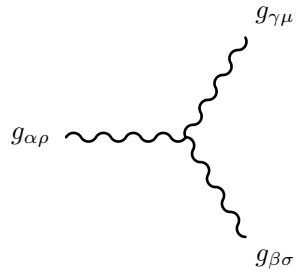
$$iV^*_{i2}\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)\delta_{\beta\gamma}\delta_{jk}Y_{u,jk}\Big(\frac{1-\gamma_5}{2}\Big) \tag{288}$$

$$+\ i\delta_{\beta\gamma}\delta_{jk}\Big(-g_2\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)U_{i1}+Y^*_{d,kj}\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big)U_{i2}\Big)\Big(\frac{1+\gamma_5}{2}\Big) \tag{289}$$

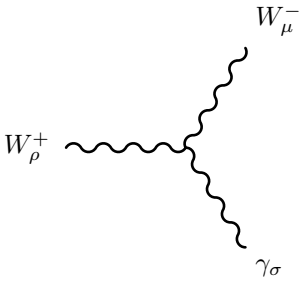


$$+ \; i\delta_{jk}\Big(- g_2 \mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3}, 1\Big)\Big) U_{i1} + Y^*_{e,kj} \mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3}, 2\Big)\Big) U_{i2}\Big)\Big(\frac{1+\gamma_5}{2}\Big) \tag{291}$$

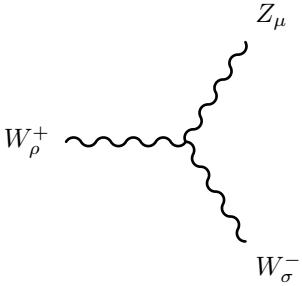
8.6 Three Vector Boson-Interaction



$$g_3 f_{\alpha,\beta,\gamma}\Big(g_{\rho\mu}\Big(- p_\sigma^{g_{\gamma\mu}} + p_\sigma^{g_{\alpha\rho}}\Big) + g_{\rho\sigma}\Big(- p_\mu^{g_{\alpha\rho}} + p_\mu^{g_{\beta\sigma}}\Big) + g_{\sigma\mu}\Big(- p_\rho^{g_{\beta\sigma}} + p_\rho^{g_{\gamma\mu}}\Big)\Big) \tag{292}$$

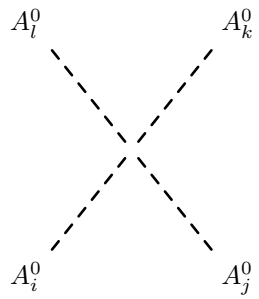


$$ig_2 \sin\Theta_W\Big(g_{\rho\mu}\Big(- p_\sigma^{W^-_\mu} + p_\sigma^{W^+_\rho}\Big) + g_{\rho\sigma}\Big(- p_\mu^{W^+_\rho} + p_\mu^{\gamma_\sigma}\Big) + g_{\sigma\mu}\Big(- p_\rho^{\gamma_\sigma} + p_\rho^{W^-_\mu}\Big)\Big) \tag{293}$$

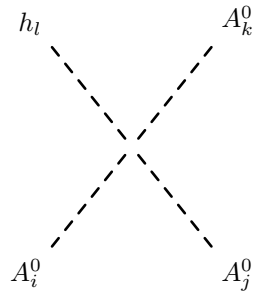


$$- ig_2 \cos\Theta_W\Big(g_{\rho\mu}\Big(- p_\sigma^{Z_\mu} + p_\sigma^{W^+_\rho}\Big) + g_{\rho\sigma}\Big(- p_\mu^{W^+_\rho} + p_\mu^{W^-_\sigma}\Big) + g_{\sigma\mu}\Big(- p_\rho^{W^-_\sigma} + p_\rho^{Z_\mu}\Big)\Big) \tag{294}$$

8.7 Four Scalar-Interaction

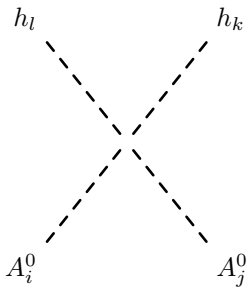


$$
\begin{aligned}
&\frac{i}{4}\Big(Z_{i1}^A\Big(Z_{j1}^A\Big(-3\Big(g_1^2+g_2^2\Big)Z_{k1}^AZ_{l1}^A+\Big(-4|\lambda|^2+g_1^2+g_2^2\Big)Z_{k2}^AZ_{l2}^A-4|\lambda|^2Z_{k3}^AZ_{l3}^A\Big)\\
&+Z_{j2}^A\Big(2\Big(\kappa\lambda^*+\lambda\kappa^*\Big)Z_{k3}^AZ_{l3}^A+\Big(-4|\lambda|^2+g_1^2+g_2^2\Big)Z_{k1}^AZ_{l2}^A+\Big(-4|\lambda|^2+g_1^2+g_2^2\Big)Z_{k2}^AZ_{l1}^A\Big)\\
&+2Z_{j3}^A\Big(\lambda^*\Big(\Big(-2\lambda Z_{k1}^A+\kappa Z_{k2}^A\Big)Z_{l3}^A+Z_{k3}^A\Big(-2\lambda Z_{l1}^A+\kappa Z_{l2}^A\Big)\Big)+\lambda\kappa^*\Big(Z_{k2}^AZ_{l3}^A+Z_{k3}^AZ_{l2}^A\Big)\Big)\Big)\\
&+Z_{i2}^A\Big(Z_{j2}^A\Big(-3\Big(g_1^2+g_2^2\Big)Z_{k2}^AZ_{l2}^A+\Big(-4|\lambda|^2+g_1^2+g_2^2\Big)Z_{k1}^AZ_{l1}^A-4|\lambda|^2Z_{k3}^AZ_{l3}^A\Big)\\
&+Z_{j1}^A\Big(2\Big(\kappa\lambda^*+\lambda\kappa^*\Big)Z_{k3}^AZ_{l3}^A+\Big(-4|\lambda|^2+g_1^2+g_2^2\Big)Z_{k1}^AZ_{l2}^A+\Big(-4|\lambda|^2+g_1^2+g_2^2\Big)Z_{k2}^AZ_{l1}^A\Big)\\
&+2Z_{j3}^A\Big(\lambda^*\Big(\Big(-2\lambda Z_{k2}^A+\kappa Z_{k1}^A\Big)Z_{l3}^A+Z_{k3}^A\Big(-2\lambda Z_{l2}^A+\kappa Z_{l1}^A\Big)\Big)+\lambda\kappa^*\Big(Z_{k1}^AZ_{l3}^A+Z_{k3}^AZ_{l1}^A\Big)\Big)\Big)\\
&+2Z_{i3}^A\Big(\lambda^*\Big(Z_{j3}^A\Big(Z_{k1}^A\Big(-2\lambda Z_{l1}^A+\kappa Z_{l2}^A\Big)+Z_{k2}^A\Big(-2\lambda Z_{l2}^A+\kappa Z_{l1}^A\Big)\Big)\\
&+Z_{j1}^A\Big(\Big(-2\lambda Z_{k1}^A+\kappa Z_{k2}^A\Big)Z_{l3}^A+Z_{k3}^A\Big(-2\lambda Z_{l1}^A+\kappa Z_{l2}^A\Big)\Big)\\
&+Z_{j2}^A\Big(\Big(-2\lambda Z_{k2}^A+\kappa Z_{k1}^A\Big)Z_{l3}^A+Z_{k3}^A\Big(-2\lambda Z_{l2}^A+\kappa Z_{l1}^A\Big)\Big)\Big)\\
&+\kappa^*\Big(Z_{j3}^A\Big(-12\kappa Z_{k3}^AZ_{l3}^A+\lambda Z_{k1}^AZ_{l2}^A+\lambda Z_{k2}^AZ_{l1}^A\Big)\\
&+\lambda\Big(Z_{j1}^A\Big(Z_{k2}^AZ_{l3}^A+Z_{k3}^AZ_{l2}^A\Big)+Z_{j2}^A\Big(Z_{k1}^AZ_{l3}^A+Z_{k3}^AZ_{l1}^A\Big)\Big)\Big)\Big)\Big)\Big)
\end{aligned}
\tag{295}
$$

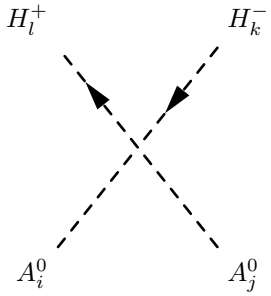


$$
-\frac{1}{2}\Big(-\kappa\lambda^*+\lambda\kappa^*\Big)\Big(Z_{i2}^A\Big(Z_{j1}^AZ_{k3}^AZ_{l3}^H+Z_{j3}^A\Big(Z_{k1}^AZ_{l3}^H-Z_{k3}^AZ_{l1}^H\Big)\Big)
$$

$$
\begin{aligned}
&- Z^A_{i3}\Big(Z^A_{j1}\Big(-Z^A_{k2}Z^H_{l3}+Z^A_{k3}Z^H_{l2}\Big)+Z^A_{j2}\Big(-Z^A_{k1}Z^H_{l3}+Z^A_{k3}Z^H_{l1}\Big)+Z^A_{j3}\Big(Z^A_{k1}Z^H_{l2}+Z^A_{k2}Z^H_{l1}\Big)\Big)\\
&+Z^A_{i1}\Big(Z^A_{j2}Z^A_{k3}Z^H_{l3}+Z^A_{j3}\Big(Z^A_{k2}Z^H_{l3}-Z^A_{k3}Z^H_{l2}\Big)\Big)\Big)
\end{aligned}
\tag{296}
$$

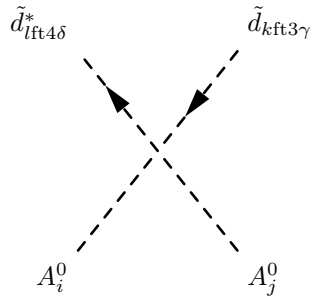


$$
\begin{aligned}
&\frac{i}{4}\Big(Z^A_{i2}\Big(Z^A_{j2}\Big(\Big(-4|\lambda|^2+g_1^2+g_2^2\Big)Z^H_{k1}Z^H_{l1}-4|\lambda|^2Z^H_{k3}Z^H_{l3}-\Big(g_1^2+g_2^2\Big)Z^H_{k2}Z^H_{l2}\Big)\\
&+2\Big(\kappa\lambda^*+\lambda\kappa^*\Big)\Big(-Z^A_{j1}Z^H_{k3}Z^H_{l3}+Z^A_{j3}\Big(Z^H_{k1}Z^H_{l3}+Z^H_{k3}Z^H_{l1}\Big)\Big)\Big)\\
&+Z^A_{i1}\Big(-Z^A_{j1}\Big(-\Big(-4|\lambda|^2+g_1^2+g_2^2\Big)Z^H_{k2}Z^H_{l2}+4|\lambda|^2Z^H_{k3}Z^H_{l3}+\Big(g_1^2+g_2^2\Big)Z^H_{k1}Z^H_{l1}\Big)\\
&+2\Big(\kappa\lambda^*+\lambda\kappa^*\Big)\Big(-Z^A_{j2}Z^H_{k3}Z^H_{l3}+Z^A_{j3}\Big(Z^H_{k2}Z^H_{l3}+Z^H_{k3}Z^H_{l2}\Big)\Big)\Big)\\
&+2Z^A_{i3}\Big(\lambda^*\Big(-Z^A_{j3}\Big(Z^H_{k1}\Big(2\lambda Z^H_{l1}+\kappa Z^H_{l2}\Big)+Z^H_{k2}\Big(2\lambda Z^H_{l2}+\kappa Z^H_{l1}\Big)\Big)\\
&+\kappa\Big(Z^A_{j1}\Big(Z^H_{k2}Z^H_{l3}+Z^H_{k3}Z^H_{l2}\Big)+Z^A_{j2}\Big(Z^H_{k1}Z^H_{l3}+Z^H_{k3}Z^H_{l1}\Big)\Big)\Big)\\
&+\kappa^*\Big(-Z^A_{j3}\Big(4\kappa Z^H_{k3}Z^H_{l3}+\lambda Z^H_{k1}Z^H_{l2}+\lambda Z^H_{k2}Z^H_{l1}\Big)\\
&+\lambda\Big(Z^A_{j1}\Big(Z^H_{k2}Z^H_{l3}+Z^H_{k3}Z^H_{l2}\Big)+Z^A_{j2}\Big(Z^H_{k1}Z^H_{l3}+Z^H_{k3}Z^H_{l1}\Big)\Big)\Big)\Big)\Big)
\end{aligned}
\tag{297}
$$

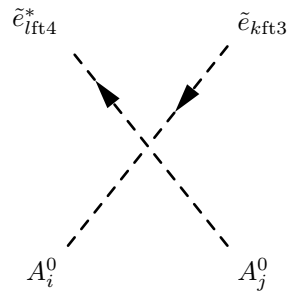


$$
\frac{i}{4}\Big(-Z^A_{i1}\Big(-\Big(-2|\lambda|^2+g_2^2\Big)Z^A_{j2}\Big(Z^+_{k1}Z^+_{l2}+Z^+_{k2}Z^+_{l1}\Big)
$$

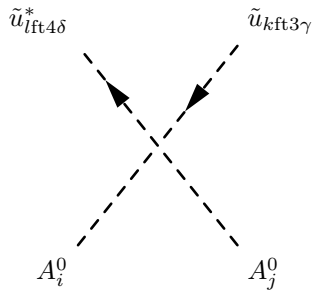
$$
\begin{aligned}
&+ Z_{j1}^{A}\Big(\Big(g_1^2 + g_2^2\Big)Z_{k1}^{+}Z_{l1}^{+} + \Big(-g_1^2 + g_2^2\Big)Z_{k2}^{+}Z_{l2}^{+}\Big)\\
&+ Z_{i2}^{A}\Big(\Big(-2|\lambda|^2 + g_2^2\Big)Z_{j1}^{A}\Big(Z_{k1}^{+}Z_{l2}^{+} + Z_{k2}^{+}Z_{l1}^{+}\Big)\\
&+ Z_{j2}^{A}\Big(-\Big(g_1^2 + g_2^2\Big)Z_{k2}^{+}Z_{l2}^{+} + \Big(-g_2^2 + g_1^2\Big)Z_{k1}^{+}Z_{l1}^{+}\Big)\Big)\\
&+ 4Z_{i3}^{A}Z_{j3}^{A}\Big(\lambda\kappa^{*}Z_{k1}^{+}Z_{l2}^{+} - \lambda^{*}\Big(\lambda Z_{k1}^{+}Z_{l1}^{+} + Z_{k2}^{+}\Big(-\kappa Z_{l1}^{+} + \lambda Z_{l2}^{+}\Big)\Big)\Big)\Big)\Big)
\end{aligned}
\tag{298}
$$



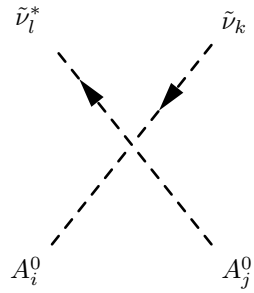
$$
\begin{aligned}
&-\frac{i}{12}\delta_{\gamma\delta}\delta_{kl}\Big(2\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big)\Big(3\lambda Y_{d,kl}^{*}\Big(Z_{i2}^{A}Z_{j3}^{A} + Z_{i3}^{A}Z_{j2}^{A}\Big)\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big) + \Big(-\Big(-6Y_{d,kk}^{*}Y_{d,lk} + g_1^2\Big)Z_{i1}^{A}Z_{j1}^{A} + g_1^2Z_{i2}^{A}Z_{j2}^{A}\Big)\mathrm{Z}\\
&+\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)\Big(-\Big(-12Y_{d,kl}^{*}Y_{d,kk} + 3g_2^2 + g_1^2\Big)Z_{i1}^{A}Z_{j1}^{A}\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big) + 6\lambda^{*}Y_{d,lk}Z_{i3}^{A}Z_{j2}^{A}\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)\\
&+ Z_{i2}^{A}\Big(\Big(3g_2^2 + g_1^2\Big)Z_{j2}^{A}\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big) + 6\lambda^{*}Y_{d,lk}Z_{j3}^{A}\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)\Big)\Big)\Big)
\end{aligned}
\tag{299}
$$



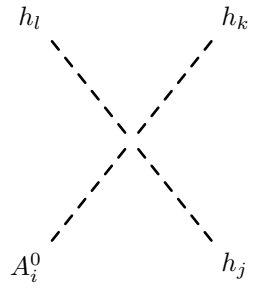
$$
\begin{aligned}
&-\frac{i}{4}\delta_{kl}\Big(2\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big)\Big(\Big(-\Big(-2Y_{e,kk}^{*}Y_{e,lk} + g_1^2\Big)Z_{i1}^{A}Z_{j1}^{A} + g_1^2Z_{i2}^{A}Z_{j2}^{A}\Big)\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big) + \lambda Y_{e,kl}^{*}\Big(Z_{i2}^{A}Z_{j3}^{A} + Z_{i3}^{A}Z_{j2}^{A}\Big)\mathrm{ZE}\Big(\mathrm{gt4}\\
&+\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)\Big(\Big(4Y_{e,kl}^{*}Y_{e,kk} - g_2^2 + g_1^2\Big)Z_{i1}^{A}Z_{j1}^{A}\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big) + 2\lambda^{*}Y_{e,lk}Z_{i3}^{A}Z_{j2}^{A}\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)\\
&+ Z_{i2}^{A}\Big(2\lambda^{*}Y_{e,lk}Z_{j3}^{A}\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big) + \Big(-g_1^2 + g_2^2\Big)Z_{j2}^{A}\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)\Big)\Big)\Big)
\end{aligned}
\tag{300}
$$



$$
\begin{aligned}
&-\frac{i}{12}\delta_{\gamma\delta}\delta_{kl}\Big(2\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big)\Big(2\Big(-\Big(-3Y^*_{u,kk}Y_{u,lk}+g_1^2\Big)Z^A_{i2}Z^A_{j2}+g_1^2Z^A_{i1}Z^A_{j1}\Big)\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)+3\lambda Y^*_{u,kl}\Big(Z^A_{i1}Z^A_{j3}+Z^A_{i3}Z^A_{j1}\Big)\Big)\\
&+\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)\Big(\Big(12Y^*_{u,kl}Y_{u,kk}-3g_2^2+g_1^2\Big)Z^A_{i2}Z^A_{j2}\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)+6\lambda^*Y_{u,lk}Z^A_{i3}Z^A_{j1}\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)\\
&+Z^A_{i1}\Big(-\Big(-3g_2^2+g_1^2\Big)Z^A_{j1}\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)+6\lambda^*Y_{u,lk}Z^A_{j3}\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)\Big)\Big)\Big)
\end{aligned}
\tag{301}
$$

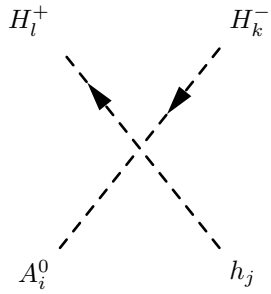


$$
-\frac{i}{4}\Big(g_1^2+g_2^2\Big)\delta_{kl}\Big(Z^A_{i1}Z^A_{j1}-Z^A_{i2}Z^A_{j2}\Big) \tag{302}
$$

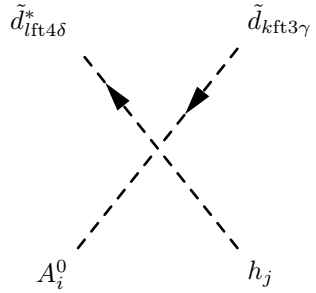


$$
-\frac{1}{2}\Big(-\kappa\lambda^*+\lambda\kappa^*\Big)\Big(Z^A_{i2}\Big(Z^H_{j1}Z^H_{k3}Z^H_{l3}+Z^H_{j3}\Big(Z^H_{k1}Z^H_{l3}+Z^H_{k3}Z^H_{l1}\Big)\Big)
$$

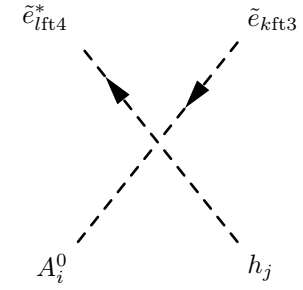
$$- Z_{i3}^{A}\Big(Z_{j1}^{H}\Big(Z_{k2}^{H}Z_{l3}^{H} + Z_{k3}^{H}Z_{l2}^{H}\Big) + Z_{j2}^{H}\Big(Z_{k1}^{H}Z_{l3}^{H} + Z_{k3}^{H}Z_{l1}^{H}\Big) + Z_{j3}^{H}\Big(Z_{k1}^{H}Z_{l2}^{H} + Z_{k2}^{H}Z_{l1}^{H}\Big)\Big)$$
$$+ Z_{i1}^{A}\Big(Z_{j2}^{H}Z_{k3}^{H}Z_{l3}^{H} + Z_{j3}^{H}\Big(Z_{k2}^{H}Z_{l3}^{H} + Z_{k3}^{H}Z_{l2}^{H}\Big)\Big)\Big) \tag{303}$$



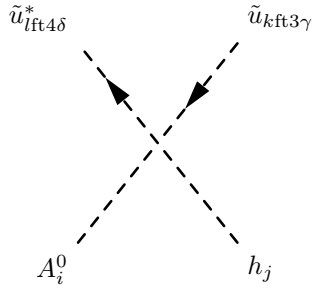
$$\frac{1}{4}\Big(\Big(-2|\lambda|^2 + g_2^2\Big)Z_{i2}^{A}Z_{j1}^{H}\Big(-Z_{k1}^{+}Z_{l2}^{+} + Z_{k2}^{+}Z_{l1}^{+}\Big)$$
$$+ \Big(-2|\lambda|^2 + g_2^2\Big)Z_{i1}^{A}Z_{j2}^{H}\Big(-Z_{k1}^{+}Z_{l2}^{+} + Z_{k2}^{+}Z_{l1}^{+}\Big)$$
$$+ 4Z_{i3}^{A}Z_{j3}^{H}\Big(\kappa\lambda^{*}Z_{k2}^{+}Z_{l1}^{+} - \lambda\kappa^{*}Z_{k1}^{+}Z_{l2}^{+}\Big)\Big) \tag{304}$$



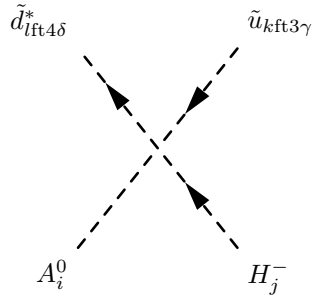
$$-\frac{1}{2}\delta_{\gamma\delta}\delta_{kl}\Big(Z_{i2}^{A}Z_{j3}^{H} + Z_{i3}^{A}Z_{j2}^{H}\Big)\Big(-\lambda^{*}\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)Y_{d,lk}\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big) + \lambda Y_{d,kl}^{*}\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big)\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)\Big) \tag{305}$$



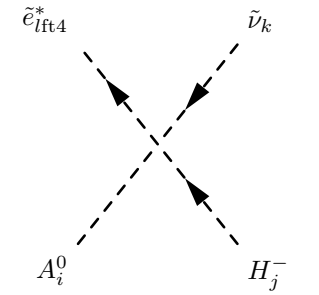
$$-\frac{1}{2}\delta_{kl}\Big(Z^A_{i2}Z^H_{j3}+Z^A_{i3}Z^H_{j2}\Big)\Big(-\lambda^*\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)Y_{e,lk}\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)+\lambda Y^*_{e,kl}\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big)\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)\Big) \tag{306}$$



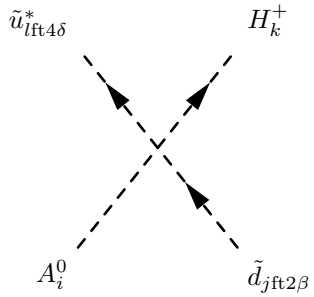
$$-\frac{1}{2}\delta_{\gamma\delta}\delta_{kl}\Big(Z^A_{i1}Z^H_{j3}+Z^A_{i3}Z^H_{j1}\Big)\Big(-\lambda^*\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)Y_{u,lk}\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)+\lambda Y^*_{u,kl}\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big)\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)\Big) \tag{307}$$



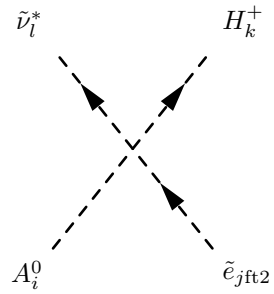
$$-\frac{1}{2}\frac{1}{\sqrt{2}}\delta_{\gamma\delta}\delta_{kl}\Big(2\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big)\Big(\lambda Y^*_{u,kl}Z^A_{i3}Z^+_{j1}\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)+Y^*_{u,kk}Y_{d,lk}\Big(Z^A_{i1}Z^+_{j2}-Z^A_{i2}Z^+_{j1}\Big)\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)\Big)$$
$$+\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)\Big(\Big(-2Y^*_{d,kl}Y_{d,kk}+g_2^2\Big)Z^A_{i1}Z^+_{j1}\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)-Z^+_{j2}\Big(2\lambda^*Y_{d,lk}Z^A_{i3}\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)+\Big(-2Y^*_{u,kl}Y_{u,kk}+g_2^2 \tag{308}$$



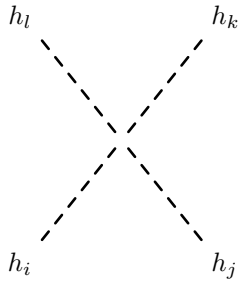
$$-\frac{1}{2}\frac{1}{\sqrt{2}}\delta_{kl}\Big(\Big(-2Y^*_{e,kl}Y_{e,kk}+g_2^2\Big)Z^A_{i1}Z^+_{j1}\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)-Z^+_{j2}\Big(2\lambda^*Y_{e,lk}Z^A_{i3}\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)+g_2^2Z^A_{i2}\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)\Big)\Big) \tag{309}$$



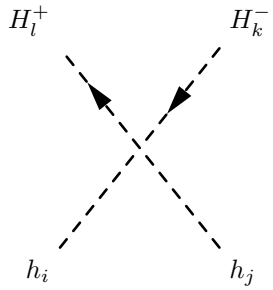
$$\frac{1}{2}\frac{1}{\sqrt{2}}\delta_{\beta\delta}\delta_{jl}\Big(\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big)\Big(2\lambda^*Y_{u,lj}Z^A_{i3}Z^+_{k1}\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)+\Big(-2Y^*_{d,jl}Y_{d,jj}+g_2^2\Big)Z^A_{i1}Z^+_{k1}\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)-\Big(-2Y^*_{u,jl}Y_u$$
$$-2\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(\lambda Y^*_{d,jl}Z^A_{i3}Z^+_{k2}\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)+Y^*_{d,jj}Y_{u,lj}\Big(-Z^A_{i1}Z^+_{k2}+Z^A_{i2}Z^+_{k1}\Big)\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)\Big)\Big) \tag{310}$$



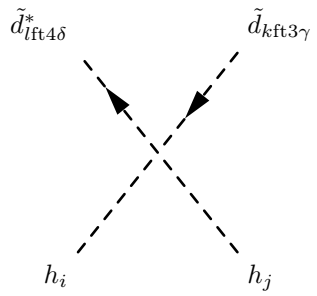
$$-\frac{1}{2}\frac{1}{\sqrt{2}}\delta_{jl}\Big(2\lambda Y^*_{e,jl}\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)Z^A_{i3}Z^+_{k2}+\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big)\Big(-\Big(-2Y^*_{e,jl}Y_{e,jj}+g_2^2\Big)Z^A_{i1}Z^+_{k1}+g_2^2Z^A_{i2}Z^+_{k2}\Big)\Big) \tag{311}$$



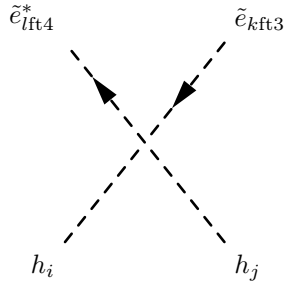
$$
\begin{aligned}
&\frac{i}{4}\Big(Z_{i1}^H\Big(Z_{j1}^H\Big(-3\Big(g_1^2+g_2^2\Big)Z_{k1}^H Z_{l1}^H+\Big(-4|\lambda|^2+g_1^2+g_2^2\Big)Z_{k2}^H Z_{l2}^H-4|\lambda|^2 Z_{k3}^H Z_{l3}^H\Big)\\
&+Z_{j2}^H\Big(2\Big(\kappa\lambda^*+\lambda\kappa^*\Big)Z_{k3}^H Z_{l3}^H+\Big(-4|\lambda|^2+g_1^2+g_2^2\Big)Z_{k1}^H Z_{l2}^H+\Big(-4|\lambda|^2+g_1^2+g_2^2\Big)Z_{k2}^H Z_{l1}^H\Big)\\
&+2Z_{j3}^H\Big(\lambda^*\Big(\Big(-2\lambda Z_{k1}^H+\kappa Z_{k2}^H\Big)Z_{l3}^H+Z_{k3}^H\Big(-2\lambda Z_{l1}^H+\kappa Z_{l2}^H\Big)\Big)+\lambda\kappa^*\Big(Z_{k2}^H Z_{l3}^H+Z_{k3}^H Z_{l2}^H\Big)\Big)\Big)\\
&+Z_{i2}^H\Big(Z_{j2}^H\Big(-3\Big(g_1^2+g_2^2\Big)Z_{k2}^H Z_{l2}^H+\Big(-4|\lambda|^2+g_1^2+g_2^2\Big)Z_{k1}^H Z_{l1}^H-4|\lambda|^2 Z_{k3}^H Z_{l3}^H\Big)\\
&+Z_{j1}^H\Big(2\Big(\kappa\lambda^*+\lambda\kappa^*\Big)Z_{k3}^H Z_{l3}^H+\Big(-4|\lambda|^2+g_1^2+g_2^2\Big)Z_{k1}^H Z_{l2}^H+\Big(-4|\lambda|^2+g_1^2+g_2^2\Big)Z_{k2}^H Z_{l1}^H\Big)\\
&+2Z_{j3}^H\Big(\lambda^*\Big(\Big(-2\lambda Z_{k2}^H+\kappa Z_{k1}^H\Big)Z_{l3}^H+Z_{k3}^H\Big(-2\lambda Z_{l2}^H+\kappa Z_{l1}^H\Big)\Big)+\lambda\kappa^*\Big(Z_{k1}^H Z_{l3}^H+Z_{k3}^H Z_{l1}^H\Big)\Big)\Big)\\
&+2Z_{i3}^H\Big(\lambda^*\Big(Z_{j3}^H\Big(Z_{k1}^H\Big(-2\lambda Z_{l1}^H+\kappa Z_{l2}^H\Big)+Z_{k2}^H\Big(-2\lambda Z_{l2}^H+\kappa Z_{l1}^H\Big)\Big)\\
&+Z_{j1}^H\Big(\Big(-2\lambda Z_{k1}^H+\kappa Z_{k2}^H\Big)Z_{l3}^H+Z_{k3}^H\Big(-2\lambda Z_{l1}^H+\kappa Z_{l2}^H\Big)\Big)\\
&+Z_{j2}^H\Big(\Big(-2\lambda Z_{k2}^H+\kappa Z_{k1}^H\Big)Z_{l3}^H+Z_{k3}^H\Big(-2\lambda Z_{l2}^H+\kappa Z_{l1}^H\Big)\Big)\Big)\\
&+\kappa^*\Big(Z_{j3}^H\Big(-12\kappa Z_{k3}^H Z_{l3}^H+\lambda Z_{k1}^H Z_{l2}^H+\lambda Z_{k2}^H Z_{l1}^H\Big)\\
&+\lambda\Big(Z_{j1}^H\Big(Z_{k2}^H Z_{l3}^H+Z_{k3}^H Z_{l2}^H\Big)+Z_{j2}^H\Big(Z_{k1}^H Z_{l3}^H+Z_{k3}^H Z_{l1}^H\Big)\Big)\Big)\Big)\Big)
\end{aligned}
\tag{312}
$$



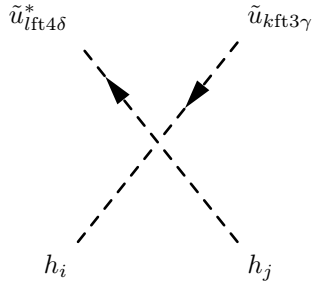
$$
\begin{aligned}
&\frac{i}{4}\Big(-Z_{i1}^H\Big(\Big(-2|\lambda|^2+g_2^2\Big)Z_{j2}^H\Big(Z_{k1}^+ Z_{l2}^+ + Z_{k2}^+ Z_{l1}^+\Big)+Z_{j1}^H\Big(\Big(g_1^2+g_2^2\Big)Z_{k1}^+ Z_{l1}^+ + \Big(-g_1^2+g_2^2\Big)Z_{k2}^+ Z_{l2}^+\Big)\Big)\\
&+Z_{i2}^H\Big(-\Big(-2|\lambda|^2+g_2^2\Big)Z_{j1}^H\Big(Z_{k1}^+ Z_{l2}^+ + Z_{k2}^+ Z_{l1}^+\Big)\\
&+Z_{j2}^H\Big(-\Big(g_1^2+g_2^2\Big)Z_{k2}^+ Z_{l2}^+ + \Big(-g_2^2+g_1^2\Big)Z_{k1}^+ Z_{l1}^+\Big)\Big)\\
&-4Z_{i3}^H Z_{j3}^H\Big(\lambda\kappa^* Z_{k1}^+ Z_{l2}^+ + \lambda^*\Big(\lambda Z_{k1}^+ Z_{l1}^+ + Z_{k2}^+\Big(\kappa Z_{l1}^+ + \lambda Z_{l2}^+\Big)\Big)\Big)\Big)
\end{aligned}
\tag{313}
$$



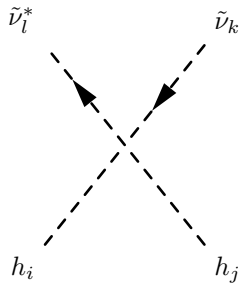
$$\frac{i}{12}\delta_{\gamma\delta}\delta_{kl}\Big(2\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big)\Big(3\lambda Y^*_{d,kl}\Big(Z^H_{i2}Z^H_{j3}+Z^H_{i3}Z^H_{j2}\Big)\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)+\Big(\Big(-6Y^*_{d,kk}Y_{d,lk}+g_1^2\Big)Z^H_{i1}Z^H_{j1}-g_1^2Z^H_{i2}Z^H_{j2}\Big)\mathrm{ZD}\Big(\mathrm{g}$$
$$+\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)\Big(\Big(-12Y^*_{d,kl}Y_{d,kk}+3g_2^2+g_1^2\Big)Z^H_{i1}Z^H_{j1}\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)+6\lambda^*Y_{d,lk}Z^H_{i3}Z^H_{j2}\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)$$
$$-Z^H_{i2}\Big(\Big(3g_2^2+g_1^2\Big)Z^H_{j2}\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)-6\lambda^*Y_{d,lk}Z^H_{j3}\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)\Big)\Big)\Big) \tag{314}$$



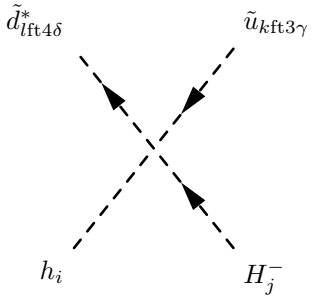
$$\frac{i}{4}\delta_{kl}\Big(2\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big)\Big(\Big(\Big(-2Y^*_{e,kk}Y_{e,lk}+g_1^2\Big)Z^H_{i1}Z^H_{j1}-g_1^2Z^H_{i2}Z^H_{j2}\Big)\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)+\lambda Y^*_{e,kl}\Big(Z^H_{i2}Z^H_{j3}+Z^H_{i3}Z^H_{j2}\Big)\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft}$$
$$+\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)\Big(\Big(-4Y^*_{e,kl}Y_{e,kk}-g_1^2+g_2^2\Big)Z^H_{i1}Z^H_{j1}\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)+2\lambda^*Y_{e,lk}Z^H_{i3}Z^H_{j2}\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)$$
$$+Z^H_{i2}\Big(2\lambda^*Y_{e,lk}Z^H_{j3}\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)+\Big(-g_2^2+g_1^2\Big)Z^H_{j2}\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)\Big)\Big)\Big) \tag{315}$$



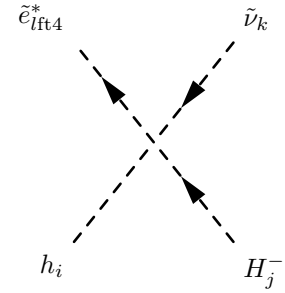
$$\frac{i}{12}\delta_{\gamma\delta}\delta_{kl}\Big(2\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big)\Big(-2\Big(-\Big(-3Y^*_{u,kk}Y_{u,lk}+g_1^2\Big)Z^H_{i2}Z^H_{j2}+g_1^2Z^H_{i1}Z^H_{j1}\Big)\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)+3\lambda Y^*_{u,kl}\Big(Z^H_{i1}Z^H_{j3}+Z^H_{i3}Z^H_{j1}$$
$$+\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)\Big(-\Big(12Y^*_{u,kl}Y_{u,kk}-3g_2^2+g_1^2\Big)Z^H_{i2}Z^H_{j2}\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)+6\lambda^*Y_{u,lk}Z^H_{i3}Z^H_{j1}\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)$$
$$+Z^H_{i1}\Big(\Big(-3g_2^2+g_1^2\Big)Z^H_{j1}\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)+6\lambda^*Y_{u,lk}Z^H_{j3}\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)\Big)\Big)\Big) \tag{316}$$



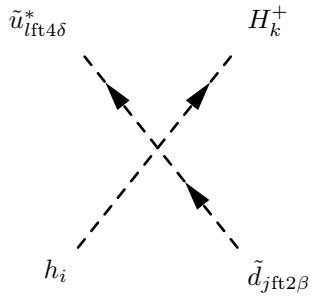
$$-\frac{i}{4}\Big(g_1^2+g_2^2\Big)\delta_{kl}\Big(Z^H_{i1}Z^H_{j1}-Z^H_{i2}Z^H_{j2}\Big) \tag{317}$$



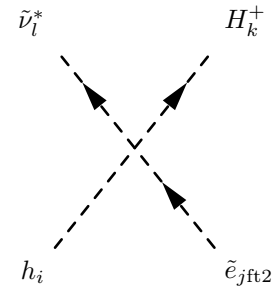
$$\frac{i}{2}\frac{1}{\sqrt{2}}\delta_{\gamma\delta}\delta_{kl}\Big(2\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big)\Big(\lambda Y^*_{u,kl}Z^H_{i3}Z^+_{j1}\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)+Y^*_{u,kk}Y_{d,lk}\Big(Z^H_{i1}Z^+_{j2}+Z^H_{i2}Z^+_{j1}\Big)\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)\Big)$$
$$+\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)\Big(-\Big(-2Y^*_{d,kl}Y_{d,kk}+g_2^2\Big)Z^H_{i1}Z^+_{j1}\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)+Z^+_{j2}\Big(2\lambda^*Y_{d,lk}Z^H_{i3}\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)-\Big(-2Y^*_{u,kl}Y_{u,kk}+$$
(318)



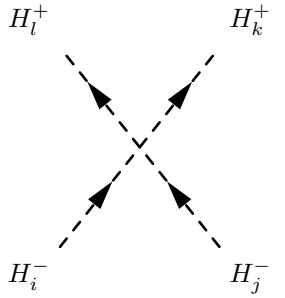
$$-\frac{i}{2}\frac{1}{\sqrt{2}}\delta_{kl}\Big(\Big(-2Y^*_{e,kl}Y_{e,kk}+g_2^2\Big)Z^H_{i1}Z^+_{j1}\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)+Z^+_{j2}\Big(-2\lambda^*Y_{e,lk}Z^H_{i3}\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)+g_2^2Z^H_{i2}\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)\Big)\Big) \tag{319}$$



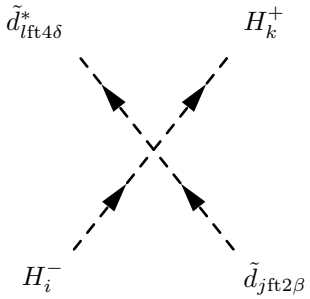
$$-\frac{i}{2}\frac{1}{\sqrt{2}}\delta_{\beta\delta}\delta_{jl}\Big(\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big)\Big(-2\lambda^*Y_{u,lj}Z^H_{i3}Z^+_{k1}\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)+\Big(-2Y^*_{d,jl}Y_{d,jj}+g_2^2\Big)Z^H_{i1}Z^+_{k1}\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)+\Big(-2$$
$$-2\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(\lambda Y^*_{d,jl}Z^H_{i3}Z^+_{k2}\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)+Y^*_{d,jj}Y_{u,lj}\Big(Z^H_{i1}Z^+_{k2}+Z^H_{i2}Z^+_{k1}\Big)\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)\Big)\Big) \tag{320}$$



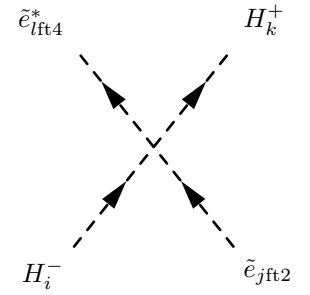
$$-\frac{i}{2}\frac{1}{\sqrt{2}}\delta_{jl}\Big(-2\lambda Y^*_{e,jl}\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)Z^H_{i3}Z^+_{k2}+\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big)\Big(\Big(-2Y^*_{e,jl}Y_{e,jj}+g_2^2\Big)Z^H_{i1}Z^+_{k1}+g_2^2Z^H_{i2}Z^+_{k2}\Big)\Big) \tag{321}$$



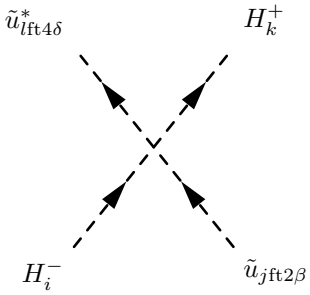
$$\frac{i}{4}\Big(Z^+_{i2}\Big(-2\Big(g_1^2+g_2^2\Big)Z^+_{j2}Z^+_{k2}Z^+_{l2}+\Big(-4|\lambda|^2+g_1^2+g_2^2\Big)Z^+_{j1}\Big(Z^+_{k1}Z^+_{l2}+Z^+_{k2}Z^+_{l1}\Big)\Big)$$
$$+Z^+_{i1}\Big(-2\Big(g_1^2+g_2^2\Big)Z^+_{j1}Z^+_{k1}Z^+_{l1}+\Big(-4|\lambda|^2+g_1^2+g_2^2\Big)Z^+_{j2}\Big(Z^+_{k1}Z^+_{l2}+Z^+_{k2}Z^+_{l1}\Big)\Big)\Big) \tag{322}$$



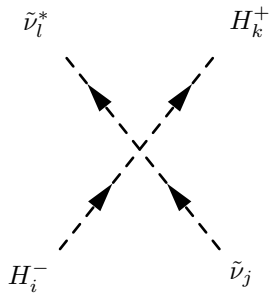
$$\frac{i}{12}\delta_{\beta\delta}\delta_{jl}\Big(\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big)\Big(-\Big(12Y^*_{u,jl}Y_{u,jj}-3g_2^2+g_1^2\Big)Z^+_{i2}Z^+_{k2}+\Big(-3g_2^2+g_1^2\Big)Z^+_{i1}Z^+_{k1}\Big)\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)$$
$$+2\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(\Big(-6Y^*_{d,jj}Y_{d,lj}+g_1^2\Big)Z^+_{i1}Z^+_{k1}-g_1^2Z^+_{i2}Z^+_{k2}\Big)\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)\Big) \tag{323}$$



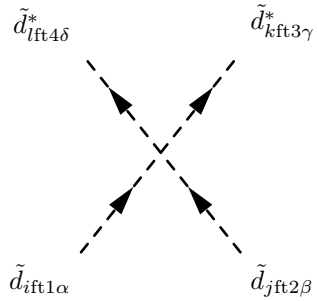
$$-\frac{i}{4}\delta_{jl}\Big(\Big(g_1^2+g_2^2\Big)\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big)\Big(Z^+_{i1}Z^+_{k1}-Z^+_{i2}Z^+_{k2}\Big)\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)$$
$$+2\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(-\Big(-2Y^*_{e,jj}Y_{e,lj}+g_1^2\Big)Z^+_{i1}Z^+_{k1}+g_1^2Z^+_{i2}Z^+_{k2}\Big)\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)\Big) \tag{324}$$



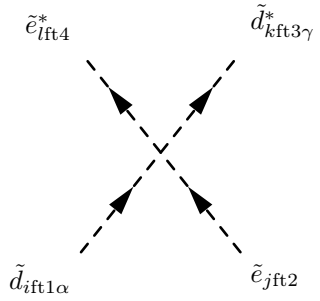
$$\frac{i}{12}\delta_{\beta\delta}\delta_{jl}\Big(\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big)\Big(\Big(-12Y^*_{d,jl}Y_{d,jj}+3g_2^2+g_1^2\Big)Z^+_{i1}Z^+_{k1}-\Big(3g_2^2+g_1^2\Big)Z^+_{i2}Z^+_{k2}\Big)\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)$$
$$-4\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(-\Big(-3Y^*_{u,jj}Y_{u,lj}+g_1^2\Big)Z^+_{i2}Z^+_{k2}+g_1^2Z^+_{i1}Z^+_{k1}\Big)\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)\Big) \tag{325}$$



$$\frac{i}{4}\delta_{jl}\Big(\Big(-4Y^*_{e,jl}Y_{e,jj}-g_1^2+g_2^2\Big)Z^+_{i1}Z^+_{k1}+\Big(-g_2^2+g_1^2\Big)Z^+_{i2}Z^+_{k2}\Big) \tag{326}$$

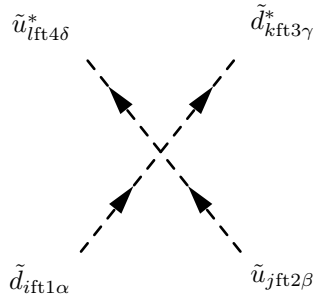


$$\frac{i}{36}\Big(-\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\Big(\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big)\Big(\delta_{\alpha\delta}\delta_{\beta\gamma}\Big(18g_3^2\delta_{ik}\delta_{jl}+\Big(-6g_3^2+9g_2^2+g_1^2\Big)\delta_{il}\delta_{jk}\Big)$$
$$+\delta_{\alpha\gamma}\delta_{\beta\delta}\Big(18g_3^2\delta_{il}\delta_{jk}+\Big(-6g_3^2+9g_2^2+g_1^2\Big)\delta_{ik}\delta_{jl}\Big)\Big)\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)$$
$$+2\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(\delta_{\alpha\gamma}\delta_{\beta\delta}\Big(-9g_3^2\delta_{il}\delta_{jk}\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)+\delta_{ik}\delta_{jl}\Big(18Y^*_{d,jl}Y_{d,ki}\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft}$$
$$+\delta_{\alpha\delta}\delta_{\beta\gamma}\Big(-9g_3^2\delta_{ik}\delta_{jl}\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)+\delta_{il}\delta_{jk}\Big(18Y^*_{d,jk}Y_{d,li}\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)+\Big(3g_3^2+g_1^2\Big)\mathrm{ZD}\Big(\mathrm{gt}$$
$$+2\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},2\Big)\Big)\Big(-\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(\delta_{\alpha\delta}\delta_{\beta\gamma}\Big(\Big(2g_1^2-3g_3^2\Big)\delta_{il}\delta_{jk}+9g_3^2\delta_{ik}\delta_{jl}\Big)$$
$$+\delta_{\alpha\gamma}\delta_{\beta\delta}\Big(\Big(2g_1^2-3g_3^2\Big)\delta_{ik}\delta_{jl}+9g_3^2\delta_{il}\delta_{jk}\Big)\Big)\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)$$
$$+\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big)\Big(\delta_{\alpha\delta}\delta_{\beta\gamma}\Big(9g_3^2\delta_{ik}\delta_{jl}\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)-\delta_{il}\delta_{jk}\Big(18Y^*_{d,il}Y_{d,kj}\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1$$
$$-\delta_{\alpha\gamma}\delta_{\beta\delta}\Big(-9g_3^2\delta_{il}\delta_{jk}\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)+\delta_{ik}\delta_{jl}\Big(18Y^*_{d,ik}Y_{d,lj}\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\mathrm{ZD}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)+\Big(3g_3^2+g_1^2\Big)\mathrm{ZD}\Big(\mathrm{gt}$$
(327)



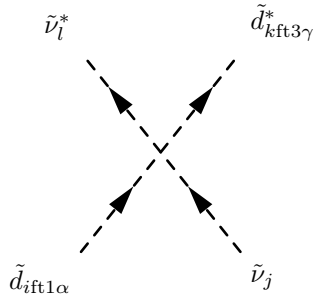
$$\frac{i}{12}\delta_{\alpha\gamma}\delta_{ik}\delta_{jl}\Big(\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\Big(-2\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(6Y^*_{e,jl}Y_{d,ki}\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)+g_1^2\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)$$
$$+2\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},2\Big)\Big)\Big(-2g_1^2\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)+\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big)\Big(-6Y^*_{d,ik}Y_{e,}$$

(328)

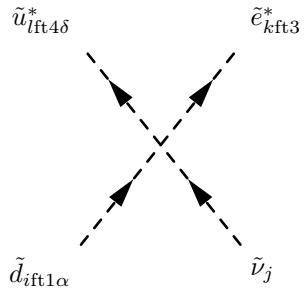


$$\frac{i}{36}\Big(2\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},2\Big)\Big)\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big(\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big)\Big(-\Big(3g_3^2+g_1^2\Big)\delta_{\alpha\gamma}\delta_{\beta\delta}\delta_{ik}\delta_{jl}+9\delta_{\alpha\delta}\delta_{\beta\gamma}\Big(-2Y^*_{d,il}\delta_{il}\delta_{jk}Y_{d,kj}+$$
$$+\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(\Big(3g_3^2+4g_1^2\Big)\delta_{\alpha\gamma}\delta_{\beta\delta}-9g_3^2\delta_{\alpha\delta}\delta_{\beta\gamma}\Big)\delta_{ik}\delta_{jl}\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)\Big)$$
$$-\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big(\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big)\Big(18\delta_{\alpha\delta}\delta_{\beta\gamma}\Big(g_2^2\delta_{il}\delta_{jk}+g_3^2\delta_{ik}\delta_{jl}\Big)+\Big(-6g_3^2-9g_2^2+g_1^2\Big)\delta_{\alpha\gamma}\delta_{\beta\delta}\delta_{ik}$$
$$+2\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(\Big(-2g_1^2+3g_3^2\Big)\delta_{\alpha\gamma}\delta_{\beta\delta}\delta_{ik}\delta_{jl}-9\delta_{\alpha\delta}\delta_{\beta\gamma}\Big(-2Y^*_{u,jk}\delta_{il}\delta_{jk}Y_{u,li}+g_3^2\delta_{ik}\delta_{jl}\Big)\Big)\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)\Big)\Big)$$

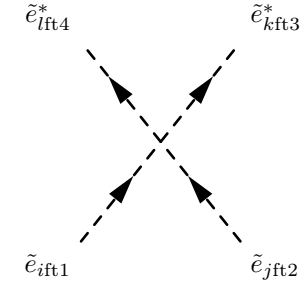
(329)



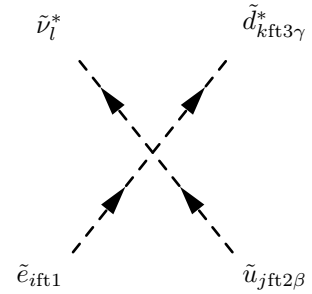
$$\frac{i}{12}\delta_{\alpha\gamma}\delta_{ik}\delta_{jl}\Big(2g_1^2\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},2\Big)\Big)\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)+\Big(3g_2^2+g_1^2\Big)\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big) \tag{330}$$



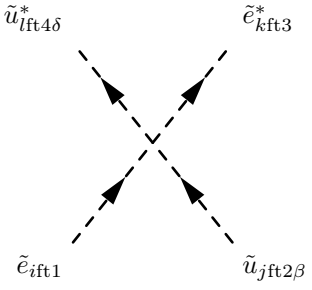
$$-\frac{i}{2}\delta_{\alpha\delta}\delta_{il}\delta_{jk}\Big(2Y^*_{d,il}\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},2\Big)\Big)Y_{e,kj}\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)+g_2^2\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\Big)\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big) \tag{331}$$



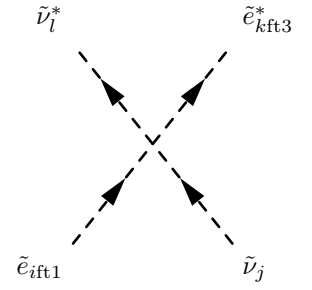
$$\begin{aligned}
&\frac{i}{4}\Big(-\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\Big(\Big(g_1^2+g_2^2\Big)\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big)\Big(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}\Big)\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)\\
&-2\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(\delta_{ik}\delta_{jl}\Big(-2Y^*_{e,jl}Y_{e,ki}\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)+g_1^2\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)\Big)+\delta_{il}\delta_{jk}\Big(\\
&+2\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},2\Big)\Big)\Big(-2g_1^2\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}\Big)\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)\\
&+\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big)\Big(\delta_{ik}\delta_{jl}\Big(-2Y^*_{e,ik}Y_{e,lj}\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)+g_1^2\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\mathrm{ZE}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)\Big)+\delta_{il}\delta_{jk}\Big(-
\end{aligned} \tag{332}$$



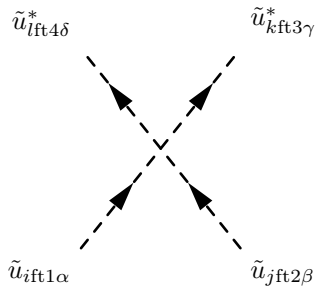
$$-\frac{i}{2}\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big)\delta_{\beta\gamma}\delta_{il}\delta_{jk}\Big(2Y^*_{e,il}\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},2\Big)\Big)Y_{d,kj}\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)+g_2^2\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\mathrm{ZD}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},\tag{333}$$



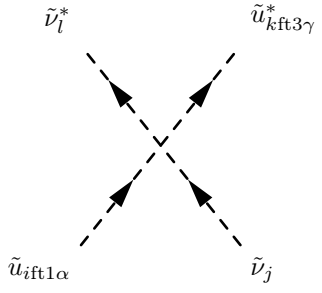
$$\frac{i}{12}\delta_{\beta\delta}\delta_{ik}\delta_{jl}\Big(-2g_1^2\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},2\Big)\Big)\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big(-4\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)+\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big)\Big)\tag{334}$$



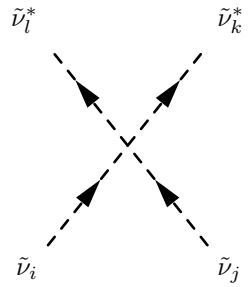
$$\frac{i}{4}\Big(2\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},2\Big)\Big)\Big(-2Y^*_{e,il}\delta_{il}\delta_{jk}Y_{e,kj}+g_1^2\delta_{ik}\delta_{jl}\Big)\mathrm{ZE}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)+\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\Big(-2g_2^2\delta_{il}\delta_{jk}+\Big(-g_1^2+g_2^2\Big)\delta\tag{335}$$



$$\frac{i}{36}\Big(-\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\Big(\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big)\Big(\delta_{\alpha\delta}\delta_{\beta\gamma}\Big(18g_3^2\delta_{ik}\delta_{jl}+\Big(-6g_3^2+9g_2^2+g_1^2\Big)\delta_{il}\delta_{jk}\Big)$$
$$+\delta_{\alpha\gamma}\delta_{\beta\delta}\Big(18g_3^2\delta_{il}\delta_{jk}+\Big(-6g_3^2+9g_2^2+g_1^2\Big)\delta_{ik}\delta_{jl}\Big)\Big)\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)$$
$$+2\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(\delta_{\alpha\gamma}\delta_{\beta\delta}\Big(-9g_3^2\delta_{il}\delta_{jk}\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)+\delta_{ik}\delta_{jl}\Big(18Y^*_{u,jl}Y_{u,ki}\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{f}$$
$$+\delta_{\alpha\delta}\delta_{\beta\gamma}\Big(-9g_3^2\delta_{ik}\delta_{jl}\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)+\delta_{il}\delta_{jk}\Big(18Y^*_{u,jk}Y_{u,li}\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)+\Big(-2g_1^2+3g_3^2\Big)\mathrm{ZU}$$
$$+2\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},2\Big)\Big)\Big(-\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(\delta_{\alpha\delta}\delta_{\beta\gamma}\Big(\Big(-3g_3^2+8g_1^2\Big)\delta_{il}\delta_{jk}+9g_3^2\delta_{ik}\delta_{jl}\Big)$$
$$+\delta_{\alpha\gamma}\delta_{\beta\delta}\Big(\Big(-3g_3^2+8g_1^2\Big)\delta_{ik}\delta_{jl}+9g_3^2\delta_{il}\delta_{jk}\Big)\Big)\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)$$
$$+\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big)\Big(\delta_{\alpha\delta}\delta_{\beta\gamma}\Big(9g_3^2\delta_{ik}\delta_{jl}\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},1\Big)+\delta_{il}\delta_{jk}\Big(-18Y^*_{u,il}Y_{u,kj}\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4}$$
$$+\delta_{\alpha\gamma}\delta_{\beta\delta}\Big(9g_3^2\delta_{il}\delta_{jk}\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)+\delta_{ik}\delta_{jl}\Big(-18Y^*_{u,ik}Y_{u,lj}\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)\mathrm{ZU}\Big(\mathrm{gt4}\Big)\Big(\mathrm{ft4},2\Big)+\Big(2g_1^2-3g_3^2\Big)\mathrm{ZU}\Big(\mathrm{g}$$
(336)

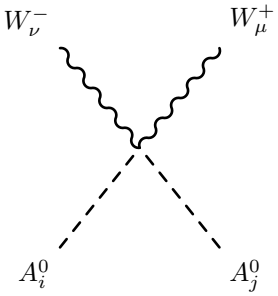


$$\frac{i}{12}\delta_{\alpha\gamma}\delta_{ik}\delta_{jl}\Big(\Big(-3g_2^2+g_1^2\Big)\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},1\Big)-4g_1^2\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},2\Big)\Big)\mathrm{ZU}\Big(\mathrm{gt3}\Big)\Big(\mathrm{ft3},2\Big)\Big)$$
(337)

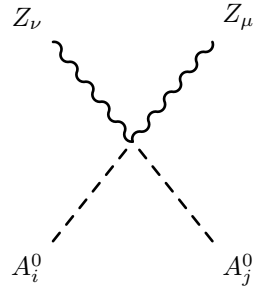


$$-\frac{i}{4}\Big(g_1^2+g_2^2\Big)\Big(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}\Big) \tag{338}$$

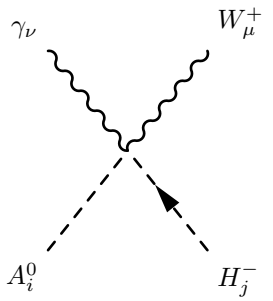
8.8 Two Scalar-Two Vector Boson-Interaction



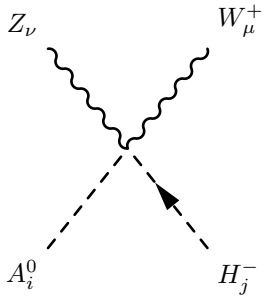
$$\Big(\frac{i}{2}g_2^2 Z_{i1}^A Z_{j1}^A+\frac{i}{2}g_2^2 Z_{i2}^A Z_{j2}^A\Big)\Big(g_{\mu\nu}\Big) \tag{339}$$



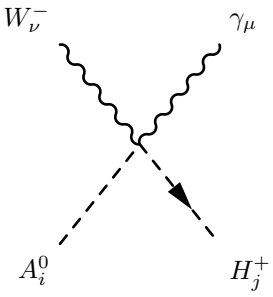
$$\begin{aligned}\Big(&+\frac{i}{2}g_2^2\cos\Theta_W^2 Z_{i1}^A Z_{j1}^A+ig_1g_2\cos\Theta_W\sin\Theta_W Z_{i1}^A Z_{j1}^A+\frac{i}{2}g_1^2\sin\Theta_W^2 Z_{i1}^A Z_{j1}^A\\&+\frac{i}{2}g_2^2\cos\Theta_W^2 Z_{i2}^A Z_{j2}^A+ig_1g_2\cos\Theta_W\sin\Theta_W Z_{i2}^A Z_{j2}^A+\frac{i}{2}g_1^2\sin\Theta_W^2 Z_{i2}^A Z_{j2}^A\Big)\Big(g_{\mu\nu}\Big)\end{aligned} \tag{340}$$



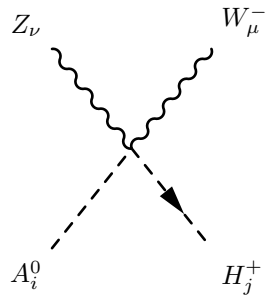
$$\Big(-\frac{1}{2} g_1 g_2 \cos\Theta_W Z^A_{i1} Z^+_{j1} - \frac{1}{2} g_1 g_2 \cos\Theta_W Z^A_{i2} Z^+_{j2} \Big) \Big(g_{\mu\nu} \Big) \tag{341}$$



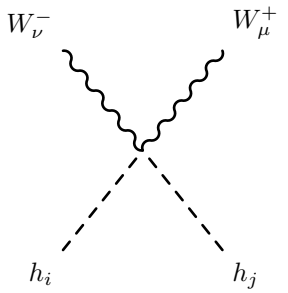
$$\Big(\frac{1}{2} g_1 g_2 \sin\Theta_W Z^A_{i1} Z^+_{j1} + \frac{1}{2} g_1 g_2 \sin\Theta_W Z^A_{i2} Z^+_{j2} \Big) \Big(g_{\mu\nu} \Big) \tag{342}$$



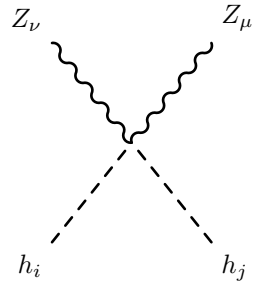
$$\Big(\frac{1}{2} g_1 g_2 \cos\Theta_W Z^A_{i1} Z^+_{j1} + \frac{1}{2} g_1 g_2 \cos\Theta_W Z^A_{i2} Z^+_{j2} \Big) \Big(g_{\mu\nu} \Big) \tag{343}$$



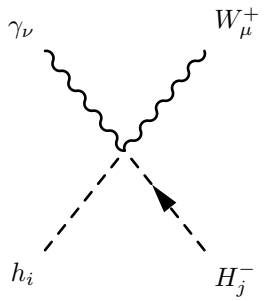
$$\Big(-\frac{1}{2} g_1 g_2 \sin\Theta_W Z^A_{i1} Z^+_{j1} - \frac{1}{2} g_1 g_2 \sin\Theta_W Z^A_{i2} Z^+_{j2} \Big) \Big(g_{\mu\nu} \Big) \tag{344}$$



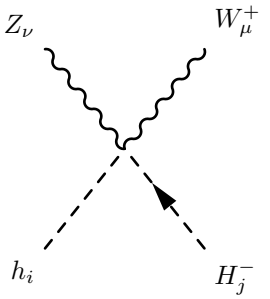
$$\Big(\frac{i}{2} g_2^2 Z^H_{i1} Z^H_{j1} + \frac{i}{2} g_2^2 Z^H_{i2} Z^H_{j2} \Big) \Big(g_{\mu\nu} \Big) \tag{345}$$



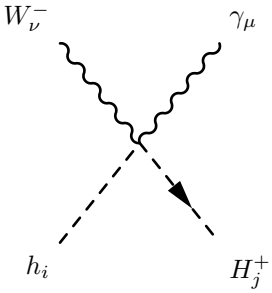
$$\begin{aligned} &\Big(+ \frac{i}{2} g_2^2 \cos\Theta_W^2 Z^H_{i1} Z^H_{j1} + i g_1 g_2 \cos\Theta_W \sin\Theta_W Z^H_{i1} Z^H_{j1} + \frac{i}{2} g_1^2 \sin\Theta_W^2 Z^H_{i1} Z^H_{j1} \\ &+ \frac{i}{2} g_2^2 \cos\Theta_W^2 Z^H_{i2} Z^H_{j2} + i g_1 g_2 \cos\Theta_W \sin\Theta_W Z^H_{i2} Z^H_{j2} + \frac{i}{2} g_1^2 \sin\Theta_W^2 Z^H_{i2} Z^H_{j2} \Big) \Big(g_{\mu\nu} \Big) \end{aligned} \tag{346}$$



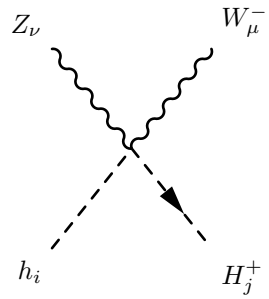
$$\Big(-\frac{i}{2} g_1 g_2 \cos\Theta_W Z_{i1}^H Z_{j1}^+ + \frac{i}{2} g_1 g_2 \cos\Theta_W Z_{i2}^H Z_{j2}^+ \Big) \Big(g_{\mu\nu} \Big) \tag{347}$$



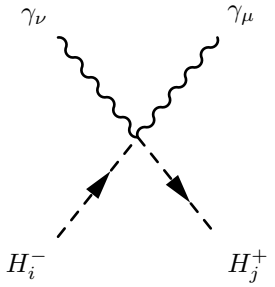
$$\Big(\frac{i}{2} g_1 g_2 \sin\Theta_W Z_{i1}^H Z_{j1}^+ - \frac{i}{2} g_1 g_2 \sin\Theta_W Z_{i2}^H Z_{j2}^+ \Big) \Big(g_{\mu\nu} \Big) \tag{348}$$



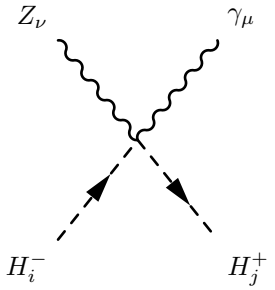
$$\Big(-\frac{i}{2} g_1 g_2 \cos\Theta_W Z_{i1}^H Z_{j1}^+ + \frac{i}{2} g_1 g_2 \cos\Theta_W Z_{i2}^H Z_{j2}^+ \Big) \Big(g_{\mu\nu} \Big) \tag{349}$$



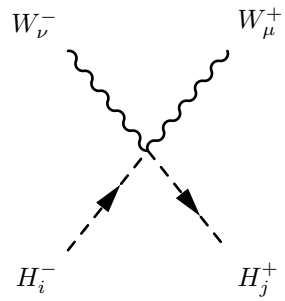
$$\Big(\frac{i}{2}g_1g_2\sin\Theta_W Z^H_{i1}Z^+_{j1} - \frac{i}{2}g_1g_2\sin\Theta_W Z^H_{i2}Z^+_{j2}\Big)\Big(g_{\mu\nu}\Big) \tag{350}$$



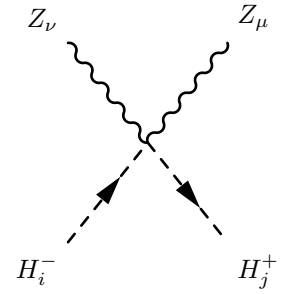
$$\begin{aligned}\Big(&+\frac{i}{2}g_1^2\cos\Theta_W^2 Z^+_{i1}Z^+_{j1} + ig_1g_2\cos\Theta_W\sin\Theta_W Z^+_{i1}Z^+_{j1} + \frac{i}{2}g_2^2\sin\Theta_W^2 Z^+_{i1}Z^+_{j1}\\ &+\frac{i}{2}g_1^2\cos\Theta_W^2 Z^+_{i2}Z^+_{j2} + ig_1g_2\cos\Theta_W\sin\Theta_W Z^+_{i2}Z^+_{j2} + \frac{i}{2}g_2^2\sin\Theta_W^2 Z^+_{i2}Z^+_{j2}\Big)\Big(g_{\mu\nu}\Big)\end{aligned} \tag{351}$$



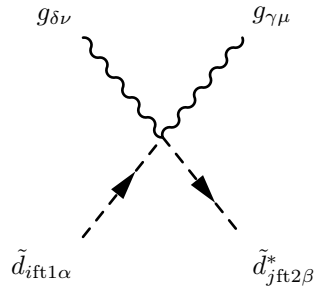
$$\begin{aligned}\Big(&+\frac{i}{2}g_1g_2\cos 2\Theta_W Z^+_{i1}Z^+_{j1} - \frac{i}{4}g_1^2\sin 2\Theta_W Z^+_{i1}Z^+_{j1} + \frac{i}{4}g_2^2\sin 2\Theta_W Z^+_{i1}Z^+_{j1}\\ &+\frac{i}{2}g_1g_2\cos 2\Theta_W Z^+_{i2}Z^+_{j2} - \frac{i}{4}g_1^2\sin 2\Theta_W Z^+_{i2}Z^+_{j2} + \frac{i}{4}g_2^2\sin 2\Theta_W Z^+_{i2}Z^+_{j2}\Big)\Big(g_{\mu\nu}\Big)\end{aligned} \tag{352}$$



$$\Big(\frac{i}{2}g_2^2 Z_{i1}^+ Z_{j1}^+ + \frac{i}{2}g_2^2 Z_{i2}^+ Z_{j2}^+\Big)\Big(g_{\mu\nu}\Big) \tag{353}$$

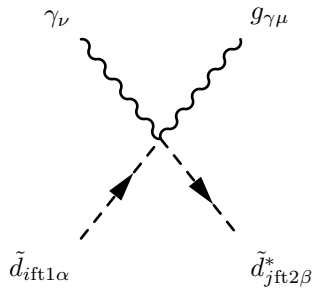


$$\begin{aligned}&\Big(+ \frac{i}{2}g_2^2 \cos\Theta_W^2 Z_{i1}^+ Z_{j1}^+ - ig_1 g_2 \cos\Theta_W \sin\Theta_W Z_{i1}^+ Z_{j1}^+ \\ &+ \frac{i}{2}g_1^2 \sin\Theta_W^2 Z_{i1}^+ Z_{j1}^+ + \frac{i}{2}g_2^2 \cos\Theta_W^2 Z_{i2}^+ Z_{j2}^+ \\ &- ig_1 g_2 \cos\Theta_W \sin\Theta_W Z_{i2}^+ Z_{j2}^+ + \frac{i}{2}g_1^2 \sin\Theta_W^2 Z_{i2}^+ Z_{j2}^+\Big)\Big(g_{\mu\nu}\Big)\end{aligned} \tag{354}$$

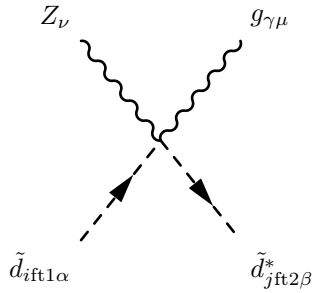


$$\Big(+ \frac{i}{4}g_3^2 \mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\delta_{ij}\sum_{a=1}^{3}\lambda_{a,\alpha}^{\gamma}\lambda_{\beta,a}^{\delta}\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big) + \frac{i}{4}g_3^2 \mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\delta_{ij}\sum_{a=1}^{3}\lambda_{\beta,a}^{\gamma}\lambda_{a,\alpha}^{\delta}\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big) + \frac{i}{4}g$$

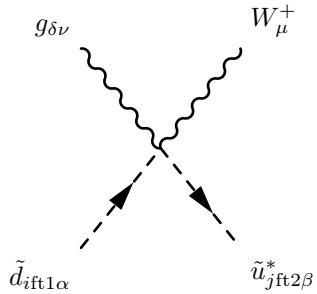
$$+ \frac{i}{4} g_3^2 \mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1}, 2\Big)\Big)\delta_{ij} \sum_{a=1}^{3} \lambda^{\gamma}_{\beta,a} \lambda^{\delta}_{a,\alpha} \mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2}, 2\Big)\Big)\Big(g_{\mu\nu}\Big) \tag{355}$$



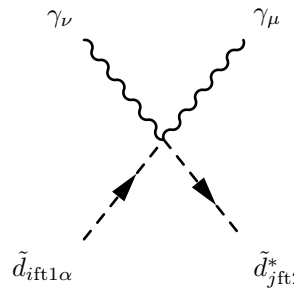
$$\Big(+ \frac{i}{6} g_1 g_3 \mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1}, 1\Big)\Big)\cos\Theta_W \delta_{ij} \lambda^{\gamma}_{\beta,\alpha} \mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2}, 1\Big) - \frac{i}{2} g_2 g_3 \mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1}, 1\Big)\Big)\delta_{ij} \lambda^{\gamma}_{\beta,\alpha} \sin\Theta_W \mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2}, 1\Big)$$
$$- \frac{i}{3} g_1 g_3 \mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1}, 2\Big)\Big)\cos\Theta_W \delta_{ij} \lambda^{\gamma}_{\beta,\alpha} \mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2}, 2\Big)\Big)\Big(g_{\mu\nu}\Big) \tag{356}$$



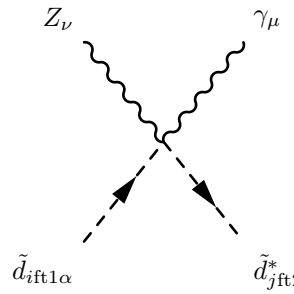
$$\Big(- \frac{i}{2} g_2 g_3 \mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1}, 1\Big)\Big)\cos\Theta_W \delta_{ij} \lambda^{\gamma}_{\beta,\alpha} \mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2}, 1\Big) - \frac{i}{6} g_1 g_3 \mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1}, 1\Big)\Big)\delta_{ij} \lambda^{\gamma}_{\beta,\alpha} \sin\Theta_W \mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2}, 1\Big)$$
$$+ \frac{i}{3} g_1 g_3 \mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1}, 2\Big)\Big)\delta_{ij} \lambda^{\gamma}_{\beta,\alpha} \sin\Theta_W \mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2}, 2\Big)\Big)\Big(g_{\mu\nu}\Big) \tag{357}$$



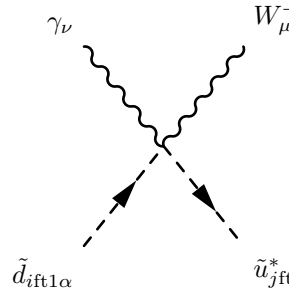
$$i\frac{1}{\sqrt{2}}g_2 g_3 \mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\delta_{ij}\lambda^{\delta}_{\beta,\alpha}\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big(g_{\mu\nu}\Big) \tag{358}$$



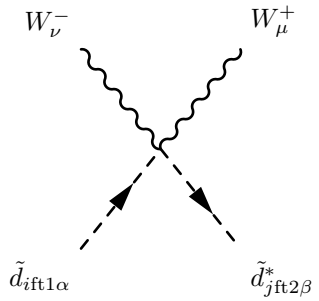
$$\Big(+\frac{i}{18}g_1^2\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\cos\Theta_W^2\delta_{\alpha\beta}\delta_{ij}\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big) - \frac{i}{3}g_1 g_2\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\cos\Theta_W\delta_{\alpha\beta}\delta_{ij}\sin\Theta_W\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},$$
$$+\frac{i}{2}g_2^2\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\delta_{\alpha\beta}\delta_{ij}\sin\Theta_W^2\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big) + \frac{2i}{9}g_1^2\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},2\Big)\Big)\cos\Theta_W^2\delta_{\alpha\beta}\delta_{ij}\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(g_{\mu\nu}\Big) \tag{359}$$



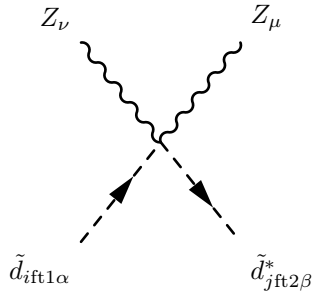
$$\Big(-\frac{i}{6}g_1 g_2\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\cos 2\Theta_W\delta_{\alpha\beta}\delta_{ij}\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big) - \frac{i}{36}g_1^2\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\delta_{\alpha\beta}\delta_{ij}\sin 2\Theta_W\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)$$
$$+\frac{i}{4}g_2^2\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\delta_{\alpha\beta}\delta_{ij}\sin 2\Theta_W\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big) - \frac{i}{9}g_1^2\mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},2\Big)\Big)\delta_{\alpha\beta}\delta_{ij}\sin 2\Theta_W\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(g_{\mu\nu}\Big) \tag{360}$$



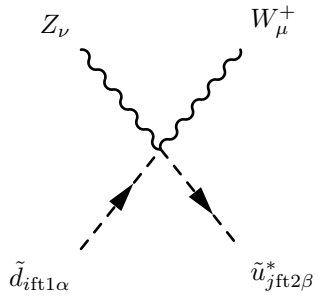
$$\frac{i}{3}\frac{1}{\sqrt{2}}g_1 g_2 \mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\cos\Theta_W \delta_{\alpha\beta}\delta_{ij}\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big(g_{\mu\nu}\Big) \tag{361}$$



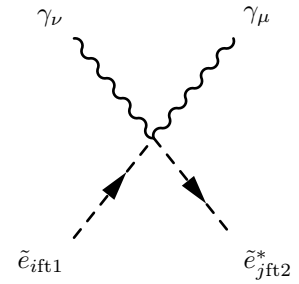
$$\frac{i}{2}g_2^2 \mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\delta_{\alpha\beta}\delta_{ij}\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big(g_{\mu\nu}\Big) \tag{362}$$



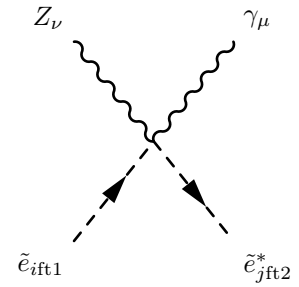
$$\begin{aligned}\Big(&+ \frac{i}{2}g_2^2 \mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\cos\Theta_W^2\delta_{\alpha\beta}\delta_{ij}\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big) + \frac{i}{3}g_1 g_2 \mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\cos\Theta_W\delta_{\alpha\beta}\delta_{ij}\sin\Theta_W\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1 \\ &+ \frac{i}{18}g_1^2 \mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\delta_{\alpha\beta}\delta_{ij}\sin\Theta_W^2\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big) + \frac{2i}{9}g_1^2 \mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},2\Big)\Big)\delta_{\alpha\beta}\delta_{ij}\sin\Theta_W^2\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(g_{\mu\nu}\Big)\end{aligned} \tag{363}$$



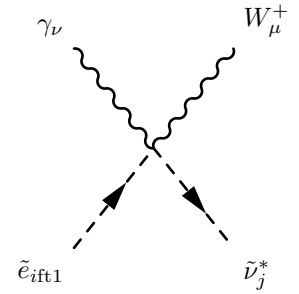
$$-\frac{i}{3}\frac{1}{\sqrt{2}}g_1 g_2 \mathrm{conj}\Big(\mathrm{ZD}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\delta_{\alpha\beta}\delta_{ij}\sin\Theta_W \mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big(g_{\mu\nu}\Big) \tag{364}$$



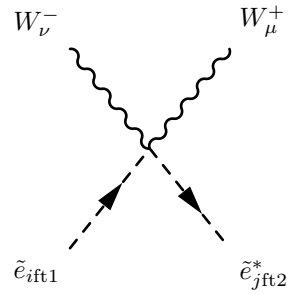
$$\Big(+\frac{i}{2}g_1^2\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\cos\Theta_W^2\delta_{ij}\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)+ig_1g_2\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\cos\Theta_W\delta_{ij}\sin\Theta_W\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)$$
$$+\frac{i}{2}g_2^2\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\delta_{ij}\sin\Theta_W^2\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)+2ig_1^2\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},2\Big)\Big)\cos\Theta_W^2\delta_{ij}\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(g_{\mu\nu}\Big) \tag{365}$$



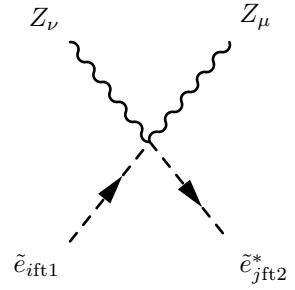
$$\Big(+\frac{i}{2}g_1g_2\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\cos 2\Theta_W\delta_{ij}\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)-\frac{i}{4}g_1^2\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\delta_{ij}\sin 2\Theta_W\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)+\frac{i}{4}g_2^2\mathrm{conj}($$
$$-ig_1^2\mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},2\Big)\Big)\delta_{ij}\sin 2\Theta_W\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(g_{\mu\nu}\Big) \tag{366}$$



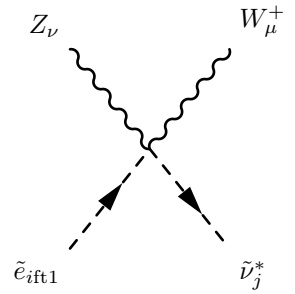
$$- i\frac{1}{\sqrt{2}}g_1 g_2 \mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1}, 1\Big)\Big)\cos\Theta_W \delta_{ij}\Big(g_{\mu\nu}\Big) \tag{367}$$



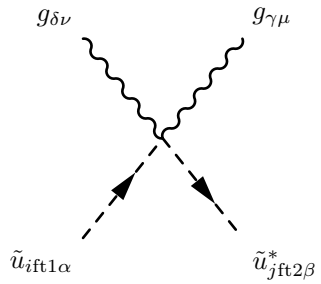
$$\frac{i}{2}g_2^2 \mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1}, 1\Big)\Big)\delta_{ij}\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2}, 1\Big)\Big(g_{\mu\nu}\Big) \tag{368}$$



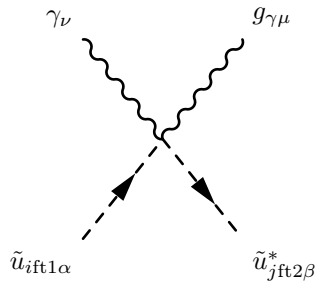
$$\Big(+\frac{i}{2}g_2^2 \mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1}, 1\Big)\Big)\cos\Theta_W^2 \delta_{ij}\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2}, 1\Big) - ig_1 g_2 \mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1}, 1\Big)\Big)\cos\Theta_W \delta_{ij}\sin\Theta_W \mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2}, 1\Big)$$
$$+\frac{i}{2}g_1^2 \mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1}, 1\Big)\Big)\delta_{ij}\sin\Theta_W^2 \mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2}, 1\Big) + 2ig_1^2 \mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1}, 2\Big)\Big)\delta_{ij}\sin\Theta_W^2 \mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2}, 2\Big)\Big)\Big(g_{\mu\nu}\Big) \tag{369}$$



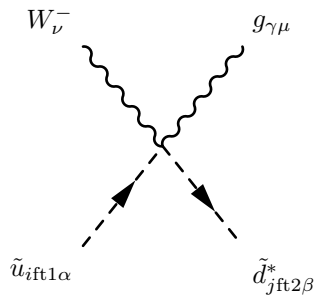
$$i\frac{1}{\sqrt{2}}g_1 g_2 \mathrm{conj}\Big(\mathrm{ZE}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\delta_{ij}\sin\Theta_W\Big(g_{\mu\nu}\Big) \tag{370}$$



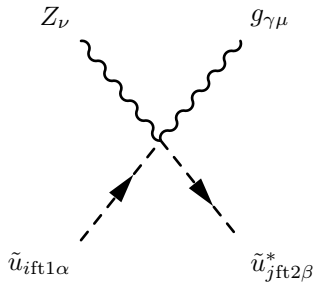
$$\Big(+\frac{i}{4}g_3^2\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\delta_{ij}\sum_{a=1}^{3}\lambda^{\gamma}_{a,\alpha}\lambda^{\delta}_{\beta,a}\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)+\frac{i}{4}g_3^2\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\delta_{ij}\sum_{a=1}^{3}\lambda^{\gamma}_{\beta,a}\lambda^{\delta}_{a,\alpha}\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)+\frac{i}{4}g$$
$$+\frac{i}{4}g_3^2\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},2\Big)\Big)\delta_{ij}\sum_{a=1}^{3}\lambda^{\gamma}_{\beta,a}\lambda^{\delta}_{a,\alpha}\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(g_{\mu\nu}\Big) \tag{371}$$



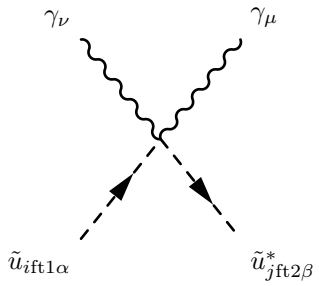
$$\Big(+\frac{i}{6}g_1 g_3\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\cos\Theta_W\delta_{ij}\lambda^{\gamma}_{\beta,\alpha}\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)+\frac{i}{2}g_2 g_3\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\delta_{ij}\lambda^{\gamma}_{\beta,\alpha}\sin\Theta_W\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)$$
$$+\frac{2i}{3}g_1 g_3\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},2\Big)\Big)\cos\Theta_W\delta_{ij}\lambda^{\gamma}_{\beta,\alpha}\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(g_{\mu\nu}\Big) \tag{372}$$



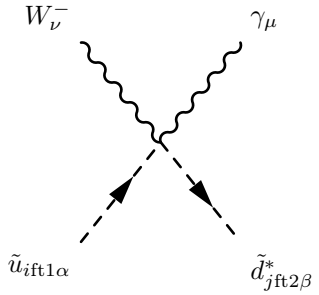
$$i\frac{1}{\sqrt{2}}g_2 g_3 \mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\delta_{ij}\lambda^{\gamma}_{\beta,\alpha}\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big(g_{\mu\nu}\Big) \tag{373}$$



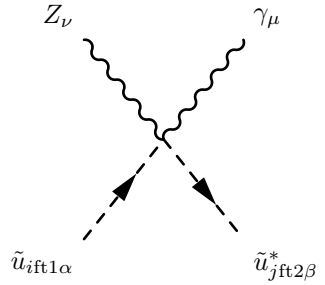
$$\Big(+\frac{i}{2}g_2 g_3 \mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\cos\Theta_W\delta_{ij}\lambda^{\gamma}_{\beta,\alpha}\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big) - \frac{i}{6}g_1 g_3 \mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\delta_{ij}\lambda^{\gamma}_{\beta,\alpha}\sin\Theta_W\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)$$
$$-\frac{2i}{3}g_1 g_3 \mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},2\Big)\Big)\delta_{ij}\lambda^{\gamma}_{\beta,\alpha}\sin\Theta_W\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(g_{\mu\nu}\Big) \tag{374}$$



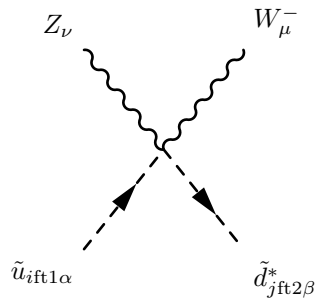
$$\Big(+\frac{i}{18}g_1^2 \mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\cos\Theta_W^2\delta_{\alpha\beta}\delta_{ij}\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big) + \frac{i}{3}g_1 g_2 \mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\cos\Theta_W\delta_{\alpha\beta}\delta_{ij}\sin\Theta_W\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},$$
$$+\frac{i}{2}g_2^2 \mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\delta_{\alpha\beta}\delta_{ij}\sin\Theta_W^2\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big) + \frac{8i}{9}g_1^2 \mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},2\Big)\Big)\cos\Theta_W^2\delta_{\alpha\beta}\delta_{ij}\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(g_{\mu\nu}\Big) \tag{375}$$



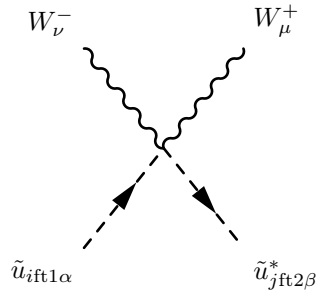
$$\frac{i}{3}\frac{1}{\sqrt{2}}g_1 g_2 \mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\cos\Theta_W \delta_{\alpha\beta}\delta_{ij}\mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big(g_{\mu\nu}\Big) \tag{376}$$



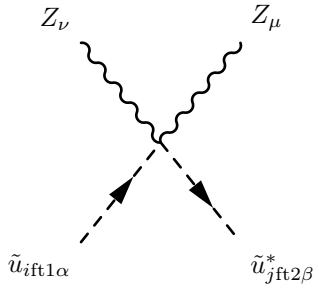
$$\Big(+\frac{i}{6}g_1 g_2 \mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\cos 2\Theta_W \delta_{\alpha\beta}\delta_{ij}\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big) - \frac{i}{36}g_1^2 \mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\delta_{\alpha\beta}\delta_{ij}\sin 2\Theta_W \mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)$$
$$+\frac{i}{4}g_2^2 \mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\delta_{\alpha\beta}\delta_{ij}\sin 2\Theta_W \mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big) - \frac{4i}{9}g_1^2 \mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},2\Big)\Big)\delta_{\alpha\beta}\delta_{ij}\sin 2\Theta_W \mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(g_{\mu\nu}\Big) \tag{377}$$



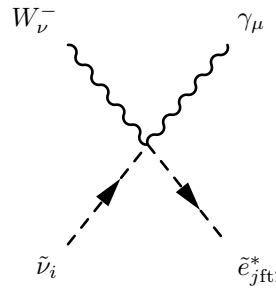
$$-\frac{i}{3}\frac{1}{\sqrt{2}}g_1 g_2 \mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\delta_{\alpha\beta}\delta_{ij}\sin\Theta_W \mathrm{ZD}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big(g_{\mu\nu}\Big) \tag{378}$$



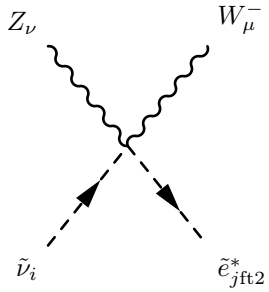
$$\frac{i}{2}g_2^2\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\delta_{\alpha\beta}\delta_{ij}\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big(g_{\mu\nu}\Big) \tag{379}$$



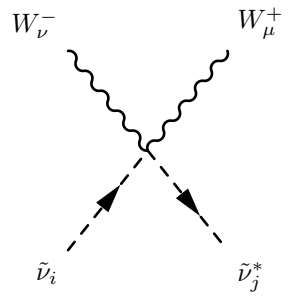
$$\Big(+\frac{i}{2}g_2^2\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\cos\Theta_W^2\delta_{\alpha\beta}\delta_{ij}\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)-\frac{i}{3}g_1g_2\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\cos\Theta_W\delta_{\alpha\beta}\delta_{ij}\sin\Theta_W\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1$$
$$+\frac{i}{18}g_1^2\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},1\Big)\Big)\delta_{\alpha\beta}\delta_{ij}\sin\Theta_W^2\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)+\frac{8i}{9}g_1^2\mathrm{conj}\Big(\mathrm{ZU}\Big(\mathrm{gt1}\Big)\Big(\mathrm{ft1},2\Big)\Big)\delta_{\alpha\beta}\delta_{ij}\sin\Theta_W^2\mathrm{ZU}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},2\Big)\Big)\Big(g_{\mu\nu}\Big) \tag{380}$$



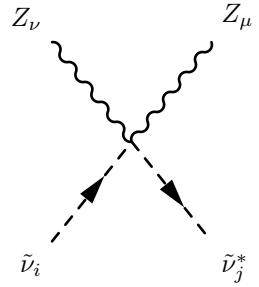
$$-i\frac{1}{\sqrt{2}}g_1g_2\cos\Theta_W\delta_{ij}\mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big(g_{\mu\nu}\Big) \tag{381}$$



$$i\frac{1}{\sqrt{2}}g_1 g_2 \delta_{ij} \sin\Theta_W \mathrm{ZE}\Big(\mathrm{gt2}\Big)\Big(\mathrm{ft2},1\Big)\Big(g_{\mu\nu}\Big) \tag{382}$$

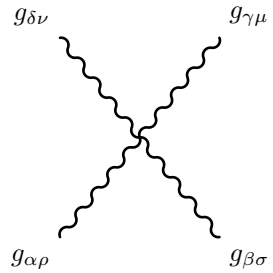


$$\frac{i}{2}g_2^2\delta_{ij}\Big(g_{\mu\nu}\Big) \tag{383}$$



$$\Big(\frac{i}{2}g_1^2\delta_{ij}\sin\Theta_W^2 + \frac{i}{2}g_2^2\cos\Theta_W^2\delta_{ij} + ig_1 g_2\cos\Theta_W\delta_{ij}\sin\Theta_W\Big)\Big(g_{\mu\nu}\Big) \tag{384}$$

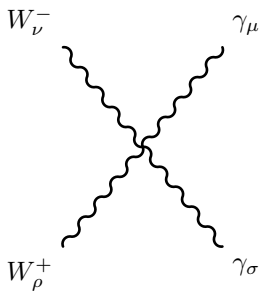
8.9 Four Vector Boson-Interaction



$$ig_3^2\Big(-\sum_{a=1}^{8} f_{\alpha,\delta,a}f_{\beta,\gamma,a} - \sum_{a=1}^{8} f_{\alpha,\gamma,a}f_{\beta,\delta,a}\Big)\Big(g_{\rho\sigma}g_{\mu\nu}\Big) \tag{385}$$

$$+\ ig_3^2\Big(-\sum_{a=1}^{8} f_{\alpha,\beta,a}f_{\gamma,\delta,a} + \sum_{a=1}^{8} f_{\alpha,\delta,a}f_{\beta,\gamma,a}\Big)\Big(g_{\rho\mu}g_{\sigma\nu}\Big) \tag{386}$$

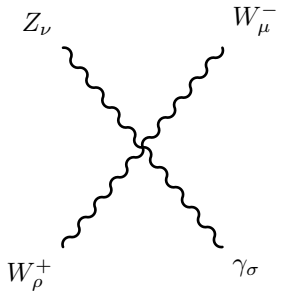
$$+\ ig_3^2\Big(\sum_{a=1}^{8} f_{\alpha,\gamma,a}f_{\beta,\delta,a} + \sum_{a=1}^{8} f_{\alpha,\beta,a}f_{\gamma,\delta,a}\Big)\Big(g_{\rho\nu}g_{\sigma\mu}\Big) \tag{387}$$



$$ig_2^2 \sin\Theta_W^2\Big(g_{\rho\sigma}g_{\mu\nu}\Big) \tag{388}$$

$$+\ ig_2^2 \sin\Theta_W^2\Big(g_{\rho\mu}g_{\sigma\nu}\Big) \tag{389}$$

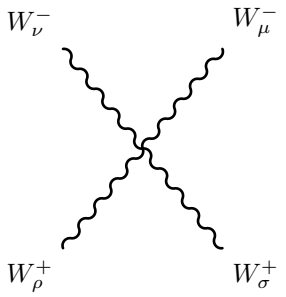
$$+\ -2ig_2^2 \sin\Theta_W^2\Big(g_{\rho\nu}g_{\sigma\mu}\Big) \tag{390}$$



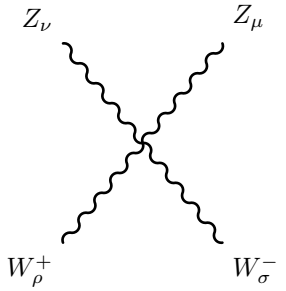
$$\frac{i}{2}g_2^2 \sin 2\Theta_W\Big(g_{\rho\sigma}g_{\mu\nu}\Big) \tag{391}$$

$$+\ -ig_2^2 \sin 2\Theta_W\Big(g_{\rho\mu}g_{\sigma\nu}\Big) \tag{392}$$

$$+\ \frac{i}{2}g_2^2 \sin 2\Theta_W\Big(g_{\rho\nu}g_{\sigma\mu}\Big) \tag{393}$$

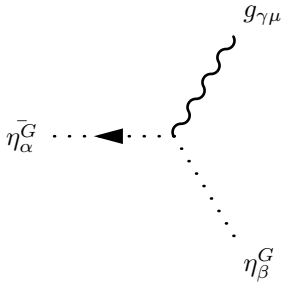


$$2ig_2^2\Big(g_{\rho\sigma}g_{\mu\nu}\Big) \tag{394}$$
$$+\ -ig_2^2\Big(g_{\rho\mu}g_{\sigma\nu}\Big) \tag{395}$$
$$+\ -ig_2^2\Big(g_{\rho\nu}g_{\sigma\mu}\Big) \tag{396}$$

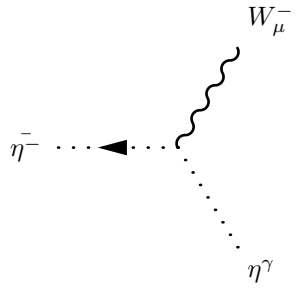


$$-\ 2ig_2^2\cos\Theta_W^2\Big(g_{\rho\sigma}g_{\mu\nu}\Big) \tag{397}$$
$$+\ ig_2^2\cos\Theta_W^2\Big(g_{\rho\mu}g_{\sigma\nu}\Big) \tag{398}$$
$$+\ ig_2^2\cos\Theta_W^2\Big(g_{\rho\nu}g_{\sigma\mu}\Big) \tag{399}$$

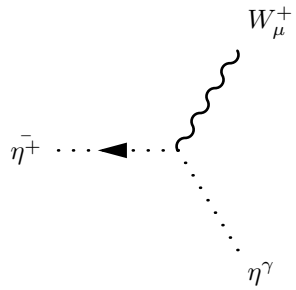
8.10 Two Ghosts-One Vector Boson-Interaction



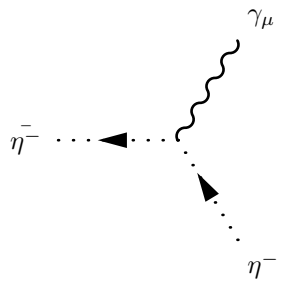
$$g_3 f_{\alpha,\beta,\gamma}\left(p_\mu^{\eta_\beta^G}\right) \tag{400}$$



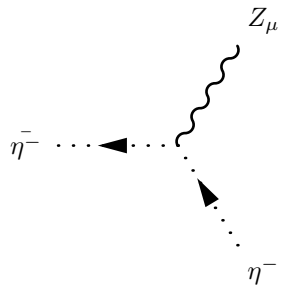
$$ig_2 \sin\Theta_W\left(p_\mu^{\eta^\gamma}\right) \tag{401}$$



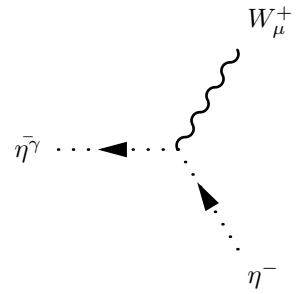
$$-ig_2 \sin\Theta_W\left(p_\mu^{\eta^\gamma}\right) \tag{402}$$



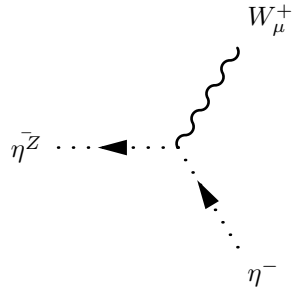
$$-ig_2 \sin\Theta_W\left(p_\mu^{\eta^-}\right) \tag{403}$$



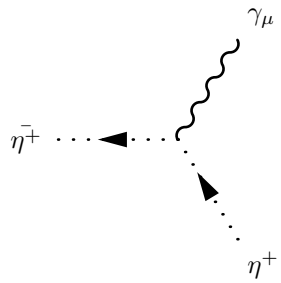
$$-ig_2 \cos\Theta_W \left(p_\mu^{\eta^-}\right) \tag{404}$$



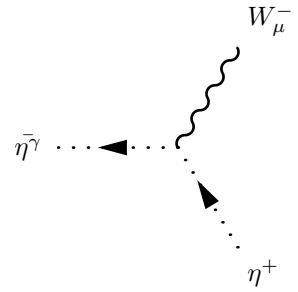
$$ig_2 \sin\Theta_W \left(p_\mu^{\eta^-}\right) \tag{405}$$



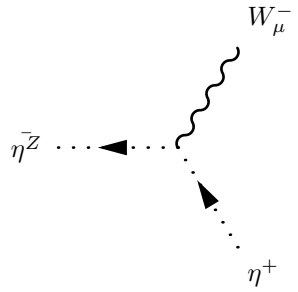
$$ig_2 \cos\Theta_W \left(p_\mu^{\eta^-}\right) \tag{406}$$



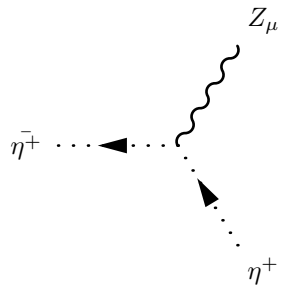
$$ig_2 \sin\Theta_W \left(p_\mu^{\eta^+}\right) \tag{407}$$



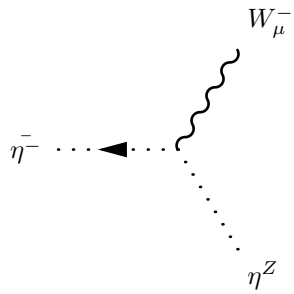
$$-ig_2 \sin\Theta_W \left(p_\mu^{\eta^+}\right) \tag{408}$$



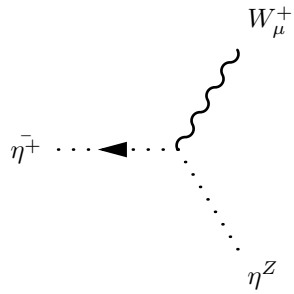
$$-ig_2 \cos\Theta_W \left(p_\mu^{\eta^+}\right) \tag{409}$$



$$ig_2 \cos\Theta_W \left(p_\mu^{\eta^+}\right) \tag{410}$$

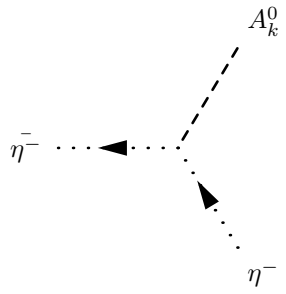


$$ig_2 \cos\Theta_W \left(p_\mu^{\eta^Z}\right) \tag{411}$$

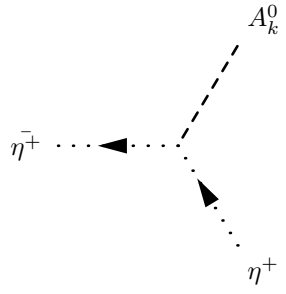


$$-ig_2 \cos\Theta_W \left(p_\mu^{\eta^Z}\right) \tag{412}$$

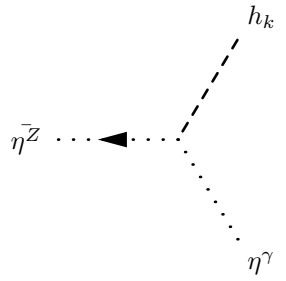
8.11 Two Ghosts-One Scalar-Interaction



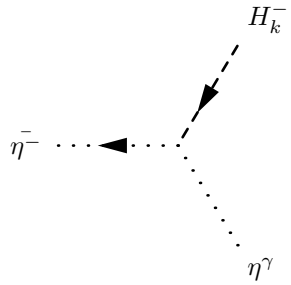
$$\frac{1}{4}g_2^2\xi_{W^-}\Big(v_d Z_{k1}^A - v_u Z_{k2}^A\Big) \tag{413}$$



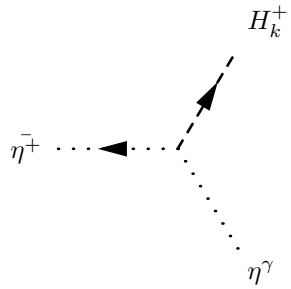
$$-\frac{1}{4}g_2^2\xi_{W^-}\Big(v_d Z_{k1}^A - v_u Z_{k2}^A\Big) \tag{414}$$



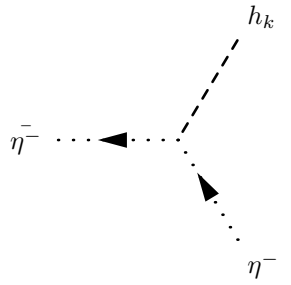
$$\frac{i}{8}\xi_Z\Big(2g_1g_2\cos 2\Theta_W + \Big(-g_2^2+g_1^2\Big)\sin 2\Theta_W\Big)\Big(v_d Z_{k1}^H + v_u Z_{k2}^H\Big) \tag{415}$$



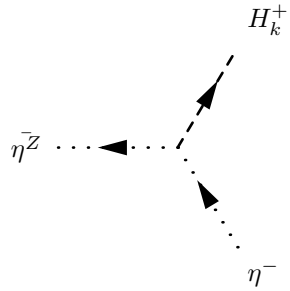
$$\frac{i}{4}g_2\xi_{W^-}\Big(g_1\cos\Theta_W + g_2\sin\Theta_W\Big)\Big(v_d Z_{k1}^+ - v_u Z_{k2}^+\Big) \tag{416}$$



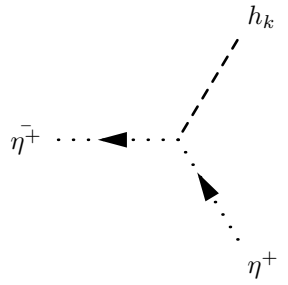
$$\frac{i}{4}g_2\xi_{W^-}\Big(g_1\cos\Theta_W + g_2\sin\Theta_W\Big)\Big(v_d Z_{k1}^+ - v_u Z_{k2}^+\Big) \tag{417}$$



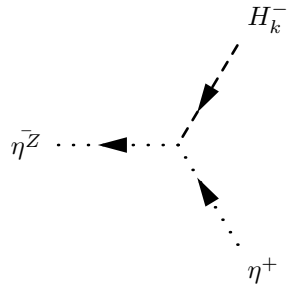
$$-\frac{i}{4}g_2^2\xi_{W^-}\Big(v_d Z_{k1}^H + v_u Z_{k2}^H\Big) \tag{418}$$



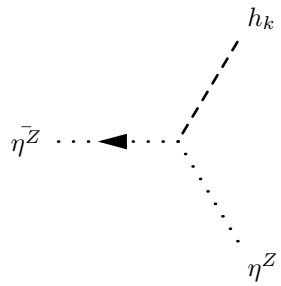
$$-\frac{i}{4}g_2\xi_Z\Big(g_1\sin\Theta_W+g_2\cos\Theta_W\Big)\Big(v_dZ_{k1}^+-v_uZ_{k2}^+\Big) \tag{419}$$



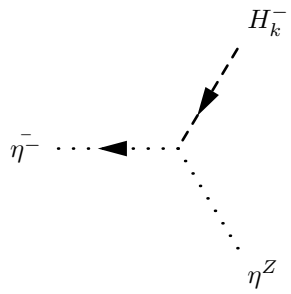
$$-\frac{i}{4}g_2^2\xi_{W^-}\Big(v_dZ_{k1}^H+v_uZ_{k2}^H\Big) \tag{420}$$



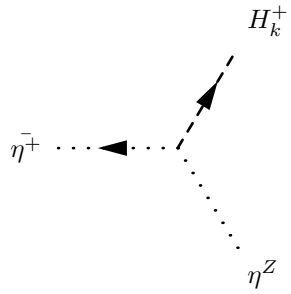
$$-\frac{i}{4}g_2\xi_Z\Big(g_1\sin\Theta_W+g_2\cos\Theta_W\Big)\Big(v_dZ_{k1}^+-v_uZ_{k2}^+\Big) \tag{421}$$



$$-\frac{i}{4}\xi_Z\Big(g_1\sin\Theta_W+g_2\cos\Theta_W\Big)^2\Big(v_dZ^H_{k1}+v_uZ^H_{k2}\Big) \tag{422}$$



$$\frac{i}{4}g_2\xi_{W^-}\Big(-g_1\sin\Theta_W+g_2\cos\Theta_W\Big)\Big(v_dZ^+_{k1}-v_uZ^+_{k2}\Big) \tag{423}$$



$$\frac{i}{4}g_2\xi_{W^-}\Big(-g_1\sin\Theta_W+g_2\cos\Theta_W\Big)\Big(v_dZ^+_{k1}-v_uZ^+_{k2}\Big) \tag{424}$$

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9 Clebsch-Gordan Coefficients